Computational Intelligence: Methods and Applications

Lecture 18
Decision trees

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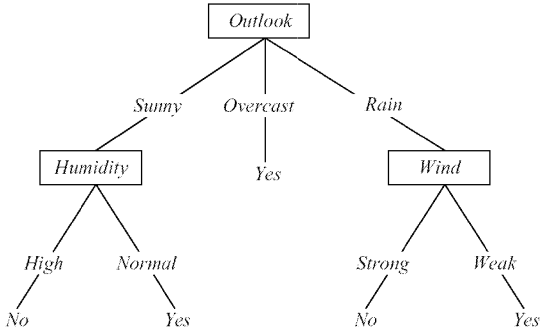
Multi-level DT

First split on the best test/attribute, then continue recursively.



Children nodes may be split using different test/attributes, sometimes repeating those already used in the parent nodes.

General DT properties

DT: first general, later more specific decisions
node ⇔ test on attribute, selecting subsets or intervals
branching ⇔ splitting data vectors into subsets
leaves of the tree are associated with decisions (classes)

Tests: on a single attribute, or their combination
attribute = {value$_1$,value$_2$ ..} or attribute < value$_1$

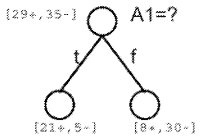
Criteria: maximize information gain, maximize the purity of new nodes; maximize separability of subset vectors

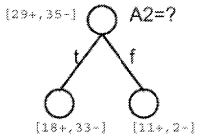
Pruning: remove branches that contain only a few cases, simple trees may generalize better (lower variance, higher bias) evaluation optimal tree complexity on validation set.

Stop criterion: node purity, accuracy, tree complexity.

Attribute selection

Which attribute should be taken next, A1 or A2? Which test?





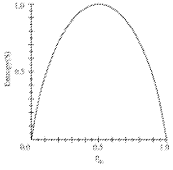
P_+ and P_- are *a priori* $\omega_\pm$ class probabilities in the node S, test divides the S set into S_t and S_f.

How much information is gained by splitting? Calculate entropy!
Optimal no. of bits to code message with P_+ is ~ $\lg_2 P_+$, same for P_-

$$H(\boldsymbol{\omega}\,|\,\mathrm{S}) = -P_+ \lg_2 P_+ - P_- \lg_2 P_-$$

$$G(\boldsymbol{\omega},\mathrm{A}|\mathrm{S}) = H(\boldsymbol{\omega}\,|\,\mathrm{S}) - \frac{|\mathrm{S}_t|}{|\mathrm{S}|} H(\boldsymbol{\omega}\,|\,\mathrm{S}_t) - \frac{|\mathrm{S}_f|}{|\mathrm{S}|} H(\boldsymbol{\omega}\,|\,\mathrm{S}_f)$$

Entropy of "pure" nodes (vectors from one class) is 0;
Max. entropy is for a node with mixed samples P_i=1/2.

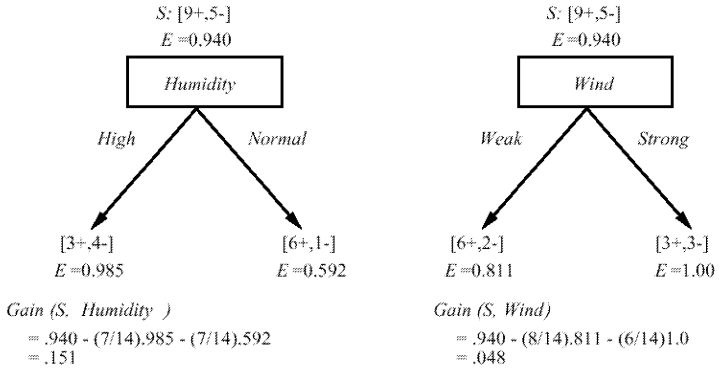


Plot of $H(P_+/\mathrm{S})$ for $P_+ = 1\text{-}P_-$

Entropy content

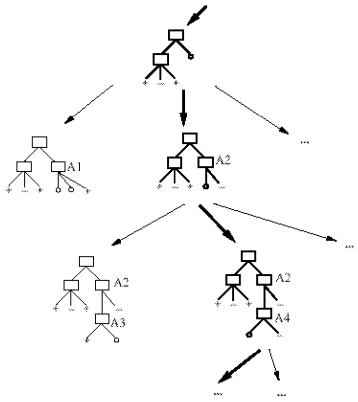
Information is just negative entropy *E*, thus entropy change = info gain:

$$H(p_1, p_2, \ldots p_n) = -\sum_i p_i \lg_2 p_i$$



Weather example (from WEKA book)

Creation of a decision tree



Creation of a tree ⇔ search in the hypothesis space for the simplest set of hierarchical rules (most compact tree).

ID3 tree – split criterion based on information gain.

Bias: smaller trees are better, if equal information gain select split with lower number of branches.

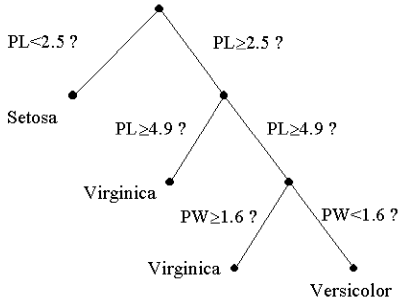
No backtracking.

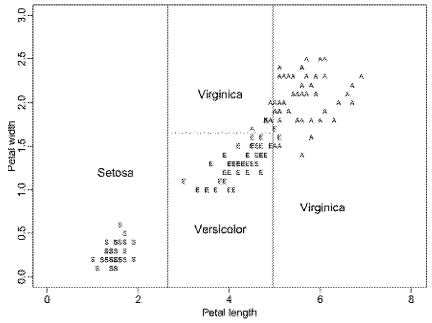
Rather robust in the presence of noise, although local (greedy) search does not guarantee that the final tree will be optimal.

Decision borders

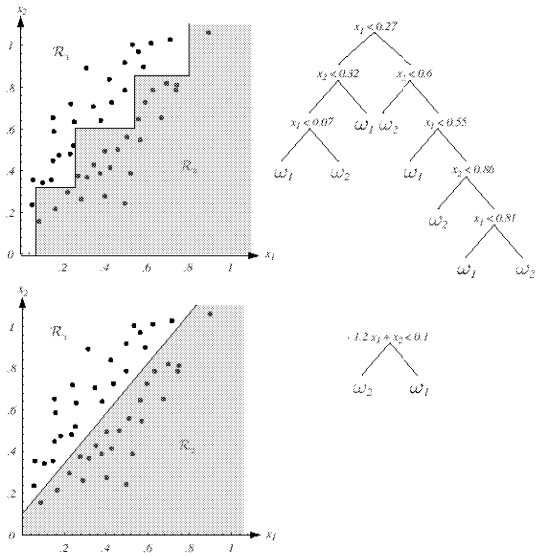
Hierarchical partitioning of feature space into hyper-rectangles.

Example: Iris flowers data, with 4 features; displayed in 2-D.





Uni and multi-variate criterions



Univariate, or monothetic trees,

mult-variate, or oblique trees.

Figure from Duda, Hart & Stork, Chap. 8

Oblique decision borders

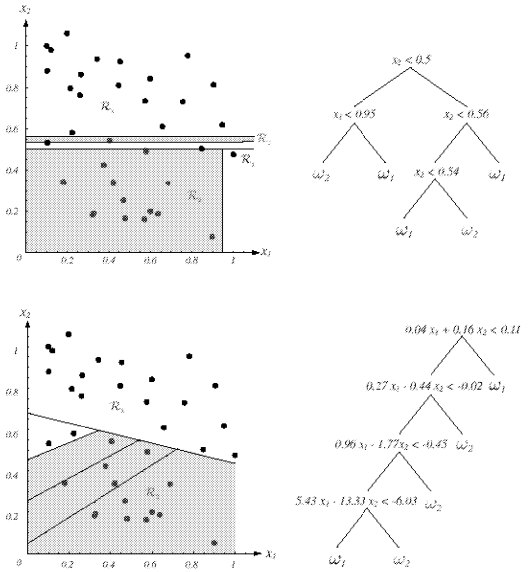
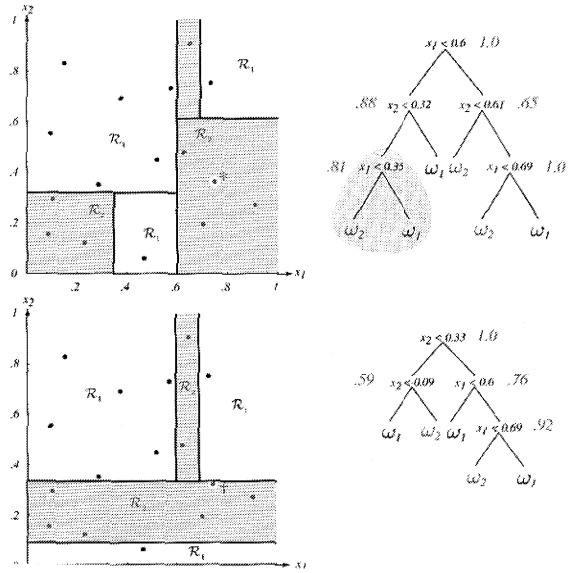


Figure from
Duda, Hart & Stork,
Chap. 8

DTs are not stable



Moving just one example slightly may lead to quite different trees and space partition!

Lack of stability against small perturbation of data.

Figure from
Duda, Hart & Stork,
Chap. 8

Ockham's razor

Why simple trees should be preferred?

1. The number of simple hypothesis that may accidentally fit the data is small, so chances that simple hypothesis uncover some interesting knowledge about the data are larger.
2. Simpler trees have higher bias and thus lower variance, they should not overfit the data that easily.
3. Simpler trees do not partition the feature space into too many small boxes and may generalize better; complex trees may create a separate box for each training data sample.

Still, even if the tree is small ...

sufficiently long search by pure chance may find false solution;

for small datasets with many attributes several equivalent (from the accuracy point of view) descriptions may exist.

=> One tree is not sufficient, we need a forest of healthy trees!

Overfitting

A model *H* overfitts the data if:

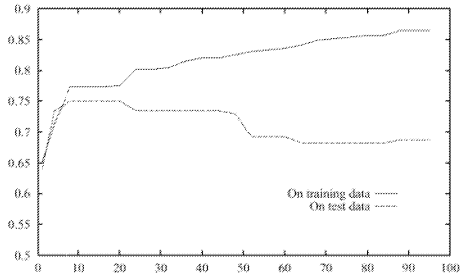
A model *H'* exists such that:

Training-error(*H*) < Training-error(*H'*)

Test-error(*H*) > Test-error(*H'*)

Model *H* draws conclusions (makes hypothesis) that are too detailed for the amount of evidence available.

Accuracy as a function of the number of tree nodes: on the training data it may grow up to 100%, but the final results may be worse than for the majority classifier!



Random data example

Generate and label randomly data samples as class ω_1 or class ω_2, with the a priori probability of the majority class $P(\omega_1)=p>0.5$

Majority classifier makes $E_{maj}=1-p$ percent of errors.

Overfitted tree that classifies correctly all training data has:

$N \cdot p$ nodes from ω_1 class

$N-N \cdot p$ nodes from ω_2 class.

A random X is assigned to ω_1 class with prob. p and ω_2 with $1-p$.

Confusion matrix:

$$\begin{pmatrix} p^2 & p(1-p) \\ (1-p)p & (1-p)^2 \end{pmatrix}$$

Tree error/Majority classifier error

$$\frac{2p(1-p)}{(1-p)} = 2p > 1 \text{ for } p > 0.5$$

For p=0.75 overfitted tree makes 37.5% errors,
while the majority classifier will make only 25% errors.

Some examples

Please run a few example of decision tree solutions using YALE or WEKA on benchmark data using wither WEKA knowledge explorer or YALE.