

Zestaw zadań do kolowkium nr 3

**Zadanie 1.** Oblicz równania różniczkowe

$$\frac{dy}{dx} + y \cos x = \frac{1}{2} \sin 2x \quad (1)$$

$$\frac{dy}{dx} + y \tan x = \sin 2x \quad (2)$$

$$\frac{dy}{dx} - y \tan x = 2 \cos^2 x \quad (3)$$

$$\frac{dy}{dx} + \frac{y}{x} = 2 \quad (4)$$

$$\frac{dy}{dx} - \frac{2y}{x+1} = (x+1)^3 \quad (5)$$

$$\frac{dy}{dx} - \frac{y}{\sin x} = \tan \frac{1}{2}x \quad (6)$$

$$x \frac{dy}{dx} - 2y = x + 1 \quad (7)$$

$$x \frac{dy}{dx} + y = x \sin x \quad (8)$$

$$x \frac{dy}{dx} - 2y = x e^{-\frac{1}{x}} \quad (9)$$

$$2x \frac{dy}{dx} - y = \frac{3}{2}x^2 \quad (10)$$

$$\frac{dy}{dx} - 2xy = x - x^3 \quad (11)$$

$$\frac{dy}{dx} + 2xy = x e^{-x^2} \quad (12)$$

**Zadanie 2.** Rozwiąż równania różniczkowe

$$y'' = y' + e^x \quad (13)$$

$$y'' = -2y' + e^x \quad (14)$$

$$xy'' + y' = 0 \quad (15)$$

$$xy'' - y' = 0 \quad (16)$$

$$xy'' + y' = 2x \quad (17)$$

$$xy'' + y' = x^2 + 1 \quad (18)$$

$$xy'' - y' = e^x x^2 \quad (19)$$

$$(1 + 2x)y'' + 2y' = 0 \quad (20)$$

$$y'' = \frac{1}{1 + x^2} \quad (21)$$

$$y'' = x + \sin x \quad (22)$$

**Zadanie 3.**

$$y'' + 9y = \sin 3x \quad (23)$$

$$y'' + y = 2 \sin x - \cos x \quad (24)$$

$$y'' + 4y = 2 \cos x + \sin x \quad (25)$$

$$y'' + 9y = 2 \cos 3x + 5 \sin 3x \quad (26)$$

$$y'' - 2y' + 2y = 2 \cos x + \sin x \quad (27)$$

$$y'' - 4y = 10e^{3x} \quad (28)$$

$$y'' - a^2y = e^{bx} \quad (29)$$

$$y'' - 4y' + 4y - e^{2x} = 0 \quad (30)$$

$$y'' + 9y = x \cos x \quad (31)$$

$$y'' - 2y' + y = \frac{e^x}{x} \quad (32)$$

$$y'' + y' - 2y = 4x \quad (33)$$

$$y'' - 3y' + 2y = x^2 \quad (34)$$

$$y'' + y' = e^x \quad (35)$$

**Zadanie 4.** Rozwiń funkcję w szereg Taylora w okolicy punktu  $x_0 = 0$  i uprość wyrażenie

$$f(x) = \sqrt{a - ax} \quad (36)$$

$$f(x) = \sin x \quad (37)$$

$$f(x) = \cos x \quad (38)$$

**Zadanie 5.** Niech

$$E = m_0 c \gamma \quad (39)$$

gdzie

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}} \quad , \quad \beta = \frac{v}{c} \quad (40)$$

rozwiń funkcję  $E$  względem  $\beta^2$  zinterpretuj 2 pierwsze wyrazy.