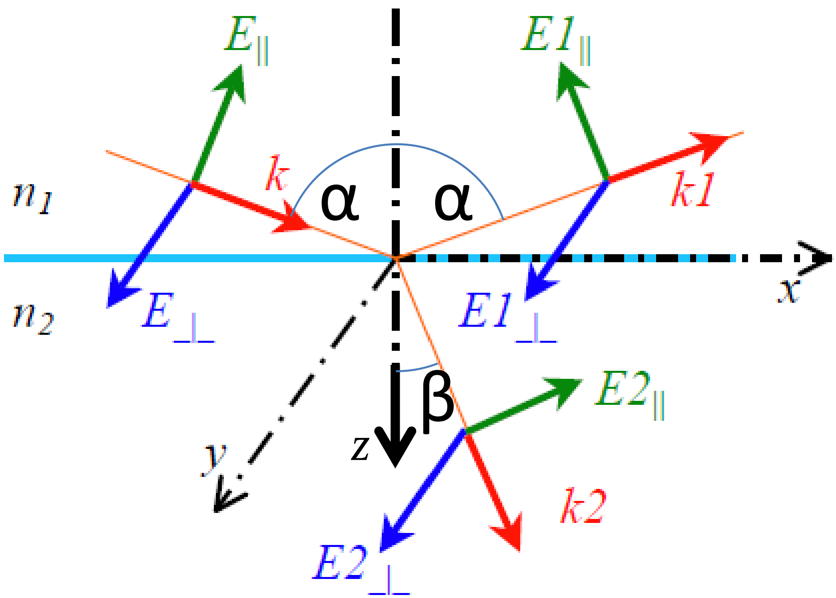
Wzory Fresnela



$$\vec{k} = \vec{s}\,\frac{\omega}{v}$$

$$\vec{s} = [\sin(\alpha), 0, \cos(\alpha)]$$

$$\vec{s1} = [\sin(\alpha), 0, -\cos(\alpha)]$$

$$\vec{s2} = [\sin(\beta), 0, \cos(\beta)]$$

Z warunków granicznych:

$$E_x + E1_x = E2_x \qquad (A)$$

$$E_y + E1_y = E2_y \qquad (B)$$

$$H_x + H1_x = H2_x \qquad (C)$$

$$H_y + H1_y = H2_y \qquad (D)$$

Rozpisując składowe

$$E_x = E_{\parallel} \cdot \cos(\alpha) \qquad E1_x = -E1_{\parallel} \cdot \cos(\alpha) \qquad E2_x = E2_{\parallel} \cdot \cos(\beta)$$

$$E_y = E_{\perp} \qquad E1_y = E1_{\perp} \qquad E2_y = E2_{\perp}$$

$$E_z = -E_{\parallel} \cdot \sin(\alpha) \qquad E1_z = -E1_{\parallel} \cdot \sin(\alpha) \qquad E2_z = -E2_{\parallel} \cdot \sin(\beta)$$

$$\vec{k}\times\vec{E}=\omega\mu_0\vec{H} \qquad \vec{\mathcal{S}}\times\vec{E}=v\mu_0\vec{H} \qquad \vec{H}=\frac{c}{v}\sqrt{\frac{\varepsilon_0}{\mu_0}}\cdot\vec{\mathcal{S}}\times\vec{E}$$

A więc:

$$H_x=\frac{c}{v}\sqrt{\frac{\varepsilon_0}{\mu_0}}\left[\mathcal{S}_y E_z-\mathcal{S}_z E_y\right]=-\frac{c}{v}\sqrt{\frac{\varepsilon_0}{\mu_0}}\left[E_y\cos(\alpha)\right]$$

$$H_y=-\frac{c}{v}\sqrt{\frac{\varepsilon_0}{\mu_0}}\left[\mathcal{S}_x E_z-\mathcal{S}_z E_x\right]=\frac{c}{v}\sqrt{\frac{\varepsilon_0}{\mu_0}}\left[E_x\cos(\alpha)-E_z\sin(\alpha)\right]$$

$$H1_x=-\frac{c}{v}\sqrt{\frac{\varepsilon_0}{\mu_0}}\left[E1_y\cos(\alpha)\right]$$

$$H1_y=-\frac{c}{v}\sqrt{\frac{\varepsilon_0}{\mu_0}}\left[E1_x\cos(\alpha)+E1_z\sin(\alpha)\right]$$

$$H2_x = -\frac{c}{v_2}\sqrt{\frac{\mu_0}{\varepsilon_0}}\left[E2_y \cos(\beta)\right]$$

$$H2_y = -\frac{c}{v_2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\left[E2_x \cos(\beta) - E2_z \sin(\beta)\right]$$

Teraz można skorzystać z ciągłości:

$$(C):\quad H_x + H1_x = H2_x$$

$$-\frac{c}{v}\sqrt{\frac{\varepsilon_0}{\mu_0}}\left[E_y \cos(\alpha)\right] + -\frac{c}{v}\sqrt{\frac{\varepsilon_0}{\mu_0}}\left[E1_y \cos(\alpha)\right] = -\frac{c}{v_2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\left[E2_y \cos(\beta)\right]$$

$$\left(E_y - E1_y\right)\cos(\alpha) = \frac{v}{v_2} E2_y \cos(\beta) = \frac{n_2}{n_1} E2_y \cos(\beta)$$

$$(D):\quad H_y + H1_y = H2_y$$

$$\left(E_x - E1_x\right)\cos(\alpha) - \left(E_z + E1_z\right)\sin(\alpha) = \frac{n_2}{n_1}\left[E2_x \cos(\beta) - E2_z \sin(\beta)\right]$$

Dla składowych **y** mamy więc dwa równania:

Bezpośrednio z ciągłości:

$$E_y + E1_y = E2_y \text{, czyli: } \quad E_{\perp} + E1_{\perp} = E2_{\perp}$$

oraz z R.M.:

$$\left(E_y - E1_y\right)\cos(\alpha) = \frac{n_2}{n_1} E2_y \cos(\beta)$$

$$\left(E_{\perp} - E1_{\perp}\right)\cos(\alpha) = \frac{n_2}{n_1} E2_{\perp} \cos(\beta)$$

$$\frac{n_2}{n_1} = \frac{\sin(\alpha)}{\sin(\beta)}$$

Ten układ równań można rozwiązać ze względu E1 i E2

$$E1_{\perp} = E_{\perp} \frac{\sin(\beta-\alpha)}{\sin(\beta+\alpha)}$$

$$E2_{\perp} = 2E_{\perp} \frac{\sin(\beta)\cos(\alpha)}{\sin(\beta+\alpha)}$$

Dla składowych **x** mamy więc dwa równania:

Bezpośrednio z ciągłości:

$$E_x + E1_x = E2_x \text{, czyli: } \quad E_{\|}\cos(\alpha) + E1_{\|}\cos(\alpha) = E2_{\|}\cos(\beta)$$

oraz z R.M., było:

$$\left(E_x - E1_x\right)\cos(\alpha) - \left(E_z + E1_z\right)\sin(\alpha) = \frac{n_2}{n_1}\left[E2_x\cos(\beta) - E2_z\sin(\beta)\right]$$

wstawiając wyrażenia na $E_{||}$ i $E_{\_|\_}$

$$\left(E_{\|}\cos(\alpha) - E1_{\|}\cos(\alpha)\right)\cos(\alpha) - \left(-E_{\|}\sin(\alpha) + -E1_{\|}\sin(\alpha)\right)\sin(\alpha) =$$
$$= \frac{n_2}{n_1}\left[\left(E2_{\|}\cos(\beta)\right)\cos(\beta) - \left(-E2_{\|}\sin(\beta)\right)\sin(\beta)\right]$$

Ten układ równań można rozwiązać ze względu E1 i E2

$$\frac{n_2}{n_1} = \frac{\sin(\alpha)}{\sin(\beta)}$$

$$\frac{n_2}{n_1} = \frac{\sin(\alpha)}{\sin(\beta)}$$

ze składowych **x**

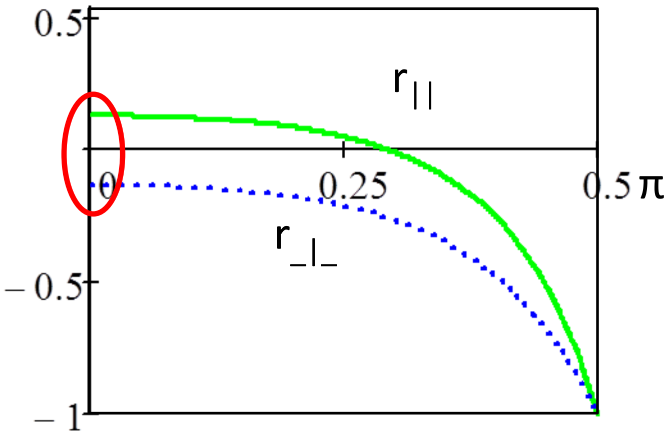
$$r_{\|} = \frac{E1_{\|}}{E_{\|}} = -\frac{tg(\beta-\alpha)}{tg(\beta+\alpha)} = \frac{n_2/n_1\cos(\alpha)-\cos(\beta)}{n_2/n_1\cos(\alpha)+\cos(\beta)}$$

$$t_{\|} = \frac{E2_{\|}}{E_{\|}} = 2\frac{\sin(\beta)\cos(\alpha)}{\sin(\beta+\alpha)\cos(\beta-\alpha)} = 2\frac{\cos(\alpha)}{n_2/n_1\cos(\alpha)+\cos(\beta)}$$

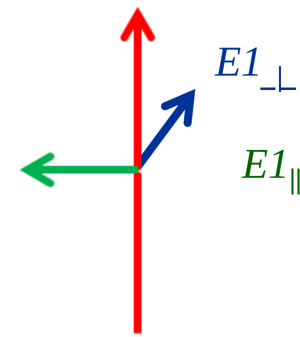
ze składowych **y**

$$r_{\perp} = \frac{E1_{\perp}}{E_{\perp}} = \frac{\sin(\beta-\alpha)}{\sin(\beta+\alpha)} = \frac{\cos(\alpha)-n_2/n_1\cos(\beta)}{\cos(\alpha)+n_2/n_1\cos(\beta)}$$

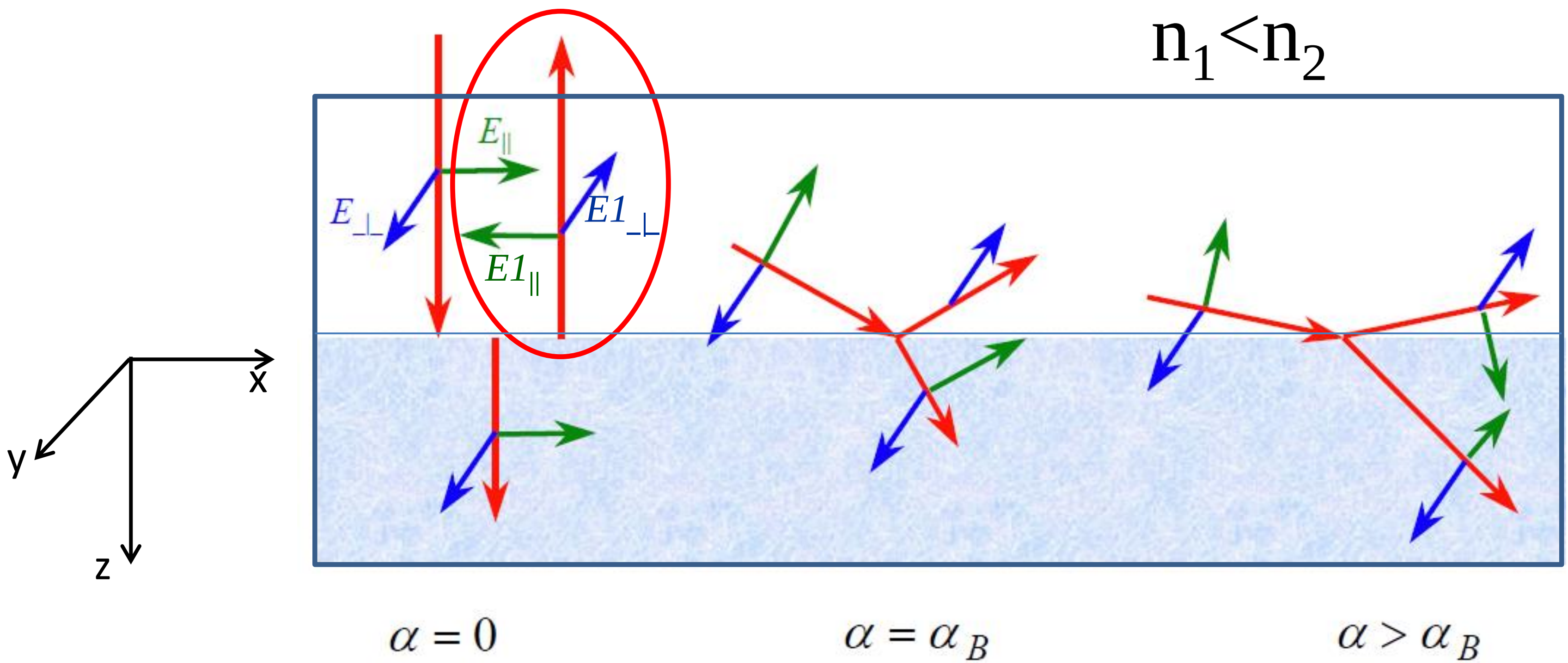
$$t_{\perp} = \frac{E2_{\perp}}{E_{\perp}} = 2\frac{\sin(\beta)\cos(\alpha)}{\sin(\beta+\alpha)} = 2\frac{\cos(\alpha)}{\cos(\alpha)+n_2/n_1\cos(\beta)}$$



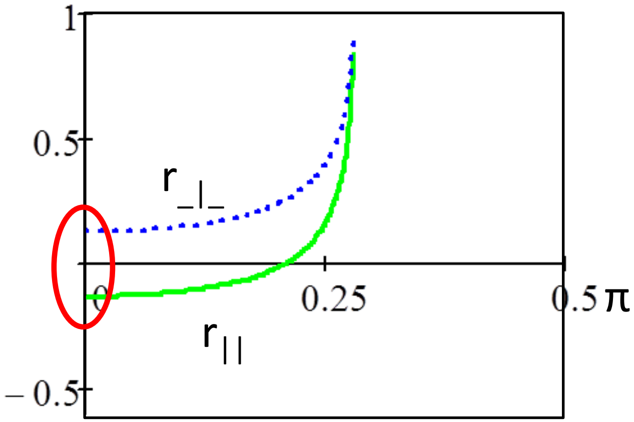
0.5
−0.5
−1
0
0.25
0.5 π
r_||
r_⊥

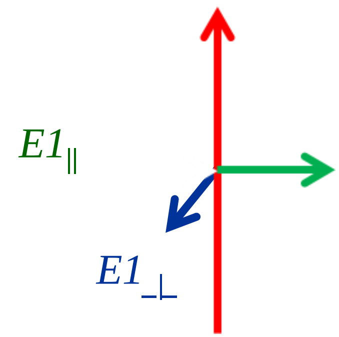


E1_⊥
E1_||

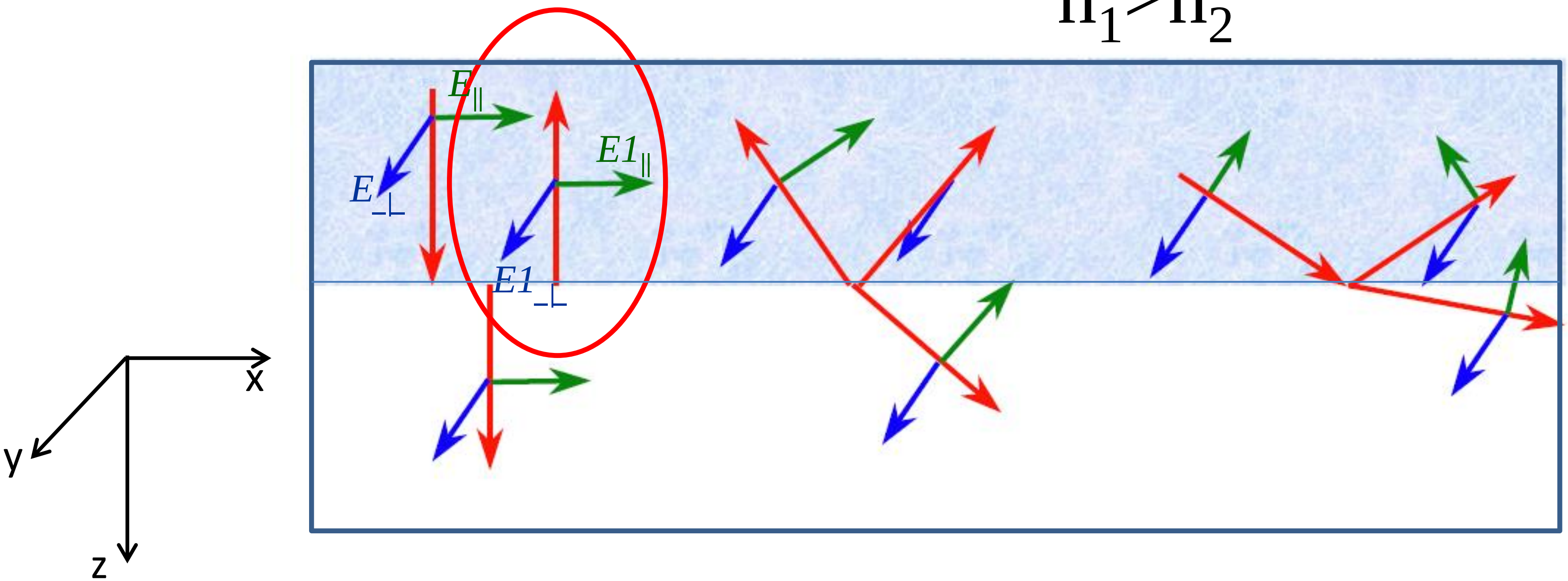


n1<n2
E_||
E_⊥
E1_⊥
E1_||
x
y
z
α = 0
α = α_B
α > α_B





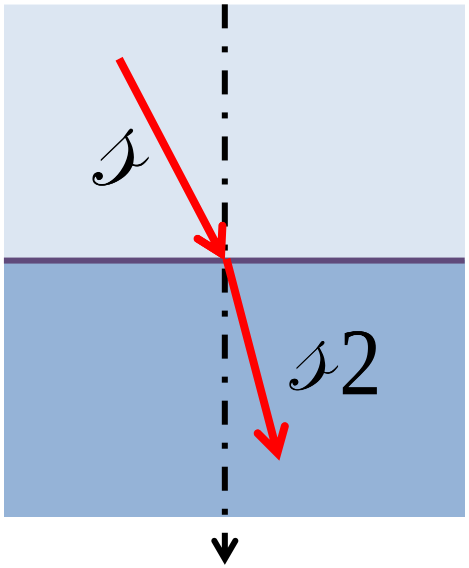
$n_1 > n_2$



	$n_1 < n_2$				$n_1 > n_2$			
$\alpha =$	$r_{\|\|}$	$r_{\perp}$	$t_{\|\|}$	$t_{\perp}$	$r_{\|\|}$	$r_{\perp}$	$t_{\|\|}$	$t_{\perp}$
0	+	-	+	+	-	+	+	+
α_{Brew}	0	-	>1	+	0	+	+	>1
$\alpha_{gran} < 90^o$ ($\beta = 90^o$)	x	x	x	x	1	1	$2\frac{n_1}{n_2}$	2
$\alpha_{gran} = 90^o$	-1	-1	0	0	x	x	x	x

$$r^2 + t^2 \neq 1$$

Transport energii przez powierzchnię



$$\vec{k} = \vec{s}\,\frac{\omega}{v} \qquad \vec{S} = \vec{E} \times \vec{H}^*$$

$$\left\langle \vec{S} \right\rangle = \frac{1}{2} E H^* = \frac{1}{2} E E^* \sqrt{\frac{\varepsilon_0}{\mu_0}}\, n\, \vec{s}$$

Transport energii prostopadle do powierzchni rozdziału $$S_n = \vec{a_n} \cdot \vec{S}$$

$$T_n = \frac{S2_n}{S_n} = \frac{S2 \cdot \cos(\beta)}{S \cdot \cos(\alpha)} = \frac{E2 \cdot E2^* n_2 \cos(\beta)}{E \cdot E^* n_1 \cos(\alpha)} = t^2 \frac{n_2 \cos(\beta)}{n_1 \cos(\alpha)}$$

$n_1 = $ [redacted] $n_2 = $ [redacted]