Metody numeryczne II (numerical methods II): tasks (2022)

1. Write a program that determines the sum of the series $S^{(a)}=\sum_{n=1}^{N} \frac{1}{n}$ and $S^{(b)}=\sum_{n=N}^{1} \frac{1}{n}$ as the function of $N$. Print the results to a file and plot the dependency of the relative differences $\left(S^{(a)}-S^{(e x a c t)}\right) / S^{(e x a c t)}$ and $\left(S^{(b)}-S^{(e x a c t)}\right) / S^{(e x a c t)}$ on $N$. Make calculations in single and double precision using double precision calculations as exact values.
2. Write a program that determines the machine precision (unit roundoff) as well as the number of mantissa bits for a floating point number in single and double precision in the IEEE 754 standard. In addition, the program is to calculate the possible range of exponents for single and double precision numbers. What is the effect of applying a gradual underflow on the results?
3. Write a program that performs LU decomposition of a square matrix A using Doolittle or Crout method. The program gets the dimension of the matrix $\mathbf{A}$ and its elements from the input. Modify the LU decomposition program using the Doolittle (or Crout) method to include partial pivoting.
4. Write a program that solves a system of linear equations $\mathbf{A x}=\mathbf{b}$ with the aid of LU decomposition of A matrix. The program gets the number of equations, the elements of $\mathbf{A}$ matrix and the elements of the right-hand-side column $\mathbf{b}$ from the input.
5. Write a program that calculates the inverse matrix $\mathbf{A}^{-1}$ and its condition number cond $(\mathbf{A})$ using the LU decomposition. Check the effect of scaling of the system of linear equations $\mathbf{A x}=\mathbf{b}$ on the condition number cond $(\mathbf{A})$ and on the accuracy of the solution of the system of linear equations.
6. Write a program that approximates the function $y(x)$ by means of linear least squares method (generally $\chi^{2}$ ) via construction and solving of normal equations. Program takes as an input the 'measurement points' $\left(x_{i}, y_{i}, \sigma_{i}\right)$, where $\sigma_{i}$ estimate the errors of $y_{i}, i=1,2, \ldots, N$. Calculate the variance-covariance matrix.
7. Write a program that calculates all the eigenvalues of the square matrix $\mathbf{A}$ with the QR method.
