

Zestaw zadań do kolokwium nr 2

Zadanie 1. Oblicz iloczyn wektorowy $a \times b$

$$a = (1, \frac{1}{2}, -1) \quad b = (0, -1, \frac{2}{3}) \quad (1)$$

$$a = (\frac{2}{5}, -\frac{3}{4}, -2) \quad b = (2, -1, 3) \quad (2)$$

$$a = (-\frac{1}{4}, 2, 0) \quad b = (1, 0, 5) \quad (3)$$

$$a = (\frac{2}{3}, \frac{3}{5}, -3) \quad b = (1, -5, 3) \quad (4)$$

$$a = (\frac{1}{2}, \frac{3}{2}, -1) \quad b = (-\frac{1}{4}, -1, \frac{3}{4}) \quad (5)$$

(6)

Zadanie 2. Oblicz długość iloczynu wektorowego $|a \times b|$

$$|a| = 3 \quad |b| = \frac{2}{\sqrt{2}} \quad \varphi = 45^\circ \quad (7)$$

$$|a| = \sqrt{3} \quad |b| = 2 \quad \varphi = \frac{7\pi}{6} \quad (8)$$

$$|a| = 1 \quad |b| = \sqrt{2} \quad \varphi = \frac{3\pi}{2} \quad (9)$$

$$|a| = \sqrt{8} \quad |b| = 3 \quad \varphi = \frac{5\pi}{4} \quad (10)$$

$$|a| = 2 \quad |b| = \frac{\sqrt{2}}{4} \quad \varphi = 135^\circ \quad (11)$$

Zadanie 3. Oblicz objętość równoległościanu skonstruowanego na wektorach

$$a = (1, \frac{1}{2}, -1) \quad c = (1, \frac{1}{3}, -2) \quad b = (0, -1, \frac{2}{3}) \quad (12)$$

$$a = (\frac{2}{5}, -\frac{3}{4}, -2) \quad c = (2, \frac{3}{5}, 0) \quad b = (2, -1, 3) \quad (13)$$

$$a = (-\frac{1}{4}, 2, 0) \quad c = (\frac{1}{4}, -\frac{1}{3}, 1) \quad b = (1, 0, 5) \quad (14)$$

$$a = (\frac{2}{3}, \frac{3}{5}, -3) \quad c = (-\frac{2}{3}, 2, 3) \quad b = (1, -5, 3) \quad (15)$$

Zadanie 4. Wyznacz równania prostej przechodzącej przez punkty A, B i wyznacz wektor kierunkowy i prostopadły do tej prostej

$$A = (1, 2) \quad B = (-1, \frac{2}{3}) \quad (16)$$

$$A = (-2, 3) \quad B = (2, -1) \quad (17)$$

$$A = (\frac{1}{4}, \frac{2}{3}) \quad B = (-1, -\frac{2}{5}) \quad (18)$$

$$A = (-1, -4) \quad B = (2, \frac{3}{5}) \quad (19)$$

Zadanie 5. Wyznacz prostą o wektorze kierunkowym $\vec{\alpha}$, która przechodzi przez punkt A

$$\vec{\alpha} = (2, \frac{1}{3}) \quad A = (1, 2) \quad (20)$$

$$\vec{\alpha} = (-2, 2) \quad A = (\frac{3}{2}, -1) \quad (21)$$

$$\vec{\alpha} = (-1, \frac{2}{5}) \quad A = (0, 9) \quad (22)$$

$$\vec{\alpha} = (\frac{1}{4}, -\frac{2}{3}) \quad A = (3, -2) \quad (23)$$

Zadanie 6. Wyznacz równanie płaszczyzny przechodzącej przez trzy punkty P_1, P_2, P_3 i oblicz wektor normalny do tej płaszczyzny

$$P_1 = (-2, \frac{1}{3}, 2) \quad P_2 = (0, \frac{1}{3}, 1) \quad P_3 = (1, 1, 1) \quad (24)$$

$$P_1 = (1, 0, \frac{1}{2}) \quad P_2 = (0, -1, \frac{1}{3}) \quad P_3 = (-\frac{1}{4}, -2, 0) \quad (25)$$

$$P_1 = (0, \frac{2}{3}, -1) \quad P_2 = (1, -2, -1) \quad P_3 = (1, \frac{2}{3}, -1) \quad (26)$$

$$P_1 = (-\frac{1}{4}, 3, \frac{1}{2}) \quad P_2 = (-1, -\frac{2}{3}, 2) \quad P_3 = (2, -1, -1) \quad (27)$$

Zadanie 7. Oblicz całki

$$\int a^x e^x dx = \quad (28)$$

$$\int e^{\sin x} \cos x dx = \quad (29)$$

$$\int x \cos x dx = \quad (30)$$

$$\int \frac{\ln^2 x}{x} dx = \quad (31)$$

$$\int e^2 x^2 dx = \quad (32)$$

$$\int \ln x dx = \quad (33)$$

$$\int \frac{e^x}{1 - e^x} dx = \quad (34)$$

$$\int (x^2 + 1 - 2x)e^{x-1} dx = \quad (35)$$

$$\int e^{\sin^2 x} \sin 2x dx = \quad (36)$$

$$\int \arctan x dx = \quad (37)$$

$$\int \arcsin x dx = \quad (38)$$

$$\int \sin^2 x dx = \quad (39)$$

$$\int \cos^2 x dx = \quad (40)$$

$$\int \tan^2 x dx = \quad (41)$$

Zadanie 8. Oblicz całki oznaczone

$$\int_{-3}^{-2} \frac{dx}{x^2 + 2x + 1} = \quad (42)$$

$$\int_{-3}^{-2} \frac{x dx}{x^2 - x - 2} = \quad (43)$$

$$\int_{-\frac{2}{5}}^{\frac{2}{5}} \frac{dx}{4 + 25x^2} = \quad (44)$$

$$\int_2^3 \frac{dx}{2x^2 + 3x - 2} = \quad (45)$$

$$\int_0^1 \frac{dx}{x^2 + 4x + 5} = \quad (46)$$

$$\int_0^{\frac{\pi}{2}} x \cos x dx = \quad (47)$$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{x}{\sin^2 x} dx = \quad (48)$$

$$\int_0^a \frac{adx}{(x-a)(x-2a)} = \quad (49)$$

Zadanie 9. Pokaż, że zachodzi

$$\int_{-\frac{1}{2}}^{+\frac{1}{2}} \ln \left(\frac{1+x}{1-x} \right) dx = 0 \quad (50)$$

Zadanie 10. Oblicz całki niewłaściwe

$$\int_0^{\infty} x e^{-x^2} dx = \quad (51)$$

$$\int_{-\infty}^{+\infty} \frac{dx}{x^2 + 2x + 2} = \quad (52)$$

$$\int_2^{+\infty} \frac{x}{(1-x^2)^3} dx = \quad (53)$$

$$\int_0^{\infty} \arctan x dx = \quad (54)$$

$$\int_0^{\infty} \frac{e^x}{e^{2x} + 1} dx = \quad (55)$$

$$\int_{-\infty}^0 \frac{dx}{x^2 + 4} = \quad (56)$$