Teoria i metody optymalizacji

Oleksandr Sokolov
Wydział Fizyki, Astronomii i Informatyki Stosowanej
UMK
http://fizyka.umk.pl/~osokolov/TMO/

Szczególne przypadki równania Eulera

Równanie Eulera–Lagrange'a jest równaniem różniczkowym drugiego rzędu.

$$F_y - \frac{d}{dx}F_{y'} = 0$$

Funkcja podcałkowa nie zależy od x: $F = F(y, y')$

$$J(y) = \int_a^b F(x, y(x), y'(x))dx$$

$$F - y'F_{y'} = C$$

Dowód

$$\frac{d}{dx}\left(F - y'F_{y'}\right) = F_y y' + \underbrace{F_{y'}y'' - y''F_{y'}}_{0} - y'\frac{d}{dx}F_{y'} =$$

Funkcja złożona

$$y'\underbrace{\left(F_y - \frac{d}{dx}F_{y'}\right)}_{0} = 0$$

To jest równanie różniczkowe 1 rzędu

Przykład

$$J = \int_0^1 \left(y^2 + y'^2 \right) dx$$

$$F = y^2 + y'^2$$

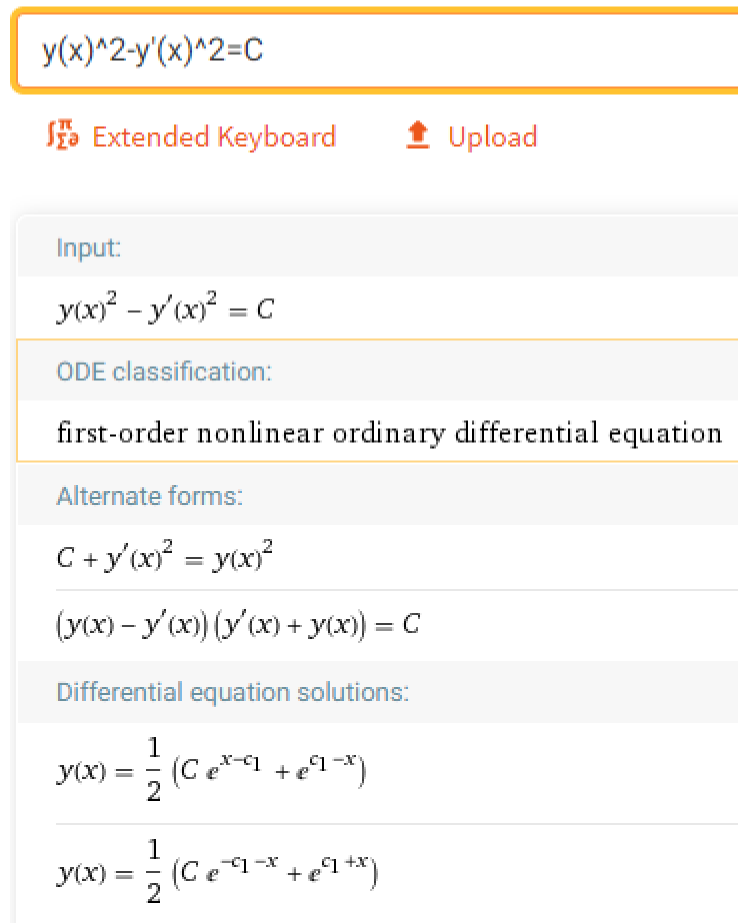
$$y(0) = 3$$

$$y(1) = 2$$

$$F - y' F_{y'} = C$$

$$F_{y'} = 2y'$$

$$F = y^2 + y'^2 - 2y'^2 = y^2 - y'^2 = C$$



Szczególne przypadki równania Eulera

Równanie Eulera–Lagrange'a jest równaniem różniczkowym drugiego rzędu.

$$F_y - \frac{d}{dx} F_{y'} = 0$$

0

Funkcja podcałkowa nie zależy od **y**:

$$J(y) = \int_a^b F(x, y(x), y'(x))dx$$

$$F = F\left(x, y'(x)\right)$$

$$F_y = \frac{\partial F}{\partial y}, \quad F_{y'} = \frac{\partial F}{\partial y'}.$$

$$-\frac{d}{dx} F_{y'} = 0 \Rightarrow F_{y'} = C$$

To jest równanie różniczkowe 1 rzędu

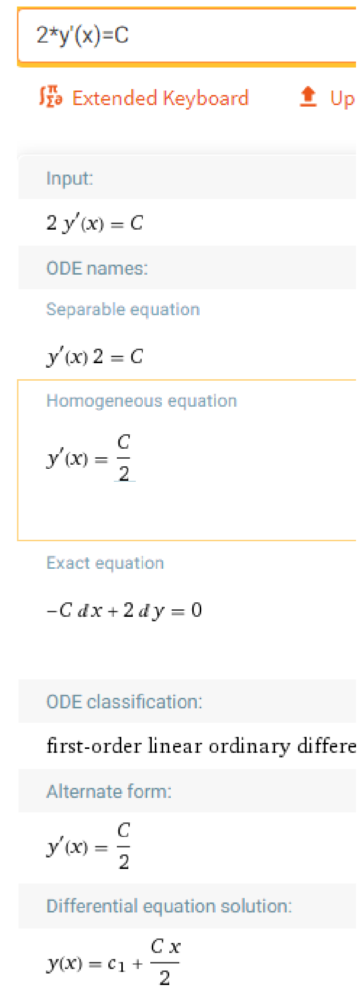
Przykład

$$J = \int_0^1 \left(x^2 + y'^2\right) dx \qquad y(0) = 3 \qquad y(1) = 2$$

$$F = x^2 + y'^2$$

$$\boxed{F_{y'} = C}$$

$$F_{y'} = 2y' \qquad 2y' = C$$



Szczególne przypadki równania Eulera

Funkcja podcałkowa nie zależy od **y'**:

$$F = F(x, y)$$

$$F_y - \frac{d}{dx} F_{y'} = 0$$

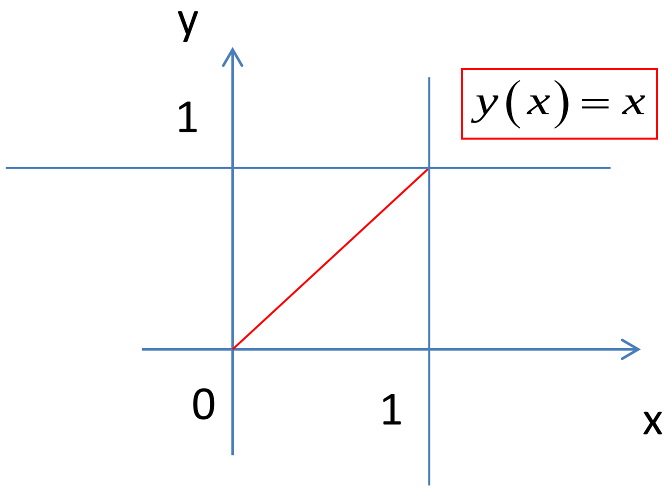
$$F_{y'} = 0 \Rightarrow F_y = 0$$

To nie jest równanie różniczkowe

Przykład

$$J = \int_0^1 (y - x)^2 \, dx$$

$$y(0) = 0$$
$$y(1) = 1$$



$$F = (y - x)^2$$

$$F_y = 2(y - x)$$

$$F_y = 0$$

$$y(x) = x$$

$$J = \int_0^1 (x - x)^2 \, dx = 0$$

Szczególne przypadki równania Eulera

Funkcja podcałkowa zależy wyłącznie od **y'**:

$$F_y - \frac{d}{dx} F_{y'} = 0$$

$$F = F\left(y'\right)$$

$$\frac{d}{dx} F_{y'} = 0 \Rightarrow F_{y'} = C \Rightarrow y\left(x\right) = C_1 x + C_2$$

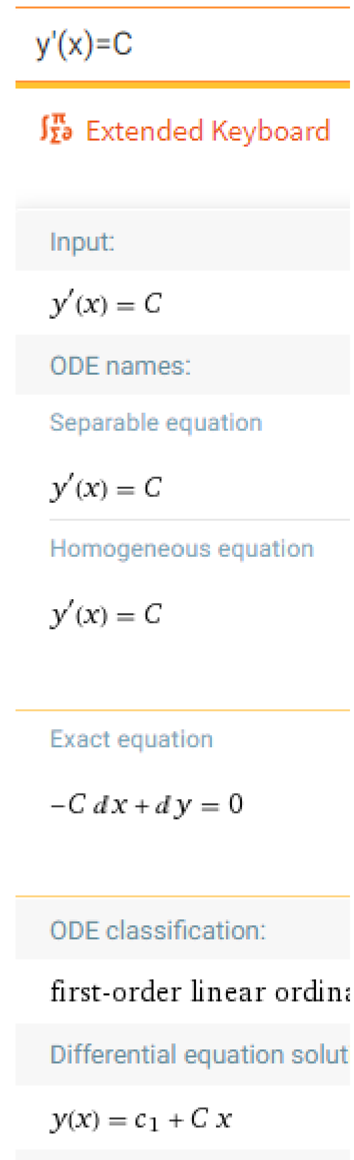
$$y\left(x\right) = C_1 x + C_2$$

Przykład

$$J=\int_0^1 \left(y'^2\right)dx \qquad \begin{aligned} y(0)&=3 \\ y(1)&=2 \end{aligned}$$

$$F=y'^2$$

$$\frac{d}{dx}F_{y'}=0 \Rightarrow F_{y'}=C \Rightarrow y(x)=C_1x+C_2$$



Równania dla kilku funkcji

$$J(x_1,\dots,x_n)=\int_{t_0}^{t_1} F\,(x_1,\dots,x_n;\dot{x}_1,\dots,\dot{x}_n,t)dt$$

Układ równań Eulera-Lagrange/a

$$F_{x_i}-\frac{d}{dt}F_{\dot{x}_i}=0,\quad i=1,\dots,n.$$

Warunki Legendre'a

$$F_{\dot{x}_1,\dot{x}_1}\geq 0,\quad \begin{vmatrix} F_{\dot{x}_1,\dot{x}_1} & F_{\dot{x}_1,\dot{x}_2} \\ F_{\dot{x}_2,\dot{x}_1} & F_{\dot{x}_2,\dot{x}_2} \end{vmatrix}\geq 0,\dots,$$

$$\begin{vmatrix} F_{\dot{x}_1,\dot{x}_1} \dots F_{\dot{x}_1,\dot{x}_n} \\ \dots\dots\dots\dots\dots\dots \\ F_{\dot{x}_n,\dot{x}_1} \dots F_{\dot{x}_n,\dot{x}_n} \end{vmatrix}\geq 0.$$

Przykład dla funkcji 2-ch argumentów $F_{\dot{x}_1,\dot{x}_1}\geq 0,\quad F_{\dot{x}_1,\dot{x}_1}F_{\dot{x}_2,\dot{x}_2}-F_{\dot{x}_1,\dot{x}_2}F_{\dot{x}_2,\dot{x}_1}\geq 0$

Równania wyższych rzędów. Równania Eulera - Poissona

$$J\left(x(t)\right) = \int_{t_0}^{t_1} F\,[x(t), \dot{x}(t), \dots, x^{(n)}(t), t]dt.$$

$$F_x - \frac{d}{dt}F_{\dot{x}} + \frac{d^2}{dt^2}F_{\ddot{x}} + \dots + (-1)^n \frac{d^n}{dt^n}F_{x^{(n)}} = 0,$$

Warunek Legendre'a $F_{x^{(n)}, x^{(n)}} \geq 0;$

Zadanie

$$J\left(x(t)\right)=\int_{t_0}^{t_1}\left(\dot{x}^2(t)+\ddot{x}^2(t)\right)dt.$$

$$\mathbf{x(t_0)=x_0} \quad x(t_1)=x_1$$

$$\dot{x}(t_0)=\dot{x}_0 \quad \dot{x}(t_1)=\dot{x}_1$$

$$F\left(x,\dot{x},\ddot{x}\right)=\dot{x}^2(t)+\ddot{x}^2(t)$$

$$F_x=0 \qquad F_{\dot{x}}=2\dot{x}, \qquad F_{\ddot{x}}=2\ddot{x},$$

$$F_x-\frac{d}{dt}F_{\dot{x}}+\frac{d^2}{dt^2}F_{\ddot{x}}=0, \quad \Longrightarrow \quad x^{(4)}(t)-\ddot{x}(t)=0$$

Warunek Lagendre'a

$$F_{\ddot{x},\ddot{x}}\geq 0; \quad \Longrightarrow \quad F_{\ddot{x},\ddot{x}}=2$$

Rozwiązanie

$$\boldsymbol{x(t_0) = x_0}$$

$$\dot{x}(t_0) = \dot{x}_0$$

$$x(t_1) = x_1$$

$$\dot{x}(t_1) = \dot{x}_1$$

$x^{(4)}(t) - x''(t) = 0$

Autonomous equation:

True

ODE classification:

higher-order linear ordinary differential equation

Alternate form:

$x''(t) = x^{(4)}(t)$

Differential equation solution:

$x(t) = c_1 e^t + c_2 e^{-t} + c_4 t + c_3$

derivative of x(t) = c1 e^t + c2 e^(-t) + c4 t + c_3

Extended Keyboard   Upload

Derivative:

$\frac{\partial}{\partial t}(x(t) = c1\, e^t + c2\, e^{-t} + c4\, t + c_3) = c1\, e^t - c2\, e^{-t} + c4$

Alternate forms:

$c1\, e^t + c4 = c2\, e^{-t} + x'(t)$

$x'(t) = e^{-t}(c1\, e^{2t} - c2 + c4\, e^t)$

Przykład

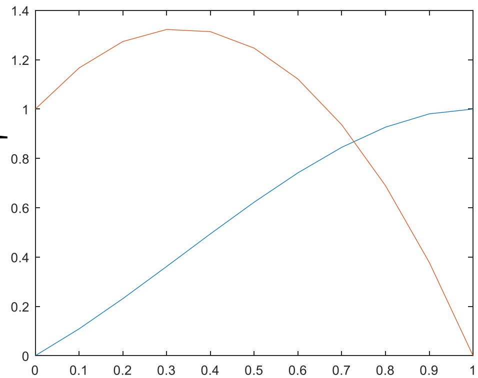
```
%exEP.m
clear all;
close all;
A=[1 1  0 1;
   1 -1  1 0;
   exp(1) exp(-1) 1 1;
   exp(1) -exp(-1) 1 0];
b=[0 1 1 0]';
C=A^-1*b;
  Out=[];
  for t=0:0.1:1
   x=C(1)*exp(t)+C(2)*exp(-t)+C(3)*t+C(4);
   dx=C(1)*exp(t)-C(2)*exp(-t)+C(3);
      Out=[Out;x dx];
  end;
  plot(Out(:,1));
  hold on;
  plot(Out(:,2));
  syms t
  x=C(1)*exp(t)+C(2)*exp(-t)+C(3)*t+C(4);
  dxdt=diff(x,t);
  d2xdt2=diff(dxdt,t);

  J=vpa(int(dxdt^2+d2xdt2^2,0,1))
```

$$J = \int_{t_0}^{t_1} \left(\dot{x}^2(t) + \ddot{x}^2(t) \right) dt.$$

$$t_0 = 0 \quad t_1 = 1$$

$$\dot{x}(t_0) = 1 \qquad x(t_1) = 1$$

$$x(t_0) = 0 \qquad \dot{x}(t_1) = 0$$

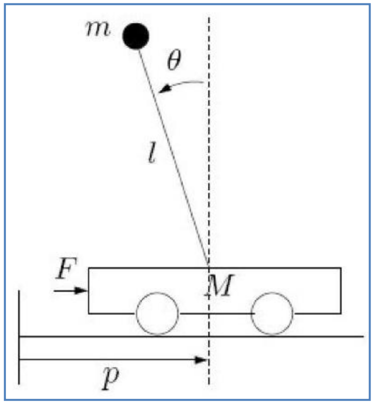


J =5.1316234851731756135551301066122

Sterowanie odwróconym wahadłem

MatLab - slcp

Układ składa się z wózka i zamocowanego na elastycznym przegubie pionowego wahadła o masie m i długości l



Zachowanie systemu opisuje równanie różniczkowe

$$-ml^2\frac{d^2\theta}{dt^2} + mlg \cdot \sin\theta = \tau = u(t)$$

gdzie $\tau = u(t)$ jest sterowaniem (moment obrotowy), które należy przyłożyć w kierunku przeciwnym do wychylenia θ aby utrzymać wahadło w pozycji pionowej.

Sterowanie odwróconym wahadłem (cd)

$$\ddot{\varphi} + a\varphi = u \qquad \int_0^T (\ddot{\varphi} + a\varphi)^2 \, dt$$

$$\ddot{x} + ax = u$$

$$x(0) = A; \quad x(T) = 0 \quad \dot{x}(0) = B \quad \dot{x}(T) = 0$$

$$F_x - \frac{d}{dt} F_{\dot{x}} + \frac{d^2}{dt^2} F_{\ddot{x}}$$

$$F = \ddot{x}^2 + 2\ddot{x} a x + a^2 x^2$$

$$F_x = 2\ddot{x} a + 2a^2 x$$

$$F_{\dot{x}} = 0$$

$$F_{\ddot{x}} = 2\ddot{x} + 2ax$$

$$2\ddot{x} a + 2a^2 x + 2x^{(4)} + 2a x^{(2)} = 0$$

$$\boxed{4a\ddot{x} + 2a^2 x + 2\ddddot{x} = 0}$$

Input

$$x^{(4)}(t) + 2\,a\,x''(t) + a^2\,x(t) = 0$$

Autonomous equation

$$2\,a\,x^{(4)}(t) = -a^2\,x(t) - x^{(4)}(t)$$

ODE classification

higher-order linear ordinary differential equation

Alternate form

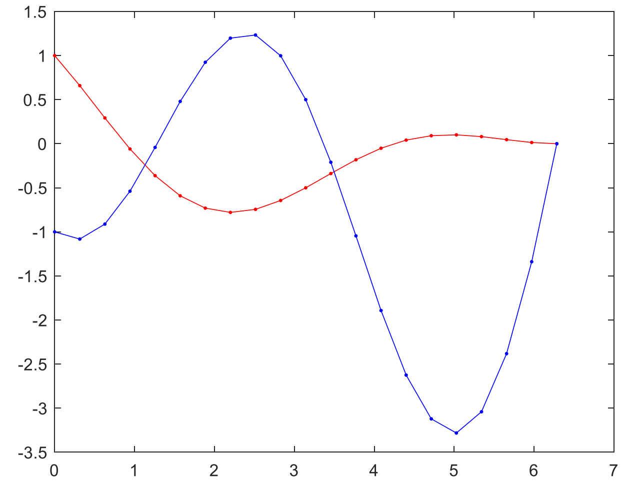
$$x^{(4)}(t) = a^2\,(-x(t)) - 2\,a\,x''(t)$$

Differential equation solution

$$x(t) = c_3 \sin(\sqrt{a}\;t) + c_4\,t \sin(\sqrt{a}\;t) + c_1 \cos(\sqrt{a}\;t) + c_2\,t \cos(\sqrt{a}\;t)$$

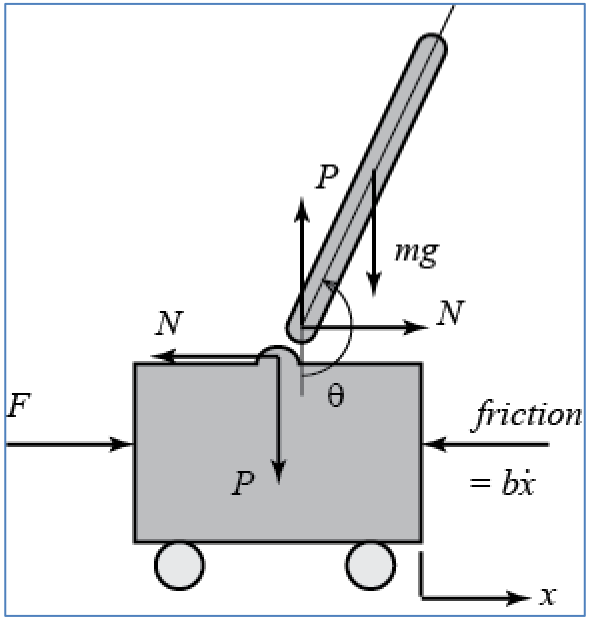
Sterowanie odwróconym wahadłem (cd)

```
close all;
clear all;
A=1;
B=-1;
T=2*pi;
C1=A;
C2=-C1/T;
C3=B-C2;
C4=-(C2+C3)/T;
Out=[];
for t=0:pi/10:2*pi
x=C3*sin(t)+C4*t*sin(t)+C1*cos(t)+C2*t*cos(t)
;
dx=cos(t)*(C2+C3+C4*t)-sin(t)*(C1+C3*t-C4);
Out=[Out;t x dx];
end;
plot(Out(:,1),Out(:,2),'r.-');
hold on;
plot(Out(:,1),Out(:,3),'b.-');
```



invPendulumEulerLagr.m

Wahadło odwrócone II



(M)	mass of the cart	0.5 kg
(m)	mass of the pendulum	0.2 kg
(b)	coefficient of friction for cart	0.1 N/m/sec
(l)	length to pendulum center of mass	0.3 m
(I)	mass moment of inertia of the pendulum	0.006 kg.m^2
(F)	force applied to the cart	
(x)	cart position coordinate	
(theta)	pendulum angle from vertical (down)	

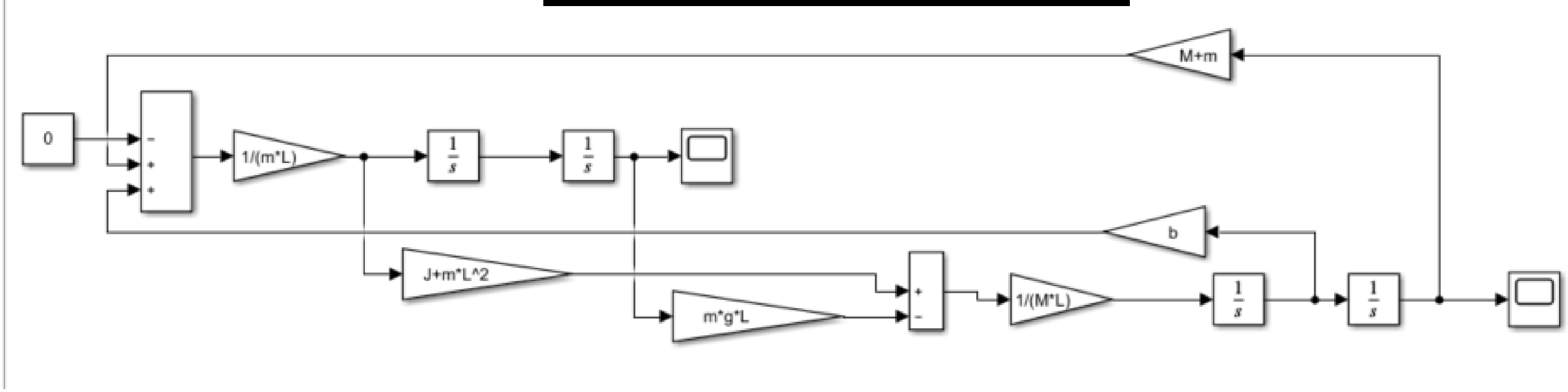
$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\phi} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-(I+ml^2)b}{I(M+m)+Mml^2} & \frac{m^2gl^2}{I(M+m)+Mml^2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-mlb}{I(M+m)+Mml^2} & \frac{mgl(M+m)}{I(M+m)+Mml^2} & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{I+ml^2}{I(M+m)+Mml^2} \\ 0 \\ \frac{ml}{I(M+m)+Mml^2} \end{bmatrix} u$$

$$\mathbf{y} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u$$

Model

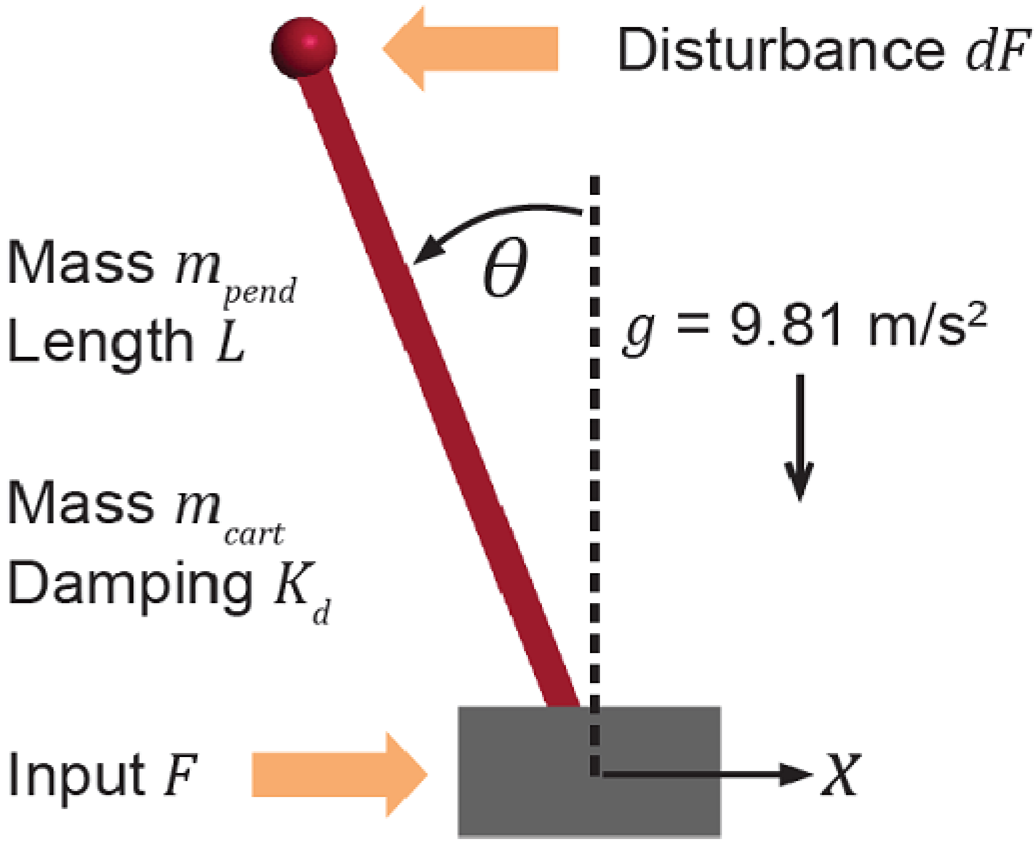
$$(J + mL^2)\ddot{\theta} - mgL\theta = mL\ddot{x}$$

$$(M + m)x + b\dot{x} - mL\ddot{\theta} = F$$



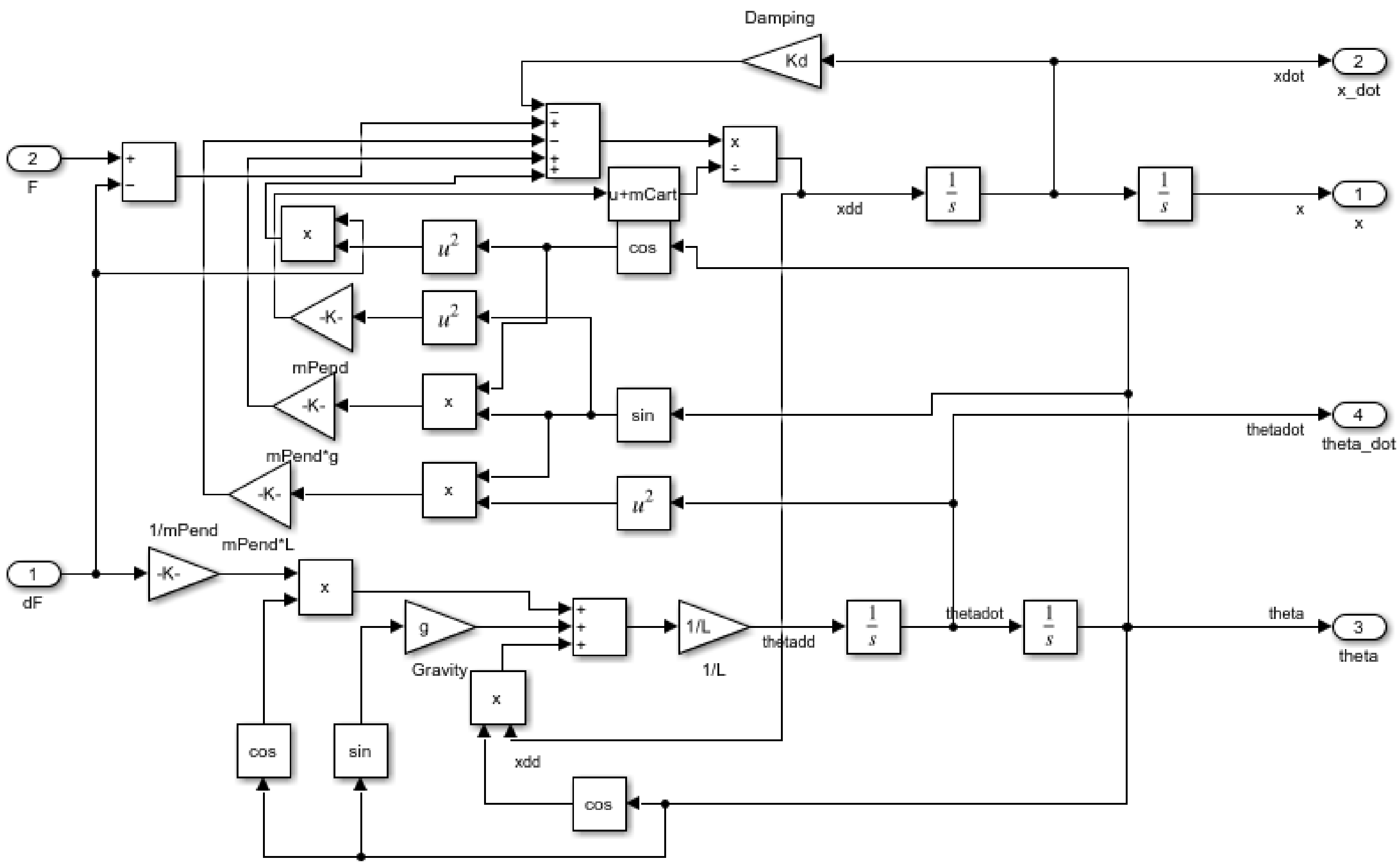
InvPendulumOS.slx

Przykład w Matlabie



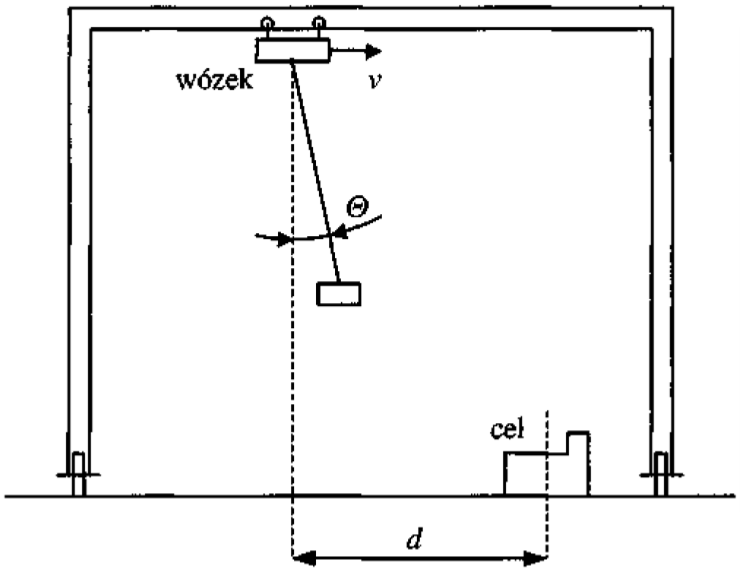
https://www.mathworks.com/help/mpc/ug/control-of-an-inverted-pendulum-on-a-cart.html

Przykład w Matlabie (cd)



Sterowanie wózkiem

Sterowanie wózkiem suwnicy przenoszącej kontenery z jednego miejsca w inne



Podczas transportu kontenerów dochodzi często do dużego ich kołysania (duże odchylenie θ od pionu). Przy ustawianiu kontenera w punkcie docelowym, np. na statku, kołysania są niedopuszczalne. Na skutek uderzenia kołyszącego się kontenera w inne, już ustawione, może nastąpić ich uszkodzenie.

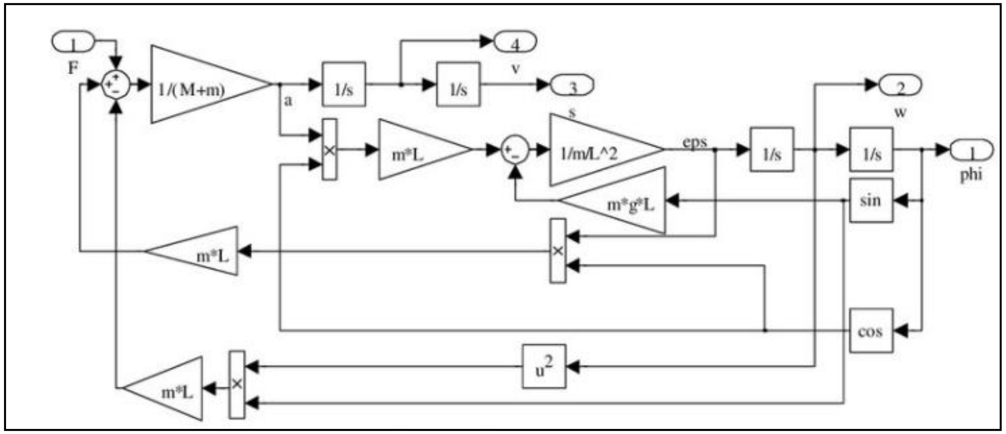
Operator suwnicy steruje prędkością v jej wózka przy pomocy dźwigni mającej dwa krańcowe położenia. Potrzebne jest tu **wyczucie reakcji wózka**, jego bezwładności i transportowanego kontenera (**zmienny ciężar**) na zmiany położenia dźwigni. Sterowanie wózkiem suwnicy mogłoby się odbywać dwoma bardzo prostymi metodami **nie wymagającymi żadnej wiedzy eksperckiej**.

Model wózka

$$(M+m)\frac{d^2s(t)}{dt^2} - mL\frac{d^2\varphi(t)}{dt^2}\cos\varphi + mL\frac{d\varphi^2(t)}{dt}\sin\varphi(t) = F(t),$$

$$-mL\frac{d^2s(t)}{dt^2}\cos\varphi(t) + mL^2\frac{d^2\varphi(t)}{dt^2} + mgL\sin\varphi(t) = 0,$$

s – odległość wózka od celu
M- masa wózka, m – masa ciężaru



Linearyzacja modelu

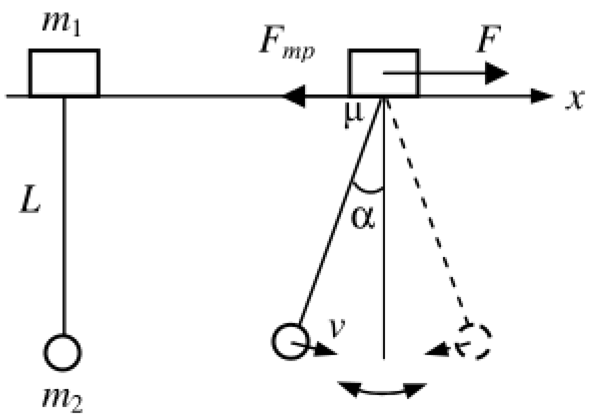
$$\frac{d^2s(t)}{dt^2} = a(t),\ \frac{ds(t)}{dt} = v(t),\ \frac{d^2\varphi(t)}{dt^2} = \varepsilon(t),\ \frac{d\varphi(t)}{dt} = \omega(t)$$

$$\varphi \approx 0,\quad \cos\varphi \approx 1,\quad \sin\varphi \approx \varphi,\quad \omega^2 \approx 0$$

$$(M+m)a(t) - mL\varepsilon(t) = F(t),$$
$$-mLa(t) + mL^2\varepsilon(t) + mLg\varphi(t) = 0.$$

Zagadnienie sterowania optymalnego

sistema-optimalnogo-upravleniya-podveshennym-gruzom.pdf



M1=100kg;
M2=200kg;
L=5m;
T=5s;
S=10m;
Mi=0.01;

$$\alpha(0)=0; \quad \dot{\alpha}(0)=0; \quad x(0)=0; \quad \dot{x}(0)=0;$$

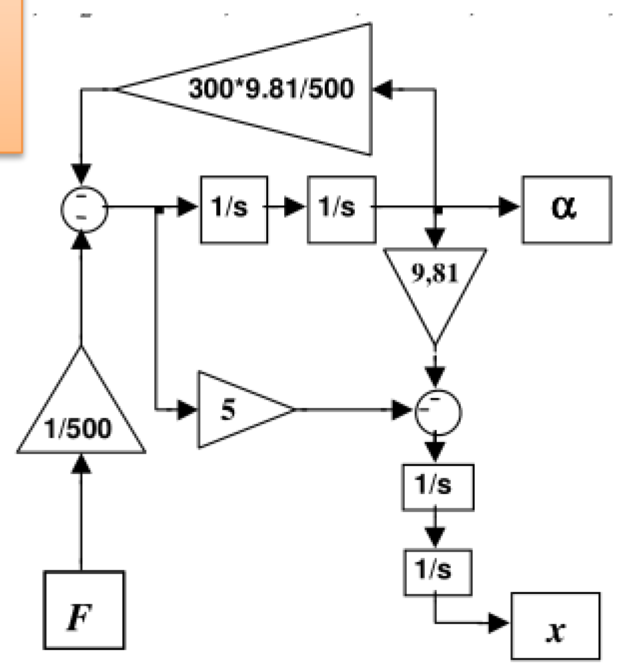
$$\alpha(T)=0; \quad \dot{\alpha}(T)=0; \quad \ddot{\alpha}(T)=0; \quad x(T)=S;$$

$$\dot{x}(T)=0; \quad \ddot{x}(T)=0.$$

$$\begin{cases} (m_1+m_2)\ddot{x}+m_2L\ddot{\alpha}=F-\mu m_2 g; \\ \ddot{x}+L\ddot{\alpha}+g\alpha=0, \end{cases}$$

$$\ddot{x}=-L\ddot{\alpha}-g\alpha$$

$$-m_1L\ddot{\alpha}-(m_1+m_2)g\alpha=F-\mu m_2 g$$



Optymalna trajektoria

$$\alpha(0)=0;\quad \dot{\alpha}(0)=0;\quad \dot{x}(0)=0;$$
$$\alpha(T)=0;\quad \dot{\alpha}(T)=0;\quad \ddot{\alpha}(T)=0;$$
$$\int_0^T \dot{x}(t)dt=S;\quad \dot{x}(T)=0.$$

$$\alpha(t)=e^{\sigma_1 t}(c_1\cos(\beta_1 t)+c_2\sin(\beta_1 t))+$$
$$+e^{-\sigma_1 t}(c_3\cos(\beta_1 t)+c_4\sin(\beta_1 t))+$$
$$+e^{\sigma_2 t}(c_5\cos(\beta_2 t)+c_6\sin(\beta_2 t))+$$
$$+e^{-\sigma_2 t}(c_7\cos(\beta_2 t)+c_8\sin(\beta_2 t)).$$

$$Q=\int_0^T(\alpha^{(4)2}+\alpha^{(3)2}+\ddot{\alpha}^2+\dot{\alpha}^2+\alpha^2)dt\to\min$$

$$\frac{\partial\Phi}{\partial\alpha}-\frac{d}{dt}\frac{\partial\Phi}{\partial\dot{\alpha}}+\frac{d^2}{dt^2}\frac{\partial\Phi}{\partial\ddot{\alpha}}-\frac{d^3}{dt^3}\frac{\partial\Phi}{\partial\alpha^{(3)}}+\frac{d^4}{dt^4}\frac{\partial\Phi}{\partial\alpha^{(4)}}=0.$$

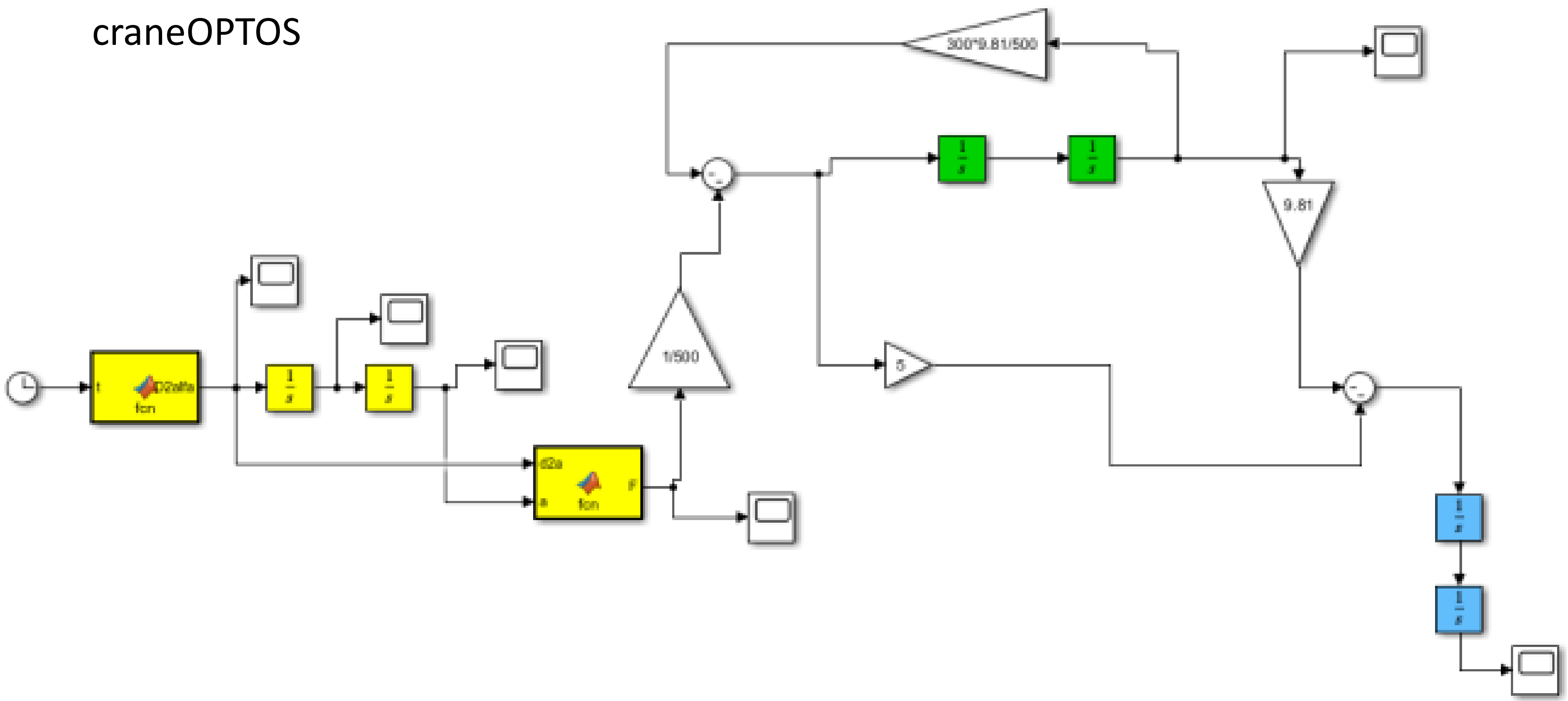
$$2\alpha-2\ddot{\alpha}+2\alpha^{(4)}-2\alpha^{(6)}+2\alpha^{(8)}=0.$$

$$p^8-p^6+p^4-p^2+1=0,$$

$p_{1,2}=\sigma_1\pm\beta_1 i;\quad p_{3,4}=-\sigma_1\pm\beta_1 i;\quad p_{5,6}=\sigma_2\pm\beta_2 i;\quad p_{7,8}=-\sigma_2\pm\beta_2 i,$

$$\sigma_1=0{,}25\sqrt{10-2\sqrt{5}};\quad \sigma_2=0{,}25\sqrt{10+2\sqrt{5}};$$
$$\beta_1=0{,}25(1+\sqrt{5});\quad \beta_2=0{,}25(-1+\sqrt{5}).$$

craneOPTOS

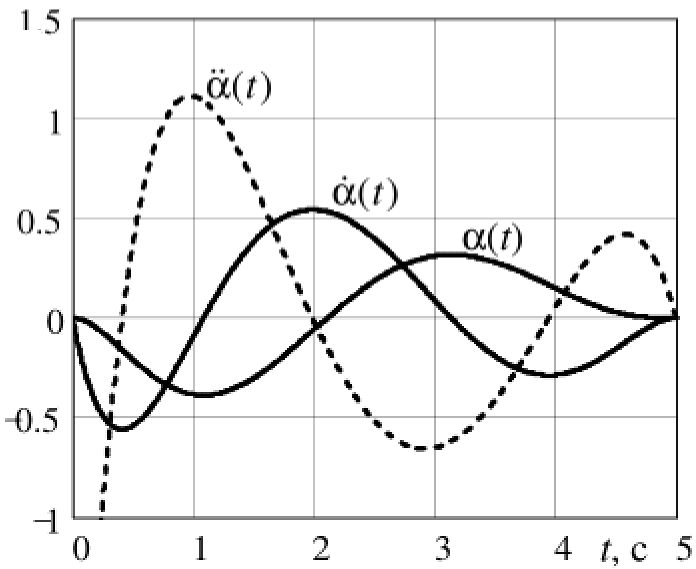


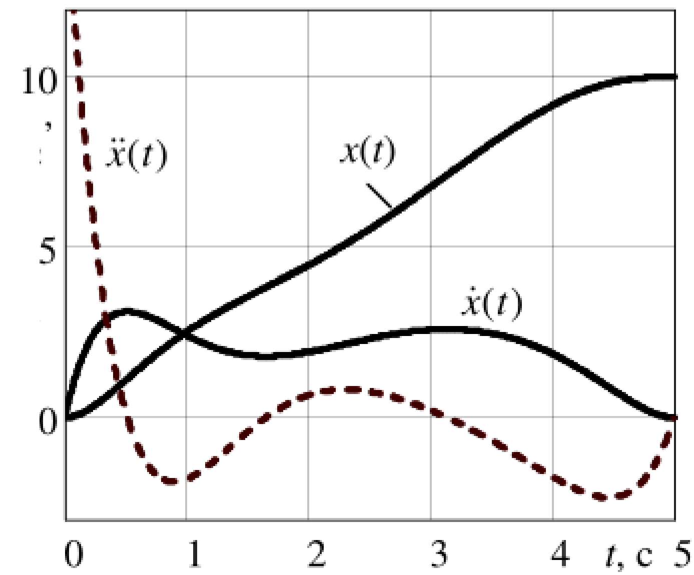
и, далее, искомые коэффициенты: c_1=0,202; c_2=-0,000723; c_3=-1,24; c_4=-7,42; c_5=0,0724; c_6=0,014; c_7=0,961; c_8=19,4.

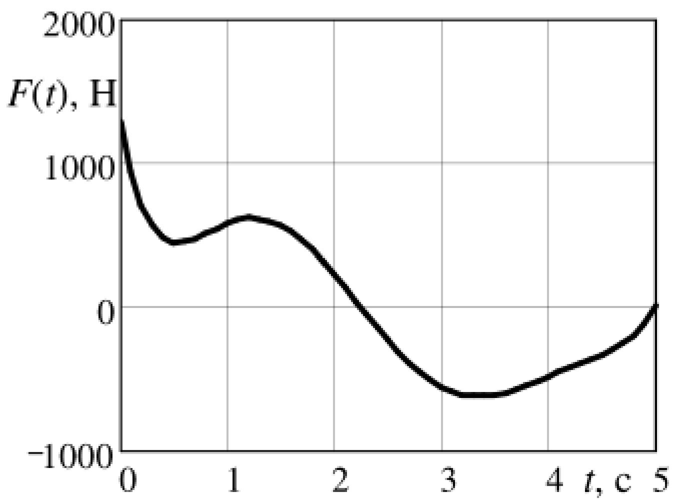
craneOPTOS

```
function D2alfa = fcn(t)
c1=0.202;c2=-0.000723;c3=-1.24;c4=-7.42;c5=0.0724;
c6=0.014;c7=0.961;c8=19.4;
s1=0.25*sqrt(10-2*sqrt(5));
s2=0.25*sqrt(10+2*sqrt(5));
b1=0.25*(1+sqrt(5));
b2=0.25*(-1+sqrt(5));
 %syms s1 s2 b1 b2 c1 c2 c3 c4 c5 c6 c7 c8 t
%alfa =exp(s1*t)*(c1*cos(b1*t)+c2*sin(b1*t))+exp(-
s1*t)*(c3*cos(b1*t)+c4*sin(b1*t))+exp(s2*t)*(c5*cos(b2*t)+c6*sin(b2*t))+exp(-
s2*t)*(c7*cos(b2*t)+c8*sin(b2*t));

%Dalfa=diff(alfa);
%D2alfa=diff(Dalfa);
D2alfa=s1^2*exp(s1*t)*(c1*cos(b1*t) + c2*sin(b1*t)) - exp(-s1*t)*(b1^2*c3*cos(b1*t)
+ b1^2*c4*sin(b1*t)) - exp(s2*t)*(b2^2*c5*cos(b2*t) + b2^2*c6*sin(b2*t)) - exp(-
s2*t)*(b2^2*c7*cos(b2*t) + b2^2*c8*sin(b2*t)) - exp(s1*t)*(b1^2*c1*cos(b1*t) +
b1^2*c2*sin(b1*t)) + s1^2*exp(-s1*t)*(c3*cos(b1*t) + c4*sin(b1*t)) +
s2^2*exp(s2*t)*(c5*cos(b2*t) + c6*sin(b2*t)) + s2^2*exp(-s2*t)*(c7*cos(b2*t) +
c8*sin(b2*t)) + 2*s1*exp(s1*t)*(b1*c2*cos(b1*t) - b1*c1*sin(b1*t)) - 2*s1*exp(-
s1*t)*(b1*c4*cos(b1*t) - b1*c3*sin(b1*t)) + 2*s2*exp(s2*t)*(b2*c6*cos(b2*t) -
b2*c5*sin(b2*t)) - 2*s2*exp(-s2*t)*(b2*c8*cos(b2*t) - b2*c7*sin(b2*t))
```

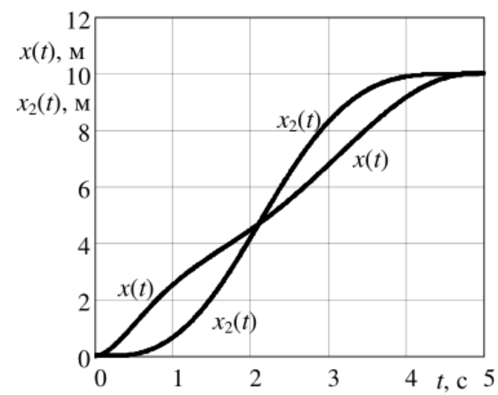
```
function F = fcn(d2a,a)
m1=100;m2=200;L=5;g=9.81;mi=0.01;
F=mi*m2*g-m1*L*d2a-(m1+m2)*g*a;
```





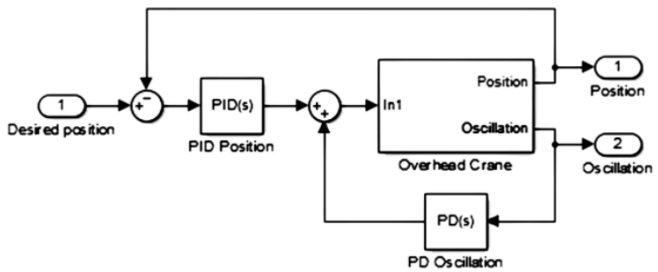


$$x_2(t) = x(t) + L \sin \alpha(t)$$

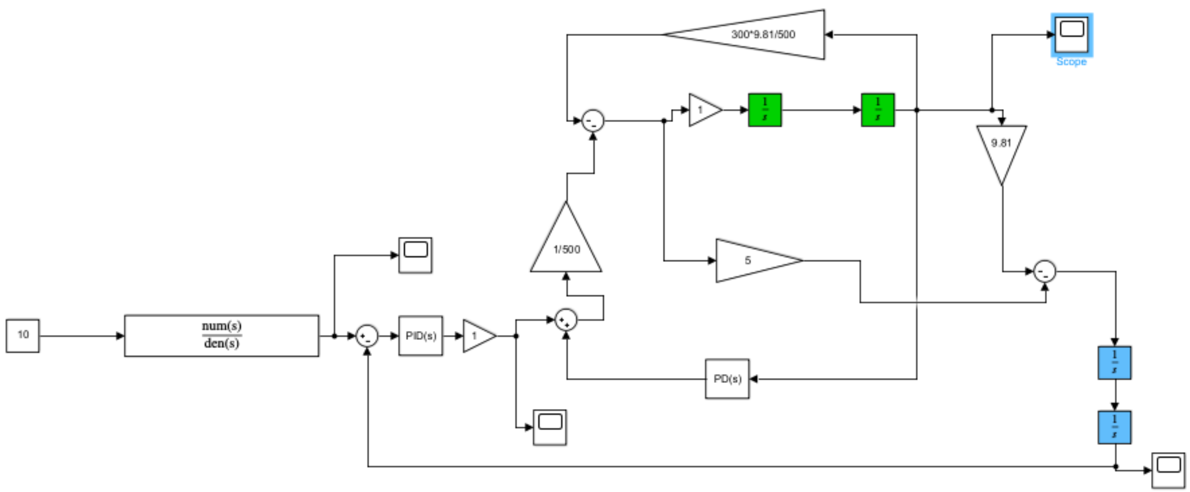


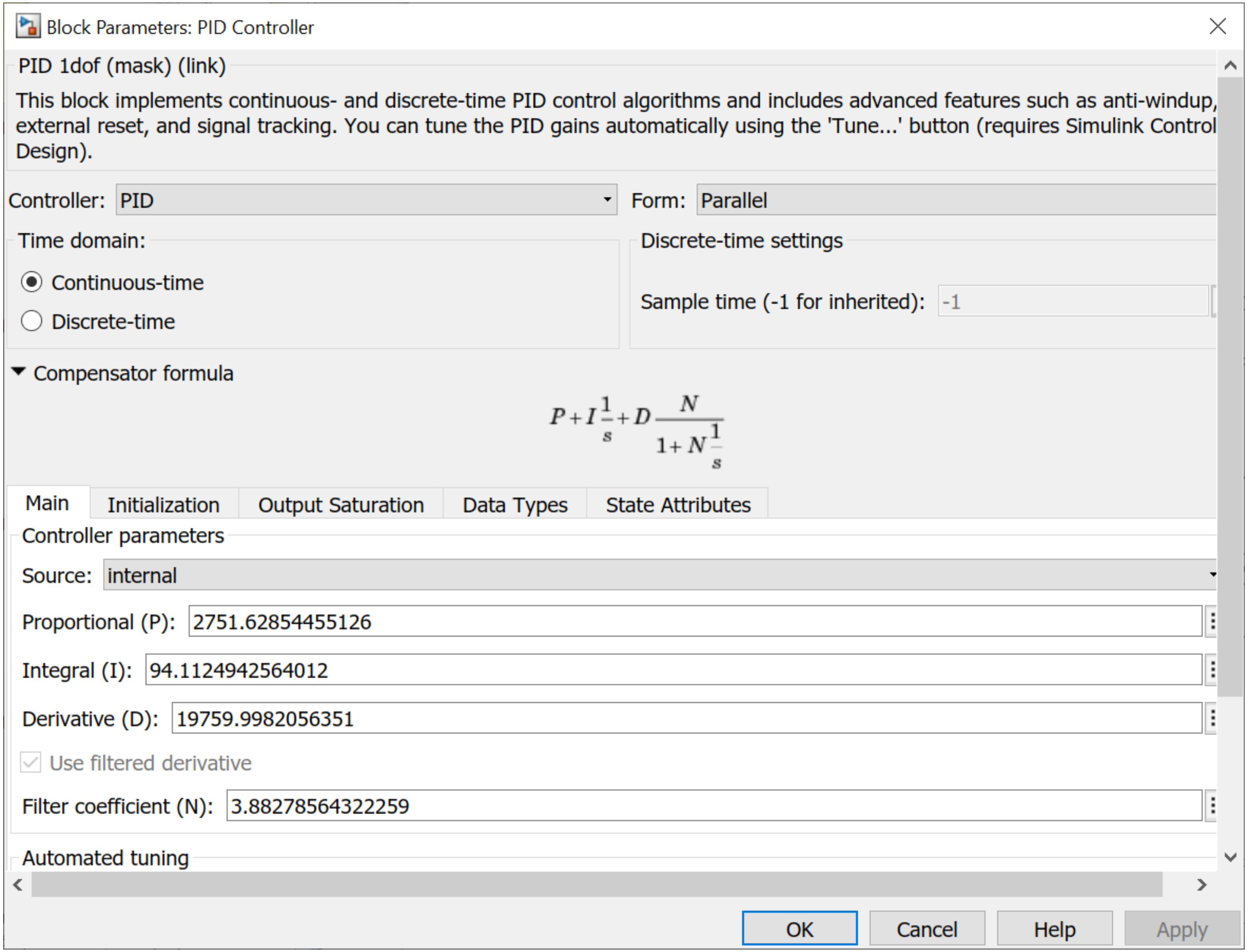
Ruch wózka i ładunku jest jednostajny: na początku ruchu (t<2,1 s) ładunek pozostaje w tyle za wózkiem, a następnie, zbliżając się do współrzędnej x=10 m, wyprzedza go iw punkcie x= 10, x 2 =10 układ bez wahania kończy ruch.

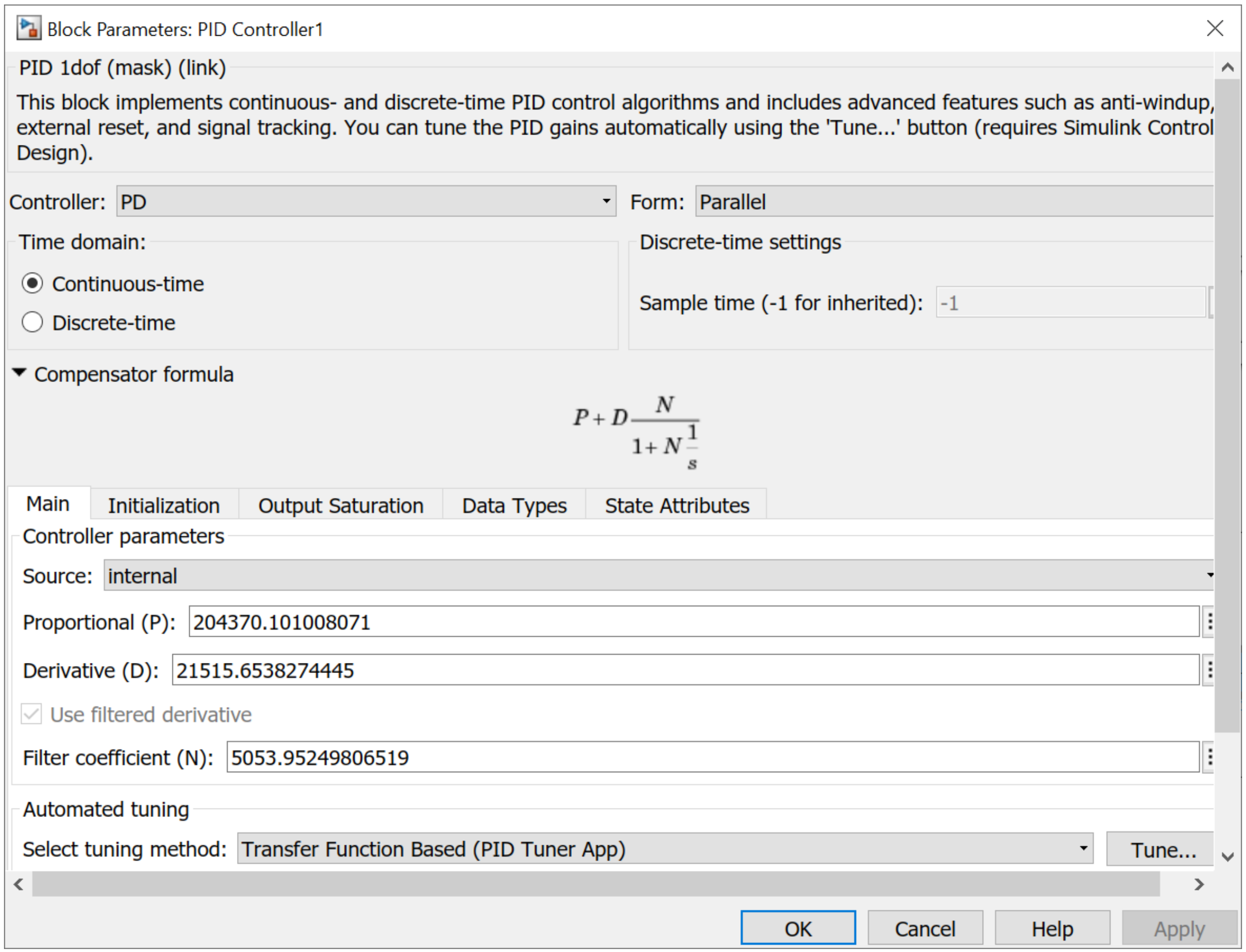
PID

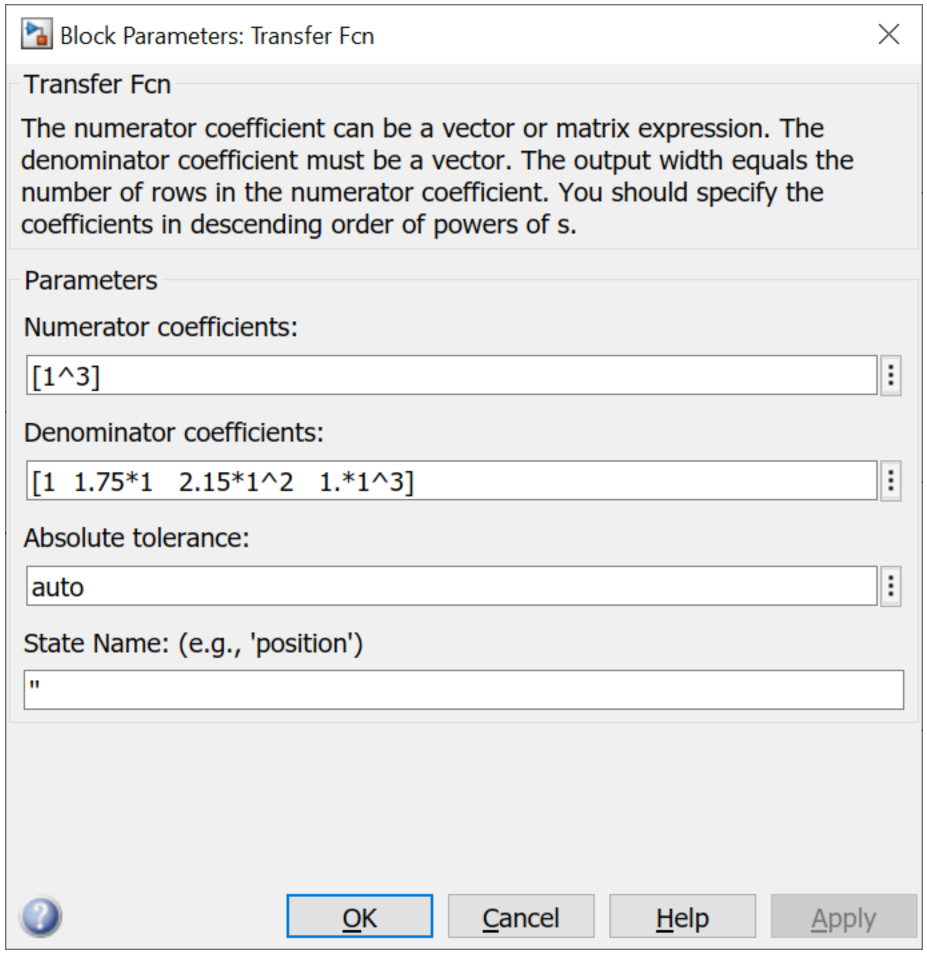


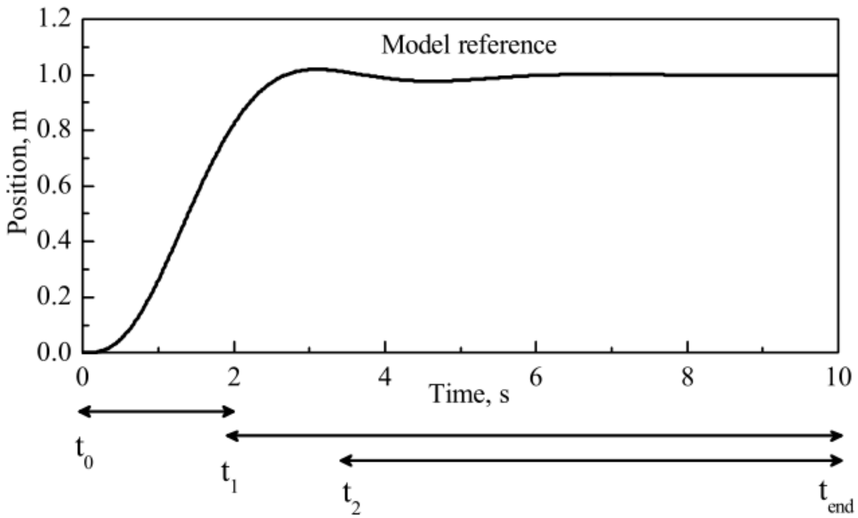
cranePIDPDOS




Block Parameters: PID Controller
PID 1dof (mask) (link)
This block implements continuous- and discrete-time PID control algorithms and includes advanced features such as anti-windup, external reset, and signal tracking. You can tune the PID gains automatically using the 'Tune...' button (requires Simulink Control Design).
Controller: PID
Form: Parallel
Time domain:
Continuous-time
Discrete-time
Discrete-time settings
Sample time (-1 for inherited): -1
Compensator formula
$P+I\frac{1}{s}+D\frac{N}{1+N\frac{1}{s}}$
Main
Initialization
Output Saturation
Data Types
State Attributes
Controller parameters
Source: internal
Proportional (P): 2751.62854455126
Integral (I): 94.1124942564012
Derivative (D): 19759.9982056351
Use filtered derivative
Filter coefficient (N): 3.88278564322259
Automated tuning
OK
Cancel
Help
Apply


Block Parameters: PID Controller1
PID 1dof (mask) (link)
This block implements continuous- and discrete-time PID control algorithms and includes advanced features such as anti-windup, external reset, and signal tracking. You can tune the PID gains automatically using the 'Tune...' button (requires Simulink Control Design).
Controller: PD
Form: Parallel
Time domain:
Continuous-time
Discrete-time
Discrete-time settings
Sample time (-1 for inherited): -1
Compensator formula
$P + D\frac{N}{1+N\frac{1}{s}}$
Main
Initialization
Output Saturation
Data Types
State Attributes
Controller parameters
Source: internal
Proportional (P): 204370.101008071
Derivative (D): 21515.6538274445
Use filtered derivative
Filter coefficient (N): 5053.95249806519
Automated tuning
Select tuning method: Transfer Function Based (PID Tuner App)
Tune...
OK
Cancel
Help
Apply


Block Parameters: Transfer Fcn
Transfer Fcn
The numerator coefficient can be a vector or matrix expression. The denominator coefficient must be a vector. The output width equals the number of rows in the numerator coefficient. You should specify the coefficients in descending order of powers of s.
Parameters
Numerator coefficients:
[1^3]
Denominator coefficients:
[1 1.75*1 2.15*1^2 1.*1^3]
Absolute tolerance:
auto
State Name: (e.g., 'position')
''
OK
Cancel
Help
Apply

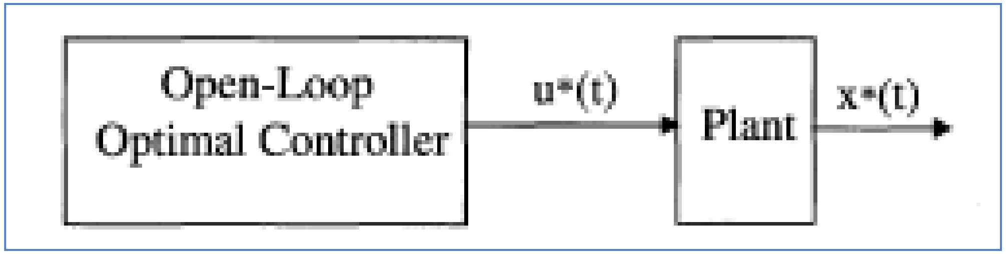


$$J = \int_0^{t_1} t(x - x_{ref})^2\, dt + \int_{t_1}^{t_{end}} t(\theta)^2\, dt + \int_{t_2}^{t_{end}} t(x - x_{ref})^2\, dt$$

$$X_{ref} = \frac{\omega^3}{s^3 + 1.75\omega s^2 + 2.15\omega^2 s + 1.5\omega^3}$$

Typy układów sterowania

Układ otwarty

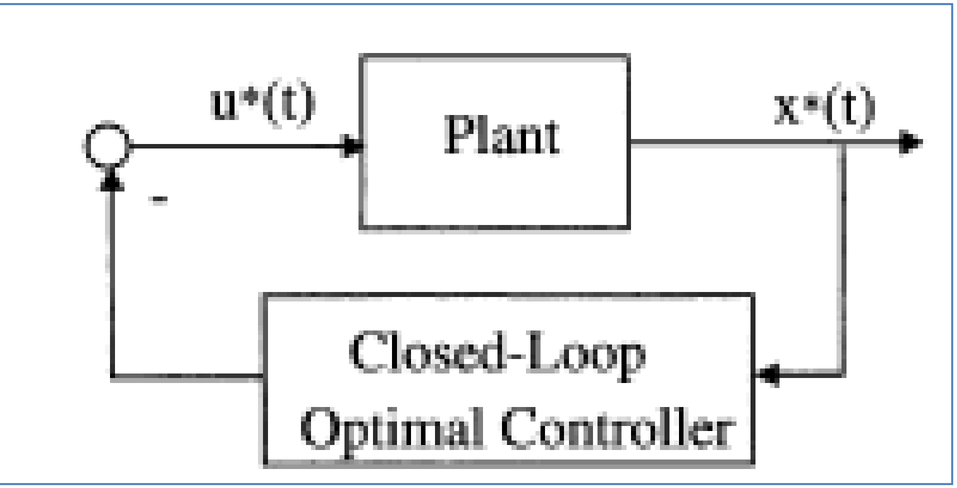


Sterowanie w układzie otwartym (ręczne lub automatyczne) polega na takim nastawieniu wielkości wejściowej, aby znając charakterystykę obiektu i przewidując możliwość działania nań zakłóceń, otrzymać na wyjściu pożądaną wartość. Ponieważ nie istnieje możliwość tłumienia nieznanych zakłóceń oraz osiągnięcie wartości zadanej nie może być zweryfikowane, układ otwarty stosowany jest w przypadku prostych obiektów, dla których znany jest dokładny model matematyczny. W przypadku znanej wartości zakłócenia (np. temperatury na zewnątrz budynku, w którym znajduje się kocioł centralnego ogrzewania) układ otwarty może być użyty do jego kompensacji.

Podstawową wadą takiego rodzaju sterowania jest wpływ dynamiki układu na wartość wyjściową. W porównaniu do układu regulacji układ otwarty jest bardziej czuły na zmiany wzmocnienia statycznego w układzie.

Typy układów sterowania

Układ zamknięty - **Regulator ze sprzężeniem zwrotnym**



Układ zamknięty (ang. closed-loop system) – układ sterowania, w którym przebieg sygnału następuje w dwóch kierunkach. Od wejścia do wyjścia przebiega sygnał realizujący wzajemne oddziaływanie elementów, natomiast od wyjścia do wejścia przebiega sygnał sprzężenia zwrotnego.

Sterowanie w układzie zamkniętym (ręczne lub automatyczne) różni się od sterowania w układzie otwartym tym, że człowiek lub regulator otrzymują dodatkowo poprzez sprzężenie zwrotne informacje o stanie wielkości wyjściowej (lub o stanie obiektu). Informacja ta (odczytana z miernika lub podana w postaci np. napięcia do regulatora) jest używana do korygowania nastaw wielkości wejściowej.

Optymalizacja układu dynamicznego

$$\frac{d^2x(t)}{dt^2} = u(t)$$

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = u(t), \end{cases}$$

$$x_1 = x, \quad x_2 = \dot{x}_1 = \dot{x}$$

$$x_1(0) = x_{10} = 1, \quad x_1(T) = x_{1T} = 0,25,$$
$$x_2(0) = x_{20} = 1, \quad x_2(T) = x_{2T} = 0,$$
$$T = 5\ s.$$

$$x_1(0) = x_{10}, \quad x_1(T) = x_{1T},$$
$$x_2(0) = x_{20}, \quad x_2(T) = x_{2T},$$
$$t_0 = 0, \quad t_1 = T.$$

$$f = m \cdot a$$

$$J = \int_0^T u^2(t)dt \rightarrow \min$$

$$J\left(x(t)\right) = \int_0^T \ddot{x}^2(t)dt.$$

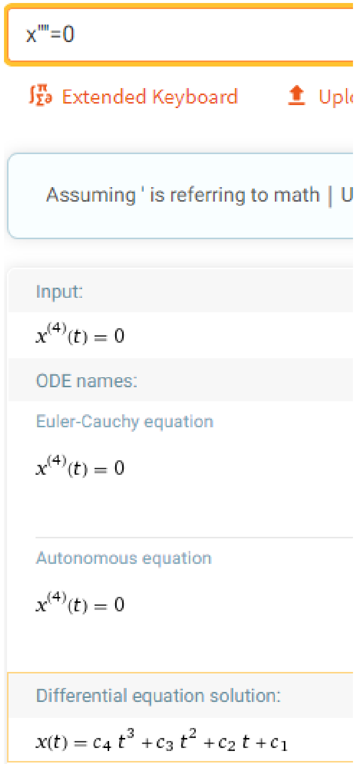
Rozwiązanie

$$F(x)=\left(\frac{d^2x}{dt^2}\right)^2; F_x=\frac{\partial F}{\partial x}=0; \quad F_{\dot{x}}=\frac{\partial F}{\partial \dot{x}}=0; \quad F_{\ddot{x}}=\frac{\partial F}{\partial \ddot{x}}=2\frac{d^2x}{\partial t^2}.$$

$$F_x-\frac{d}{dt}F_{\dot{x}}+\frac{d^2}{dt^2}F_{\ddot{x}}=0,$$

Rozwiązanie

$$2\frac{d^4x}{dt^4}=0.$$



Całkując dwa razy otrzymamy

$$\begin{cases} \dot{x}_1(t)=x_2(t), \\ \dot{x}_2(t)=u(t), \end{cases}$$

$$u(t)=\ddot{x}(t)=\dot{x}_2(t)=C_1t+C_2$$

Rozwiązanie

Całkując dalej otrzymamy

$$x_2(t) = \frac{C_1}{2}t^2 + C_2 t + C_3;$$

$$x_1(t) = \frac{C_1}{6}t^3 + \frac{C_2}{2}t^2 + C_3 t + C_4.$$

Spełnienie warunków

$$x_1(0) = x_{10} = 1, \quad x_1(T) = x_{1T} = 0,25,$$

$$x_2(0) = x_{20} = 1, \quad x_2(T) = x_{2T} = 0,$$

$$T = 5\ s.$$

$\boldsymbol{t = 0}$ $\quad \mathbf{1 = \boldsymbol{C_3}, \quad 1 = \boldsymbol{C_4};}$

$\boldsymbol{t = T = 5}$

$$\mathbf{0 = \frac{\boldsymbol{C_1}}{2}25 + \boldsymbol{C_2}5 + 1,}$$

$$\mathbf{0,25 = \frac{\boldsymbol{C_1}}{6}125 + \frac{\boldsymbol{C_2}}{2}25 + 5 + 1.}$$

Rozwiązanie systemu w MatLabie

```
>> syms c1  c2
eq1= c1*(25/2)+c2*5+1;
eq2= c1*(125/6)+c2*(25/2)+6-0.25;
 [c1,c2]= solve(eq1,  eq2)

c1 =
 39/125

c2 =
 -49/50

>> c1=39/125
c1 =
   0.3120

>> c2=-49/50
c2 =
  -0.9800
```

$$C_1 = 0{,}312$$

$$C_2 = -0{,}980$$

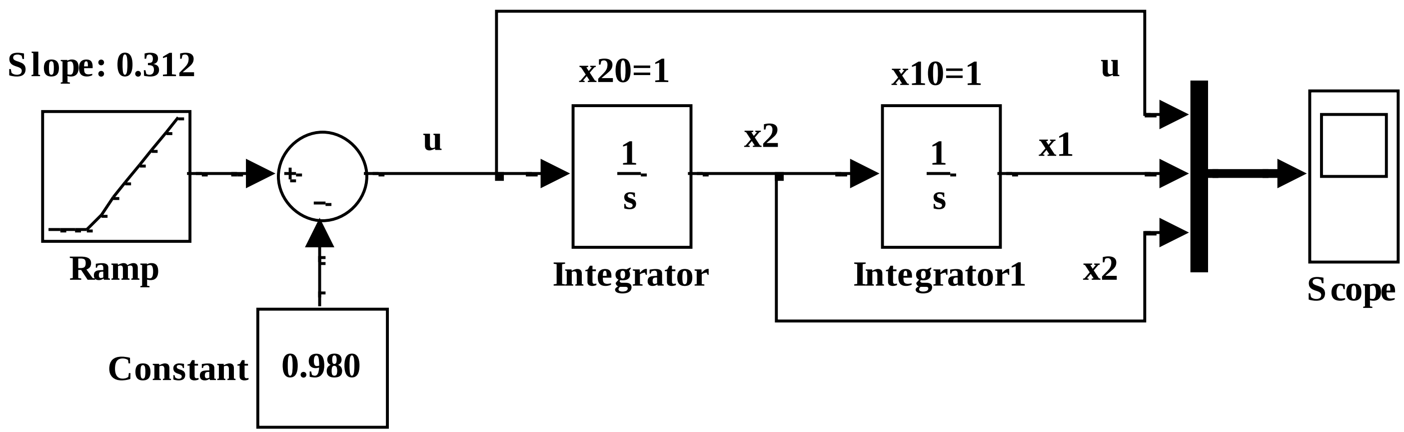
Warunek Legende'a

$$F_{\ddot{x},\ddot{x}} = \frac{\partial F_{\ddot{x}}}{\partial \ddot{x}} = 2 > 0.$$

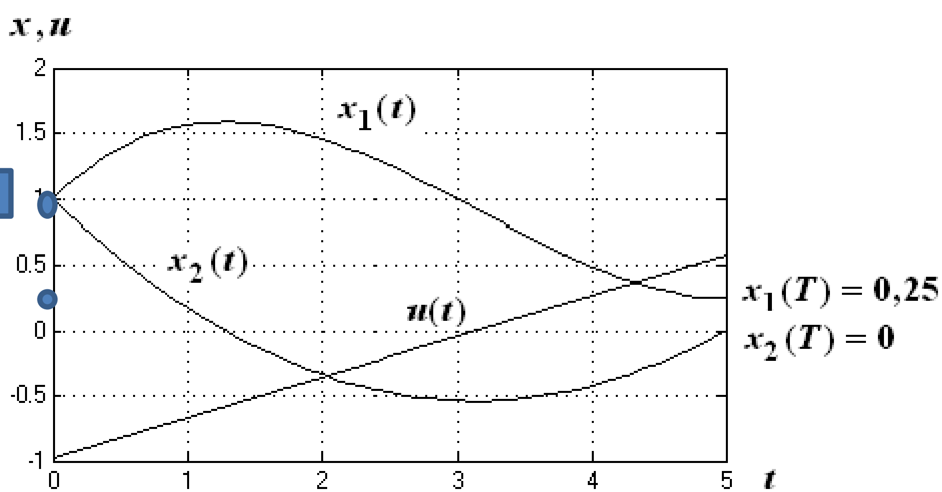
$$u(t) = 0{,}312t - 0{,}980.$$

Sterowanie programowe

Symulacja



$$J = \int_0^T u^2(t)dt \to \min$$



ster1.mdl

U=0

$$\frac{d^2x(t)}{dt^2} = u(t) = 0$$

$x''(t) = 0$

ODE names:

Euler-Cauchy equation

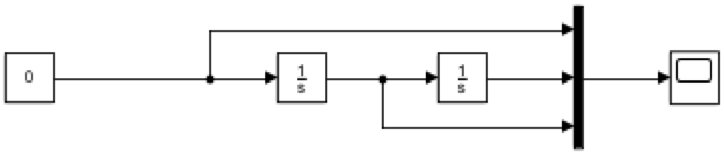
$x''(t) = 0$

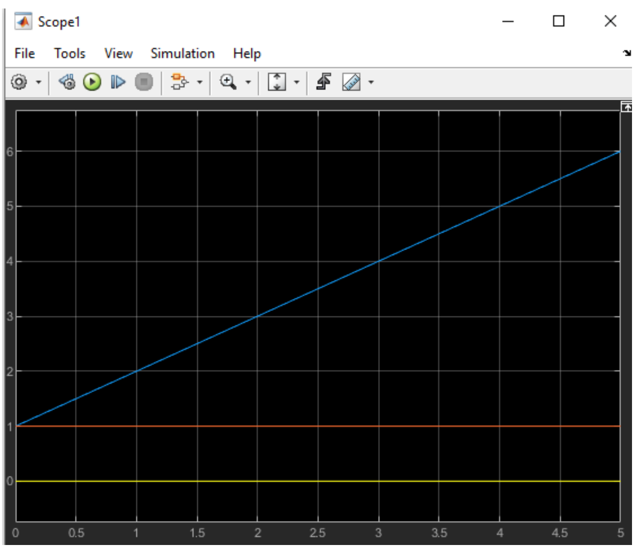
Autonomous equation

$x''(t) = 0$

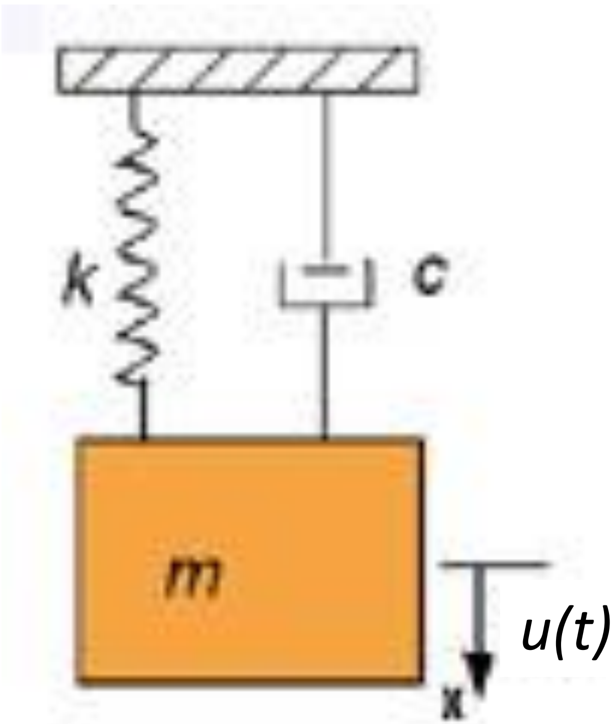
Differential equation solution:

$x(t) = c_2\, t + c_1$





Oscylator harmoniczny tłumiony



$$\frac{d^2x(t)}{dt^2} + c\frac{dx(t)}{dt} + Kx(t) = u(t)$$

$$J = \int_{t_0}^{t_1} u^2(t)\,dt \to \min$$

$\mathbf{x(t_0) = x_0}$ $\dot{x}(t_0) = \dot{x}_0$ $x(t_1) = x_1$ $\dot{x}(t_1) = \dot{x}_1$

$$J\left(x(t)\right) = \int_{t_0}^{t_1} \left(\frac{d^2 x(t)}{dt^2} + c\,\frac{dx(t)}{dt} + Kx(t) \right)^2 dt$$

$$F = \left(\ddot{x} + c\dot{x} + Kx\right)^2$$

$$F = \left(\ddot{x} + c\dot{x} + Kx\right)^2$$

$$F_x - \frac{d}{dt}F_{\dot{x}} + \frac{d^2}{dt^2}F_{\ddot{x}} = 0,$$

$$F_x = 2K\left(\ddot{x} + c\dot{x} + Kx\right)$$

$$F_{\dot{x}} = 2c\left(\ddot{x} + c\dot{x} + Kx\right)$$

$$F_{\ddot{x}} = 2\left(\ddot{x} + c\dot{x} + Kx\right)$$

$$F_x = 2K\left(\ddot{x} + c\dot{x} + Kx\right) \qquad \frac{d}{dt}F_{\dot{x}} = 2c\left(\dddot{x} + c\ddot{x} + K\dot{x}\right)$$

$$\frac{d^2}{dt^2}F_{\ddot{x}} = 2\left(\ddddot{x} + c\dddot{x} + K\ddot{x}\right)$$

$$F_x - \frac{d}{dt}F_{\dot{x}} + \frac{d^2}{dt^2}F_{\ddot{x}} = 0,$$

$$2\ddddot{x} + \left(4K - 2c^2\right)\ddot{x} + 2K^2 x = 0$$

$$\boxed{2\ddddot{x}+\left(4K-2c^{2}\right)\ddot{x}+2K^{2}x=0}$$

$$x(t)=C_{1}e^{tK_{1}}+C_{2}e^{tK_{2}}+C_{3}e^{-tK_{1}}+C_{4}e^{-tK_{2}}$$

$$K_{1}=\left(\frac{c}{2}-\frac{\left(c^{2}-4K\right)^{1/2}}{2}\right)$$

$$K_{2}=\left(\frac{c}{2}+\frac{(c^{2}-4K)^{1/2}}{2}\right)$$

Program

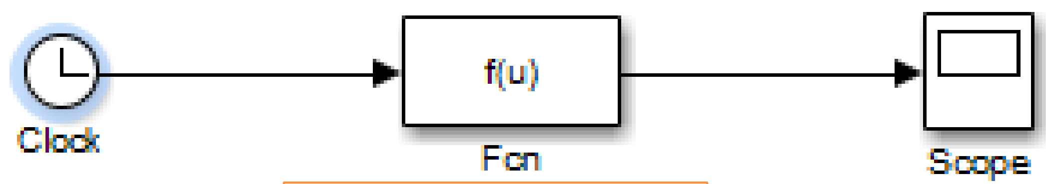
sprezynaSYM.m

```
clear all
disp('Rozwiazanie')
syms t x Dx D2x D3x D4x c K
F=(D2x+c*Dx+K*x)^2;
t1=0;
x1=0;
dx1=0;
t2=1;
x2=1;
dx2=0;
dFdx=diff(F,x);
%2*K*(D2x + Dx*c + K*x)
dFdDx=diff(F,Dx);
%2*c*(D2x + Dx*c + K*x)
dFdD2x=diff(F,D2x);
%2*D2x + 2*Dx*c + 2*K*x
dFdtdFdx=2*c*(D3x+c*D2x+K*Dx);
d2Fdt2dFdD2x=2*(D4x+c*D3x+K*D2x);
%Poisson
Poisson=simple(dFdx-dFdtdFdx+d2Fdt2dFdD2x);
%2*x*K^2 + 4*D2x*K - 2*D2x*c^2 + 2*D4x
deqPoisson=[char(Poisson) '=0'];
Sol=dsolve(deqPoisson,'t');
%C2*exp(t*(c/2 + (c^2 - 4*K)^(1/2)/2)) + C3*exp(t*(c/2 - (c^2 - 4*K)^(1/2)/2)) + C4*exp(-t*(c/2 - (c^2 - 4*K)^(1/2)/2)) + C5*exp(-t*(c/2 + (c^2 - 4*K)^(1/2)/2))
dSoldt= diff(Sol,t);
SolLeft=subs(Sol,t,t1); %  t1
%C2 + C3 + C4 + C5
SolRight=subs(Sol,t,t2); %  t2
%C2*exp(c/2 + (c^2 - 4*K)^(1/2)/2) + C3*exp(c/2 - (c^2 - 4*K)^(1/2)/2) + C4*exp((c^2 - 4*K)^(1/2)/2 - c/2) + C5*exp(- c/2 - (c^2 - 4*K)^(1/2)/2)
difSolLeft=subs(dSoldt,t,t1); %  t1
%C2*(c/2 + (c^2 - 4*K)^(1/2)/2) + C3*(c/2 - (c^2 - 4*K)^(1/2)/2) - C4*(c/2 - (c^2 - 4*K)^(1/2)/2) - C5*(c/2 + (c^2 - 4*K)^(1/2)/2)
difSolRight=subs(dSoldt,t,t2); %  t2
%C2*exp(c/2 + (c^2 - 4*K)^(1/2)/2)*(c/2 + (c^2 - 4*K)^(1/2)/2) + C3*exp(c/2 - (c^2 - 4*K)^(1/2)/2)*(c/2 - (c^2 - 4*K)^(1/2)/2) - C4*exp((c^2 - 4*K)^(1/2)/2 - c/2)*(c/2 - (c^2 - 4*K)^(1/2)/2) - C5*exp(-
c/2 - (c^2 - 4*K)^(1/2)/2)*(c/2 + (c^2 - 4*K)^(1/2)/2)
EqLeft=[char(SolLeft) '=' char(sym(x1))]; % =x1
EqRight=[char(SolRight) '=' char(sym(x2))]; % =x2
EqLeft2=[char(difSolLeft) '=' char(sym(dx1))]; % =x1
EqRight2=[char(difSolRight) '=' char(sym(dx2))]; % =x2
Con=solve(EqLeft,EqRight,EqLeft2,EqRight2,'C2,C3,C4,C5');
C2=Con.C2;
C3=Con.C3;
C4=Con.C4;
C5=Con.C5;

Sol21=vpa(eval(Sol),14);
```

```
dSol21=diff(Sol21,t);
d2Sol21=diff(dSol21,t);
```

Rozwiązanie

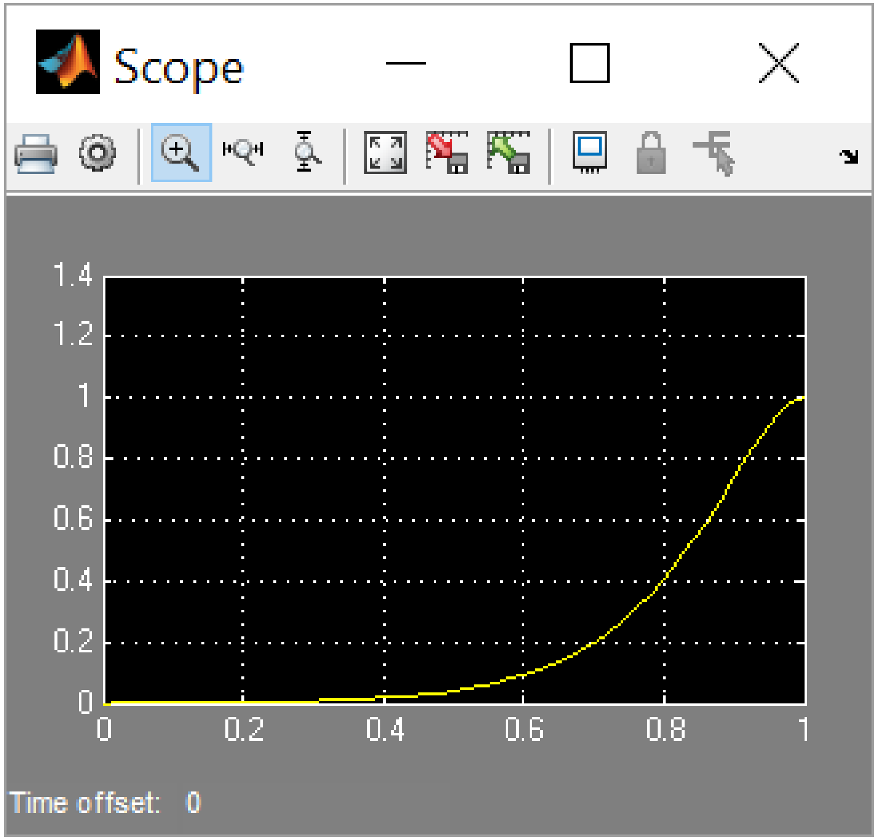
```
(0.5*exp(u(1)*(0.5*c + 0.5*(c^2 - 4.0*K)^(1/2)))*(1.0*exp(0.5*c - 0.5*(c^2 - 4.0*K)^(1/2))*(4.0*K - 1.0*c^2) - c^2*exp(- 0.5*c - 0.5*(c^2 - 4.0*K)^(1/2)) -
exp(- 0.5*c - 0.5*(c^2 - 4.0*K)^(1/2))*(4.0*K - 1.0*c^2) + 1.0*c^2*exp(0.5*(c^2 - 4.0*K)^(1/2) - 0.5*c) + 1.0*c*exp(0.5*c - 0.5*(c^2 - 4.0*K)^(1/2))*(c^2 -
4.0*K)^(1/2) - c*exp(0.5*(c^2 - 4.0*K)^(1/2) - 0.5*c)*(c^2 - 4.0*K)^(1/2)))/(c^2*exp(0.5*c + 0.5*(c^2 - 4.0*K)^(1/2))*exp(0.5*(c^2 - 4.0*K)^(1/2) - 0.5*c) -
8.0*K + c^2*exp(- 0.5*c - 0.5*(c^2 - 4.0*K)^(1/2))*exp(0.5*c - 0.5*(c^2 - 4.0*K)^(1/2)) + exp(0.5*c + 0.5*(c^2 - 4.0*K)^(1/2))*exp(0.5*c - 0.5*(c^2 -
4.0*K)^(1/2))*(4.0*K - 1.0*c^2) + exp(- 0.5*c - 0.5*(c^2 - 4.0*K)^(1/2))*exp(0.5*(c^2 - 4.0*K)^(1/2) - 0.5*c)*(4.0*K - 1.0*c^2)) - (0.5*exp(-1.0*u(1)*(0.5*c -
0.5*(c^2 - 4.0*K)^(1/2)))*(exp(0.5*c - 0.5*(c^2 - 4.0*K)^(1/2))*(4.0*K - 1.0*c^2) - 1.0*c^2*exp(0.5*c + 0.5*(c^2 - 4.0*K)^(1/2)) - 1.0*exp(- 0.5*c - 0.5*(c^2 -
4.0*K)^(1/2))*(4.0*K - 1.0*c^2) + c^2*exp(0.5*c - 0.5*(c^2 - 4.0*K)^(1/2)) - 1.0*c*exp(0.5*c + 0.5*(c^2 - 4.0*K)^(1/2))*(c^2 - 4.0*K)^(1/2) + c*exp(- 0.5*c -
0.5*(c^2 - 4.0*K)^(1/2))*(c^2 - 4.0*K)^(1/2)))/(c^2*exp(0.5*c + 0.5*(c^2 - 4.0*K)^(1/2))*exp(0.5*(c^2 - 4.0*K)^(1/2) - 0.5*c) - 8.0*K + c^2*exp(- 0.5*c -
0.5*(c^2 - 4.0*K)^(1/2))*exp(0.5*c - 0.5*(c^2 - 4.0*K)^(1/2)) + exp(0.5*c + 0.5*(c^2 - 4.0*K)^(1/2))*exp(0.5*c - 0.5*(c^2 - 4.0*K)^(1/2))*(4.0*K - 1.0*c^2) +
exp(- 0.5*c - 0.5*(c^2 - 4.0*K)^(1/2))*exp(0.5*(c^2 - 4.0*K)^(1/2) - 0.5*c)*(4.0*K - 1.0*c^2)) + (0.5*exp(-1.0*u(1)*(0.5*c + 0.5*(c^2 -
4.0*K)^(1/2)))*(exp(0.5*(c^2 - 4.0*K)^(1/2) - 0.5*c)*(4.0*K - 1.0*c^2) - 1.0*c^2*exp(0.5*c + 0.5*(c^2 - 4.0*K)^(1/2)) - 1.0*exp(0.5*c + 0.5*(c^2 -
4.0*K)^(1/2))*(4.0*K - 1.0*c^2) + c^2*exp(0.5*c - 0.5*(c^2 - 4.0*K)^(1/2)) - 1.0*c*exp(0.5*c - 0.5*(c^2 - 4.0*K)^(1/2))*(c^2 - 4.0*K)^(1/2) + c*exp(0.5*(c^2
4.0*K)^(1/2) - 0.5*c)*(c^2 - 4.0*K)^(1/2)))/(c^2*exp(0.5*c + 0.5*(c^2 - 4.0*K)^(1/2))*exp(0.5*(c^2 - 4.0*K)^(1/2) - 0.5*c) - 8.0*K + c^2*exp(- 0.5*c - 0.5*(c^
- 4.0*K)^(1/2))*exp(0.5*c - 0.5*(c^2 - 4.0*K)^(1/2)) + exp(0.5*c + 0.5*(c^2 - 4.0*K)^(1/2))*exp(0.5*c - 0.5*(c^2 - 4.0*K)^(1/2))*(4.0*K - 1.0*c^2) + exp(-
0.5*c - 0.5*(c^2 - 4.0*K)^(1/2))*exp(0.5*(c^2 - 4.0*K)^(1/2) - 0.5*c)*(4.0*K - 1.0*c^2)) - (0.5*exp(u(1)*(0.5*c - 0.5*(c^2 - 4.0*K)^(1/2)))*(1.0*exp(0.5*(c^2 -
4.0*K)^(1/2) - 0.5*c)*(4.0*K - 1.0*c^2) - c^2*exp(- 0.5*c - 0.5*(c^2 - 4.0*K)^(1/2)) - exp(0.5*c + 0.5*(c^2 - 4.0*K)^(1/2))*(4.0*K - 1.0*c^2) +
1.0*c^2*exp(0.5*(c^2 - 4.0*K)^(1/2) - 0.5*c) + 1.0*c*exp(0.5*c + 0.5*(c^2 - 4.0*K)^(1/2))*(c^2 - 4.0*K)^(1/2) - c*exp(- 0.5*c - 0.5*(c^2 - 4.0*K)^(1/2))*(c^2
4.0*K)^(1/2)))/(c^2*exp(0.5*c + 0.5*(c^2 - 4.0*K)^(1/2))*exp(0.5*(c^2 - 4.0*K)^(1/2) - 0.5*c) - 8.0*K + c^2*exp(- 0.5*c - 0.5*(c^2 - 4.0*K)^(1/2))*exp(0.5*c
0.5*(c^2 - 4.0*K)^(1/2)) + exp(0.5*c + 0.5*(c^2 - 4.0*K)^(1/2))*exp(0.5*c - 0.5*(c^2 - 4.0*K)^(1/2))*(4.0*K - 1.0*c^2) + exp(- 0.5*c - 0.5*(c^2 -
4.0*K)^(1/2))*exp(0.5*(c^2 - 4.0*K)^(1/2) - 0.5*c)*(4.0*K - 1.0*c^2))
```

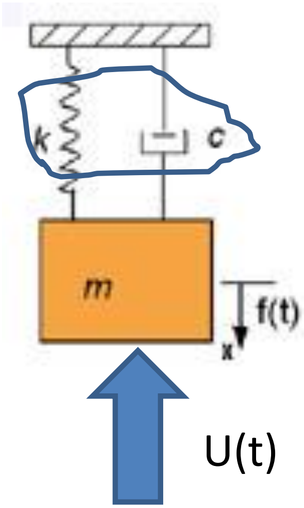


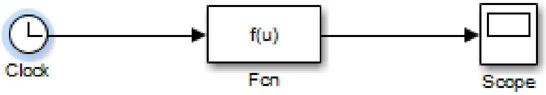
optimtrajektorja.slx

Symulacja. Trajektoria optymalna.

t1=0;
x1=0;
dx1=0;
t2=1;
x2=1;
dx2=0;

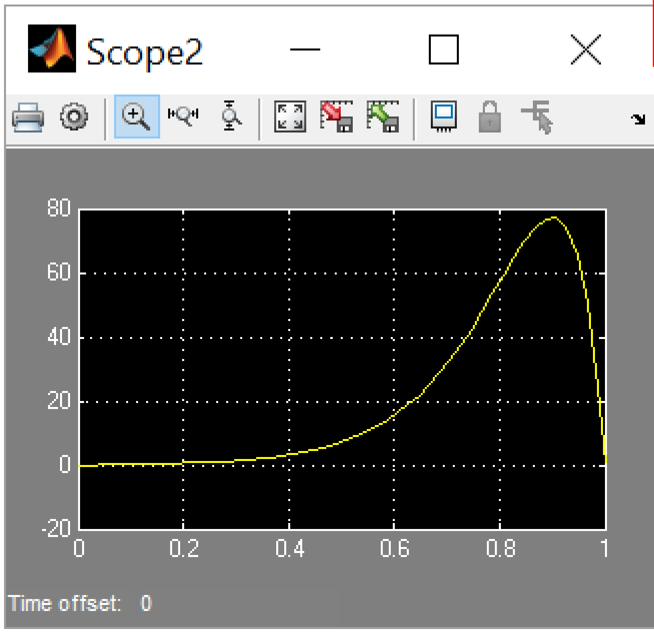






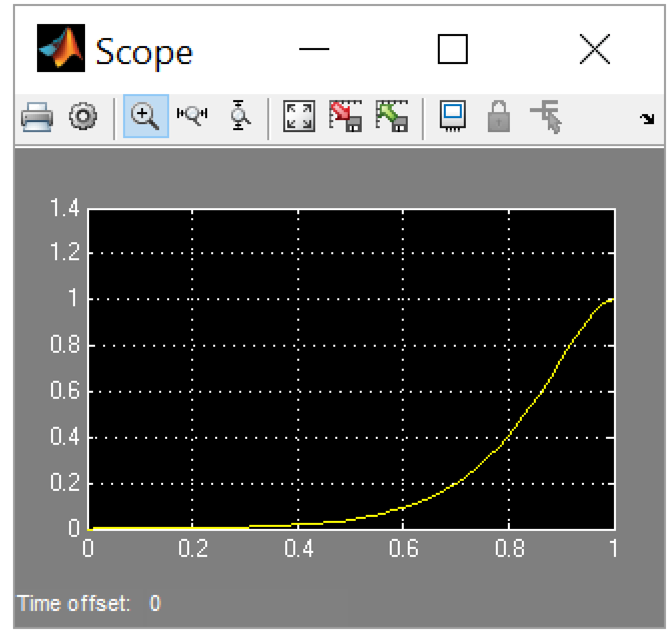
Sterowanie optymalne

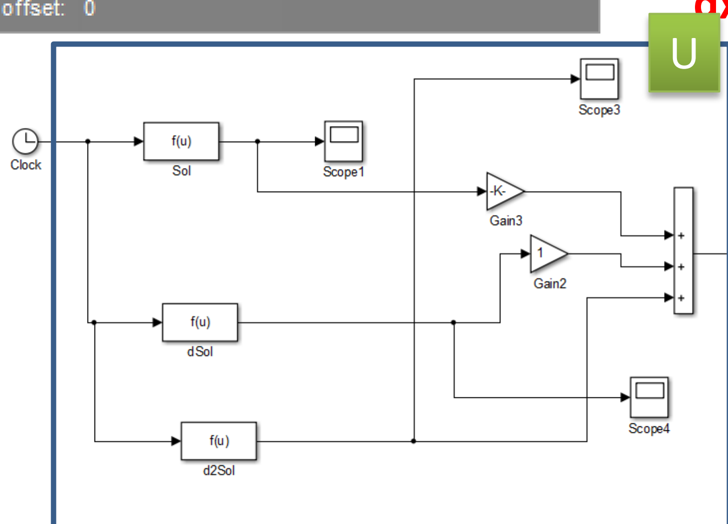
$$\frac{d^2x(t)}{dt^2}+c\frac{dx(t)}{dt}+Kx(t)=u(t)$$

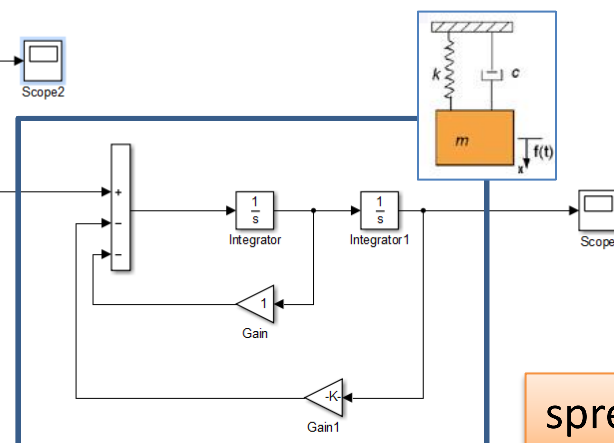


m=1kg;
c=1kg/s;
K=100kg/s^2;

t1=0;
x1=0;
dx1=0;
t2=1;
x2=1;
dx2=0;







sprezynaOpt.slx

Zagadnienie

$$\frac{d^n x(t)}{dt^n} = u(t)$$

$$J = \int_0^T [u(t)]^2 dt = \int_0^T [x^{(n)}(t)]^2 dt$$

$$x^{(n)} = u$$

$$F_x - \frac{d}{dt}F_{\dot{x}} + \frac{d^2}{dt^2}F_{\ddot{x}} + \ldots + (-1)^n \frac{d^n}{dt^n}F_{x^{(n)}} = 0,$$

$$F = [x^{(n)}(t)]^2$$

$$F_x = F_{\dot{x}} = F_{\ddot{x}} = \ldots = F_{x^{(n-1)}} = 0,$$

$$\frac{d^{2n} x(t)}{dt^{2n}} = 0,$$

$$x(t) = C_0 + C_1 t + \ldots + C_{2n-1} t^{2n-1}.$$

$$u = \sum_{i=1}^{n} q_{i-1} t^{i-1}.$$

Przykład

$$\frac{d^3 x(t)}{dt^3} = u(t) \Rightarrow J = \int_0^T [u(t)]^2 dt = \int_0^T [x^{(n)}(t)]^2 dt \Rightarrow J = \int_0^T [x^{(3)}(t)]^2 dt$$

$$x_1(0) = x(0) = 1, \quad x_2(0) = \dot{x}(0) = 1, \quad x_3(0) = \ddot{x}(0) = 1;$$

$$x_1(T) = x(T) = 0,5, \quad x_2(T) = \dot{x}(T) = 0, \quad x_3(T) = \ddot{x}(T) = 0;$$

$$T = 5\ s.$$

$$F_x - \frac{d}{dt} F_{\dot{x}} + \frac{d^2}{dt^2} F_{\ddot{x}} + \ldots + (-1)^n \frac{d^n}{dt^n} F_{x^{(n)}} = 0,$$

$$F_{\dddot{x}} = 2\dddot{x} \qquad \frac{d^3(\dddot{x})}{dt^3} = x^{(6)}$$

Rozwiązanie

$$\frac{d^{2n}x(t)}{dt^{2n}} = 0, \qquad \frac{d^6x(t)}{dt^6} = 0.$$

```
>> x1 = dsolve('D6x1=0')
 x1 =
 1/120*C1*t^5+1/24*C2*t^4+1/6*C3*t^3+1/2*C4*t^2+C5*t+C6
```

$$x_1(t) = x(t) = \frac{C_1}{120}t^5 + \frac{C_2}{24}t^4 + \frac{C_3}{6}t^3 + \frac{C_4}{2}t^2 + C_5 t + C_6.$$

Obliczenie stałych

```
>> x2 = dsolve('D5x2=0')
 x2 =

1/24*C1*t^4+1/6*C2*t^3+1/2*C3*t^2+C4*t+C5
>> x3 = dsolve('D4x3=0')
 x3 =
 1/6*C1*t^3+1/2*C2*t^2+C3*t+C4
```

$$x_2(t) = \dot{x}(t) = \frac{C_1}{24}t^4 + \frac{C_2}{6}t^3 + \frac{1}{2}C_3t^2 + C_4t + C_5;$$

$$x_3(t) = \ddot{x}(t) = \frac{C_1}{6}t^3 + \frac{C_2}{2}t^2 + C_3t + C_4.$$

$$\begin{cases} x_1(0) = C_6; \\ x_2(0) = C_5; \\ x_3(0) = C_4; \\ x_1(T) = \dfrac{C_1}{120}T^5 + \dfrac{C_2}{24}T^4 + \dfrac{C_3}{6}T^3 + \dfrac{C_4}{2}T^2 + C_5T + C_6; \\ x_2(T) = \dfrac{C_1}{24}T^4 + \dfrac{C_2}{6}T^3 + \dfrac{C_3}{2}T^2 + C_4T + C_5; \\ x_3(T) = \dfrac{C_1}{6}T^3 + \dfrac{C_2}{2}T^2 + C_3T + C_4. \end{cases}$$

Rozwiązanie systemu

```
>> syms  c1  c2  c3  c4  c5  c6
%warunki końcowe
x10=1;  x20=1;  x30=1;
x1T=0.5; x2T=0;  x3T=0;
T=5;
%system równań algebraicznych
eq1=c6-x10;
eq2=c5-x20;
eq3=c4-x30;
eq4=c1/120*T^5+c2/24*T^4+c3/6*T^3+c4/2*T^2+c5*T+c6-x1T;
eq5=c1/24*T^4+c2/6*T^3+c3/2*T^2+c4*T+c5-x2T;
eq6=c1/6*T^3+c2/2*T^2+c3*T+c4-x3T;
%rozwązanie
[c1,c2,c3,c4,c5,c6]= solve(eq1,  eq2,  eq3, eq4, eq5, eq6)
```

$$\boldsymbol{C_1 = -1{,}1712;\ C_2 = 3{,}2640;\ C_3 = -3{,}4800;\ C_4 = 1;\ C_5 = 1;\ C_6 = 1.}$$

Sposób drugi

$$\boldsymbol{x} = \boldsymbol{A}\boldsymbol{c} \qquad \boldsymbol{c} = \boldsymbol{A}^{-1}\boldsymbol{x}$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{T^5}{120} & \frac{T^4}{24} & \frac{T^3}{6} & \frac{T^2}{2} & T & 1 \\ \frac{T^4}{24} & \frac{T^3}{6} & \frac{T^2}{2} & T & 1 & 0 \\ \frac{T^3}{6} & \frac{T^2}{2} & T & 1 & 0 & 0 \end{bmatrix}; \; x = \begin{bmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \\ x_1(T) \\ x_2(T) \\ x_3(T) \end{bmatrix} = \begin{bmatrix} x(0) \\ \dot{x}(0) \\ \ddot{x}(0) \\ x(T) \\ \dot{x}(T) \\ \ddot{x}(T) \end{bmatrix}.$$

Macierz odwrotna

```
>> syms  c1  c2  c3  c4  c5  c6
%warunki końcowe i macierz
x10=1;  x20=1;  x30=1;
x1T=0.5; x2T=0;  x3T=0;
T=5;
x=[ x10;  x20;  x30;  x1T;  x2T;  x3T];
A=[0  0  0  0  0  1; 0  0  0  0  1  0; 0  0  0  1  0  0;
T^5/120  T^4/24  T^3/6  T^2/2  T  1;
T^4/24  T^3/6  T^2/2  T  1  0; T^3/6  T^2/2  T  1
0  0];
%rozwiązanie
c=inv(A)*x
```

Sterowanie optymalne

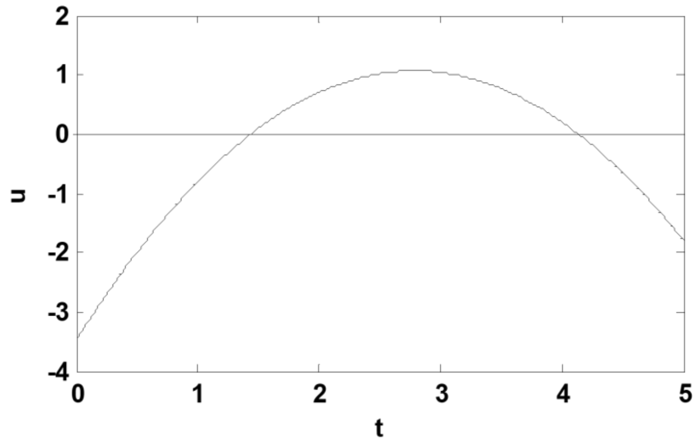
$$\frac{d^3x(t)}{dt^3} = u(t)$$

$$\ddot{x}(t) = \frac{C_1}{2}t^2 + C_2 t + C_3. \qquad u(t) = \dddot{x}(t) = q_0 + q_1 t + q_2 t^2,$$

$$q_0 = C_3 = -3{,}4800;\ q_1 = C_2 = 3{,}2640;\ q_2 = C_1/2 = -0{,}5856.$$

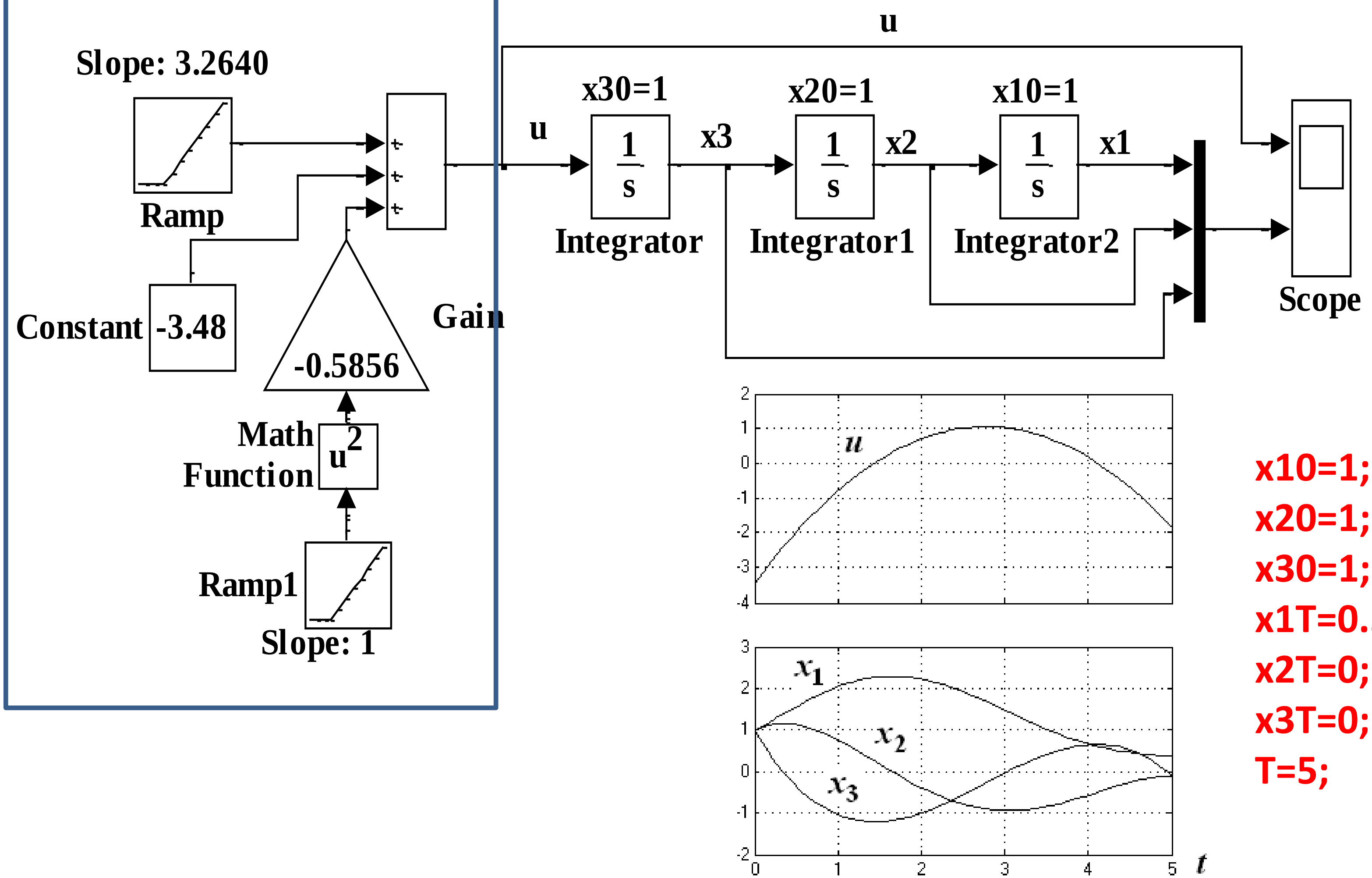
$$u(t) = -3{,}4800 + 3{,}2640t - 0{,}5856t^2.$$

```
>>t=0:0.01:5;
u=-3.4800+3.2640*t-0.5856*t.^2;
plot(t,u)
xlabel('t'), ylabel('u')
```



Symulacja

$$\frac{d^3x(t)}{dt^3} = u(t)$$



x10=1;

x20=1;

x30=1;

x1T=0.5;

x2T=0;

x3T=0;

T=5;

Przykład

$$W(s)=\frac{X(s)}{U(s)}=\frac{1}{s^2(\tau s+1)},\qquad J=\int_0^T F\,dt=\int_0^T u^2(t)\,dt=\int_0^T(\tau\dddot{x}+\ddot{x})^2\,dt.$$

$$\tau\dddot{x}(t)+\ddot{x}(t)=u(t)$$

$$\dot{x}_1=x_2;\qquad x_1=x$$

$$\dot{x}_2=x_3;$$

$$\dot{x}_3=-\frac{1}{\tau}x_3+\frac{1}{\tau}u,$$

$$x_1(0)=x(0)=1,\quad x_2(0)=\dot{x}(0)=0,\quad x_3(0)=\ddot{x}(0)=0;$$

$$x_1(T)=x(T)=0,\quad x_2(T)=\dot{x}(T)=0,\quad x_3(T)=\ddot{x}(T)=0;$$

$$\tau=1\ s;\ T=2\ s.$$

Rozwiązanie

$$F_x - \frac{d}{dt}F_{\dot{x}} + \frac{d^2}{dt^2}F_{\ddot{x}} - \frac{d^3}{dt^3}F_{\dddot{x}} = 0.$$

$$F = (\tau\,\dddot{x} + \ddot{x})^2;\ F_x = \frac{\partial F}{dx} = 0;\ F_{\dot{x}} = \frac{\partial F}{d\dot{x}} = 0;$$

$$F_{\ddot{x}} = \frac{\partial F}{d\,\ddot{x}} = 2(\tau\,\dddot{x} + \ddot{x});\quad F_{\dddot{x}} = \frac{\partial F}{d\,\dddot{x}} = 2\tau\,(\tau\,\dddot{x} + \ddot{x}).$$

$$2(\tau\,x^{(5)} + x^{(4)}) - 2\tau\,(\tau\,x^{(6)} + x^{(5)}) = 0,$$

$$\boxed{\tau^2\,x^{(6)} - x^{(4)} = 0.}$$

Rozwiązanie

```
>> x = dsolve('tau^2*D6x-D4x=0')

 x =
 C1+C2*t+C3*t^2+C4*t^3+C5*exp(-1/tau*t)+C6*exp(1/tau*t)
```

$$x(t) = C_1 + C_2 t + C_3 t^2 + C_4 t^3 + C_5 e^{-\frac{t}{\tau}} + C_6 e^{\frac{t}{\tau}}.$$

$$\dot{x}(t) = C_2 + 2C_3 t + 3C_4 t^2 - \frac{1}{\tau} C_5 e^{-\frac{t}{\tau}} + \frac{1}{\tau} C_6 e^{\frac{t}{\tau}};$$

$$\ddot{x}(t) = 2C_3 + 6C_4 t + \frac{1}{\tau^2} C_5 e^{-\frac{t}{\tau}} + \frac{1}{\tau^2} C_6 e^{\frac{t}{\tau}}.$$

Rozwiązanie

$$\begin{cases} x_1(0) = C_1 + C_2 + C_6; \\ x_2(0) = C_2 - \frac{1}{\tau}C_5 + \frac{1}{\tau}C_6; \\ x_3(0) = 2C_3 + \frac{1}{\tau^2}C_5 + \frac{1}{\tau^2}C_6; \\ x_1(T) = C_1 + C_2T + C_3T^2 + C_4T^3 + C_5e^{-\frac{T}{\tau}} + C_6e^{\frac{T}{\tau}}; \\ x_2(T) = C_2 + 2C_3T + 3C_4T^2 - \frac{1}{\tau}C_5e^{-\frac{T}{\tau}} + \frac{1}{\tau}C_6e^{\frac{T}{\tau}}; \\ x_3(T) = 2C_3 + 6C_4T + \frac{1}{\tau^2}C_5e^{-\frac{T}{\tau}} + \frac{1}{\tau^2}C_6e^{\frac{T}{\tau}}. \end{cases}$$

Reguła sterowania

$$\ddot{x}(t) = 6C_4 - \frac{1}{\tau^3} C_5 e^{-\frac{t}{\tau}} + \frac{1}{\tau^3} C_6 e^{\frac{t}{\tau}}.$$

$$u(t) = \tau\, \dddot{x}(t) + \ddot{x}(t) = \tau\left(6C_4 - \frac{1}{\tau^3} C_5 e^{-\frac{t}{\tau}} + \frac{1}{\tau^3} C_6 e^{\frac{t}{\tau}}\right) +$$

$$+ 2C_3 + 6C_4 t + \frac{1}{\tau^2} C_5 e^{-\frac{t}{\tau}} + \frac{1}{\tau^2} C_6 e^{\frac{t}{\tau}} =$$

$$= 2C_3 + 6C_4\tau + 6C_4 t + 2\frac{1}{\tau^2} C_6 e^{\frac{t}{\tau}},$$

$$q_0 = 2C_3 + 6C_4\tau;\ \ q_1 = 6C_4;\ \ q_2 = 2\frac{1}{\tau^2} C_6.$$

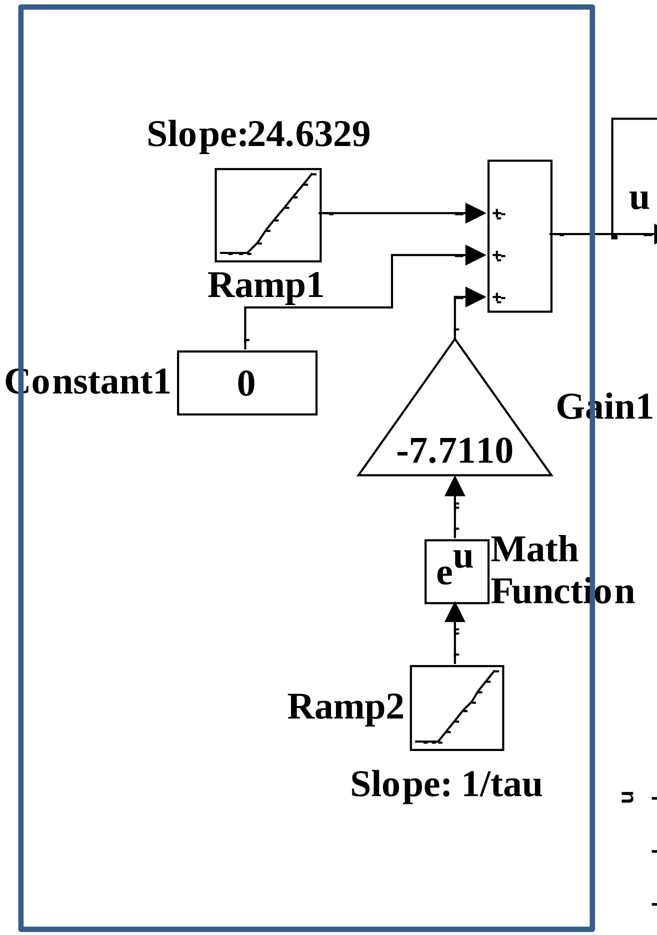
$$u(t) = q_0 + q_1 t + q_2 e^{\frac{t}{\tau}},$$

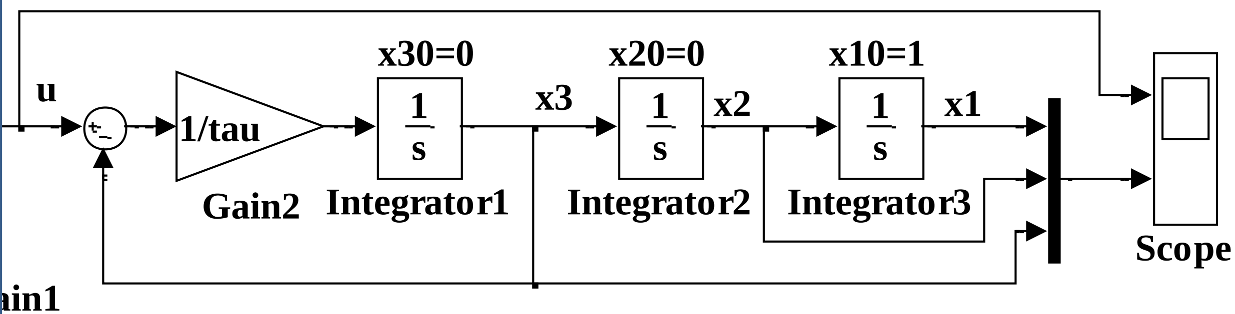
Program

```
>> syms  c1  c2  c3  c4  c5  c6
tau=1; T=2;
x10=1;  x20=0;  x30=0;
x1T=0; x2T=0;  x3T=0;
eq1=c1+c5+c6-x10;
eq2=c2-1/tau*c5+1/tau*c6-x20;
eq3=2*c3+1/tau^2*c5+1/tau^2*c6-x30;
eq4=c1+c2*T+c3*T^2+c4*T^3+c5*exp(-
T/tau)+c6*exp(T/tau)-x1T;
eq5= c2 +2*c3*T+3*c4*T^2-1/tau*c5*exp(-
T/tau)+1/tau*c6*exp(T/tau)-x2T;
eq6= 2*c3 +6*c4*T+1/tau^2*c5*exp(-
T/tau)+1/tau^2*c6*exp(T/tau)-x3T;
[c1,c2,c3,c4,c5,c6]= solve(eq1,  eq2,  eq3, eq4,
eq5, eq6)
c1 =
 -23.6329
c2 =
  32.3439
c3 =
 -12.3165
c4 =
   4.1055
c5 =
  28.4884
c6 =
  -3.8555

q0=2*c3+6*c4*tau;
q1=6*c4;
q2=2/tau^2*c6;
[q0], [q1], [q2]
q0 =
     0
q1 =
  24.6329
q2 =
  -7.7110
```

Model

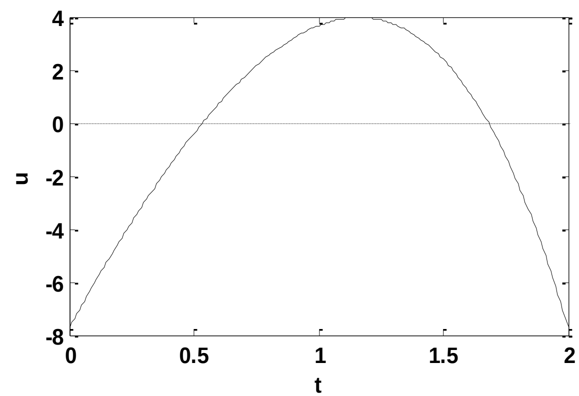
$$W(s) = \frac{X(s)}{U(s)} = \frac{1}{s^2(\tau s + 1)},$$

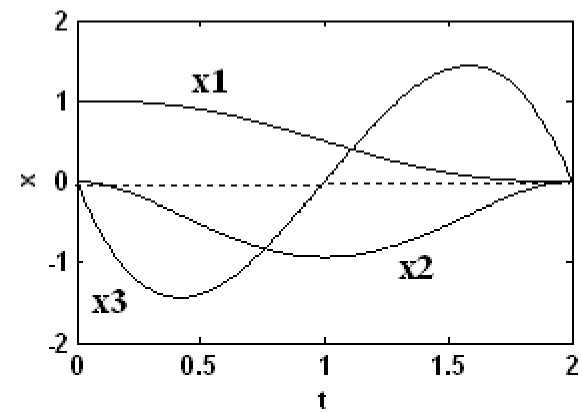




Simulation time

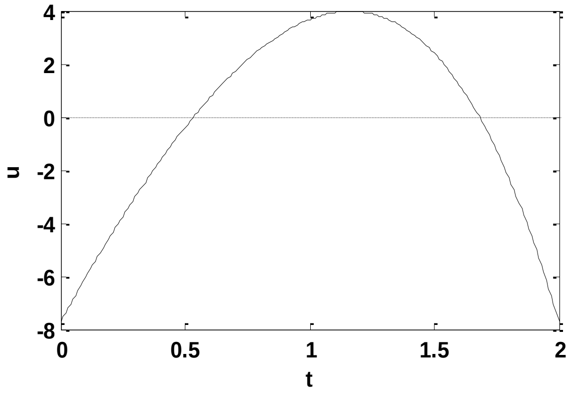
Start time: 0.0 Stop time: 2

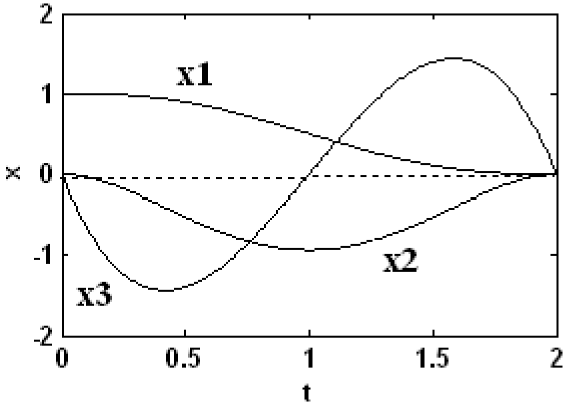




Analiza

```
>> x1=c1+c2*t+c3*t.^2+c4*t.^3+c5*exp(-1./tau*t)+c6*exp(1./tau*t);
plot(t,x1, 'k -')
hold on
x2= c2 +2*c3*t+3*c4*t.^2-1/tau*c5*exp(-1./tau*t)+1/tau*c6*exp(1./tau*t);
plot(t,x2, 'k-')
hold on
x3= 2*c3 +6*c4*t+1/tau^2*c5*exp(-1./tau*t)+1/tau^2*c6*exp(1./tau*t);
plot(t,x3, 'k-')
xlabel('t'), ylabel('x')
```





Oscylator harmoniczny tłumiony (2)

$$W(s) = \frac{X(s)}{U(s)} = \frac{\omega_0^2}{s^2 + 2\xi\omega_0 s + \omega_0^2}$$

$$\ddot{x}(t) + 2\xi\omega_0\dot{x}(t) + \omega_0^2 x(t) = \omega_0^2 u(t),$$

$$x_1(0) = x(0) = 1, \quad x_2(0) = \dot{x}(0) = 1;$$

$$x_1(T) = x(T) = 0, \quad x_2(T) = \dot{x}(T) = 0;$$

$$\xi = 0{,}1;\ \omega_0 = 0{,}5\ s^{-1};\ T = 5\ s.$$

$$J = \int_0^T F dt = \int_0^T u^2(t) dt = \int_0^T [(\ddot{x} + 2\xi\omega_0\dot{x} + \omega_0^2 x)/\omega_0^2]^2 dt,$$

Układ dynamiczny II rzędu

Układ dynamiczny II rzędu - układ dynamiczny opisany równaniem:

$$a_2\frac{d^2y(t)}{dt^2} + a_1\frac{dy(t)}{dt} + a_0y(t) = u(t)$$

Jego transmitancja dana jest wzorem:

$$G(s) = \frac{1}{a_2(s^2 + 2\tau s + \gamma^2)}$$

gdzie:

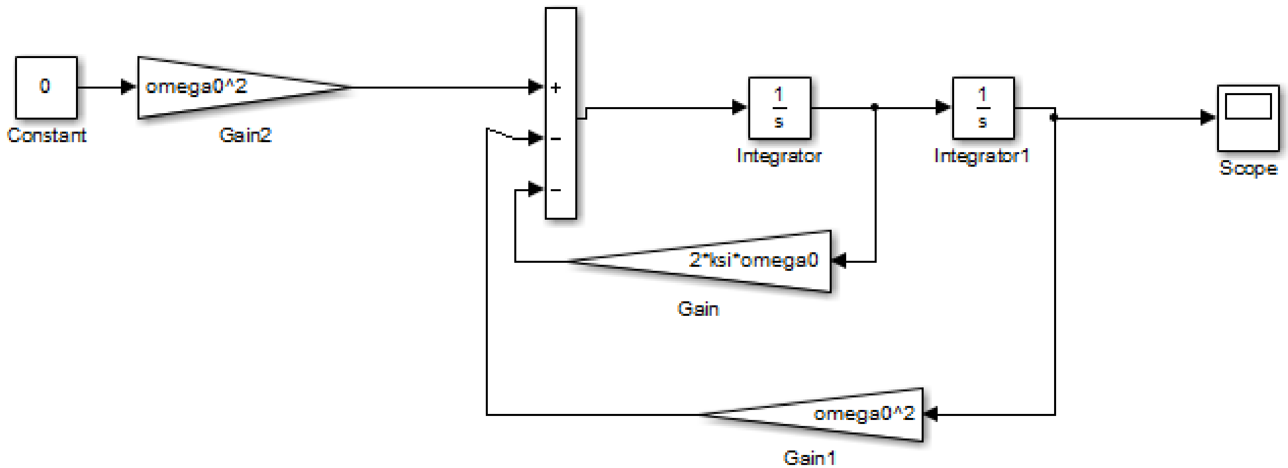
$$2\tau = \frac{a_1}{a_2}, \gamma^2 = \frac{a_0}{a_2}$$

Gdy:

- $\tau^2 > \gamma^2$, jest to **układ przetłumiony**
- $\tau^2 = \gamma^2$, jest to **układ tłumiony krytycznie**
- $\tau^2 < \gamma^2$, jest to **układ niedotłumiony**

https://pl.wikipedia.org/wiki/Uk%C5%82ad_dynamiczny_II_rz%C4%99du

Symulacja



$$x_1(0) = x(0) = 1, \quad x_2(0) = \dot{x}(0) = 1;$$

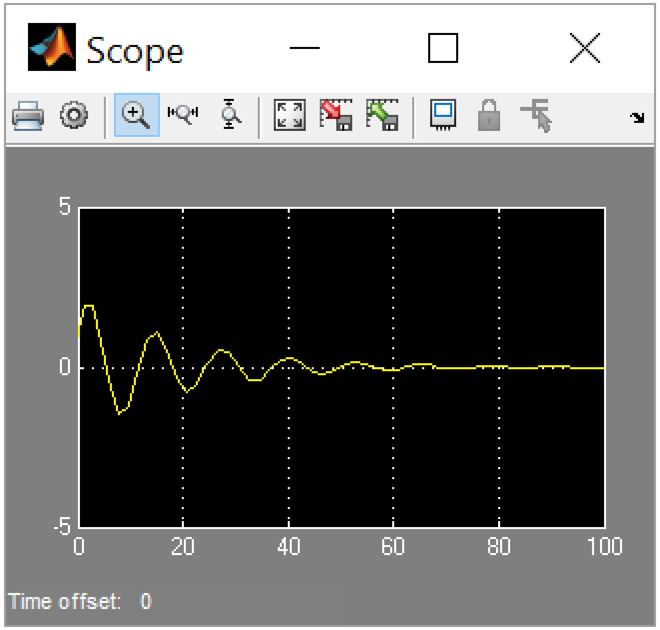
$$x_1(T) = x(T) = 0, \quad x_2(T) = \dot{x}(T) = 0;$$

$$\xi = 0,1;\ \omega_0 = 0,5\ s^{-1};\ T = 5\ s.$$

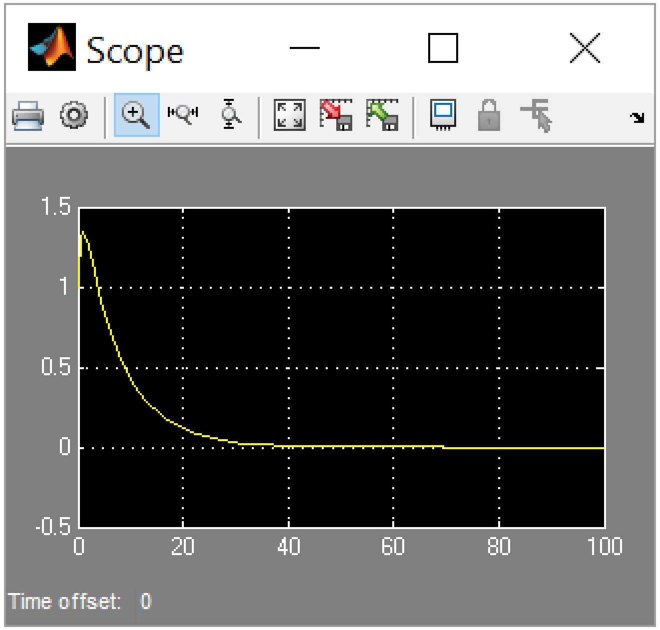
sprezynaKsiOmega.slx

Tłumienie

Ksi=0.1



Ksi=2



Rozwiązanie

$$F = [(\ddot{x} + 2\xi\omega_0\dot{x} + \omega_0^2 x)/\omega_0^2]^2.$$

$$F_x - \frac{d}{dt}F_{\dot{x}} + \frac{d^2}{dt^2}F_{\ddot{x}} + \ldots + (-1)^n \frac{d^n}{dt^n}F_{x^{(n)}} = 0,$$

$$n = 2$$

$$F_x - \frac{d}{dt}F_{\dot{x}} + \frac{d^2}{dt^2}F_{\ddot{x}} = 0.$$

Obliczenia w MatLabie

```
>> syms  ksi  omega0 x  xdot  xdotdot
F=((xdotdot+2*ksi*omega0*xdot+omega0^2*x)/omega0^2)^2;
Fx=diff(F,'x')
Fxdot=diff(F,'xdot')
Fxdotdot=diff(F,'xdotdot')

Fx = 2*(xdotdot+2*ksi*omega0*xdot+omega0^2*x)/omega0^2
 Fxdot = 4*(xdotdot+2*ksi*omega0*xdot+omega0^2*x)/omega0^3*ksi
 Fxdotdot = 2*(xdotdot+2*ksi*omega0*xdot+omega0^2*x)/omega0^4
```

$$F_x = \frac{\partial F}{dx} = 2(\ddot{x} + 2\xi\omega_0\dot{x} + \omega_0^2 x)/\omega_0^2,$$

$$F_{\dot{x}} = \frac{\partial F}{d\dot{x}} = 4\xi(\ddot{x} + 2\xi\omega_0\dot{x} + \omega_0^2 x)/\omega_0^3,$$

$$F_{\ddot{x}} = \frac{\partial F}{d\ \ddot{x}} = 2(\ddot{x} + 2\xi\omega_0\dot{x} + \omega_0^2 x)/\omega_0^4.$$

$$\frac{d}{dt}F_{\dot{x}} = 4\xi(\dddot{x} + 2\xi\omega_0\ddot{x} + \omega_0^2\dot{x})/\omega_0^3;$$

$$\frac{d^2}{dt^2}F_{\ddot{x}} = 2(x^{(4)} + 2\xi\omega_0\dddot{x} + \omega_0^2\ddot{x})/\omega_0^4.$$

Równanie Eulera-Poissona

$$2(\ddot{x}+2\xi\omega_0\dot{x}+\omega_0^2 x)/\omega_0^2-4\xi(\ddot{x}+2\xi\omega_0\ddot{x}+\omega_0^2\dot{x})/\omega_0^3+$$
$$+2(x^{(4)}+2\xi\omega_0\dddot{x}+\omega_0^2\ddot{x})/\omega_0^4=0,$$

lub

$$x^{(4)}+2\omega_0^2(1-2\xi^2)\ddot{x}+\omega_0^4 x=0.$$

```
>> x = dsolve('D4x+2*omega0^2*(1-2*ksi^2)*D2x+omega0^4*x=0');
pretty(x)
```

```
                        2    1/2
  C1 exp((-ksi + (ksi  - 1)   ) omega0 t)
                              2    1/2
       + C2 exp(-(ksi + (ksi  - 1)   ) omega0 t)
                             2    1/2
       + C3 exp((ksi + (ksi  - 1)   ) omega0 t)
                                2   1/2
       + C4 exp(-(-ksi + (ksi  - 1)   ) omega0 t)
```

$$x=C_1 e^{\left(-\xi\omega_0+\omega_0\sqrt{\xi^2-1}\right)t}+C_2 e^{-\left(\xi\omega_0+\omega_0\sqrt{\xi^2-1}\right)t}+$$
$$+C_3 e^{\left(\xi\omega_0+\omega_0\sqrt{\xi^2-1}\right)t}+C_4 e^{-\left(-\xi\omega_0+\omega_0\sqrt{\xi^2-1}\right)t}.$$

Rozwiązanie

$$x = e^{-\xi\omega_0 t}\left(C_1 e^{j\omega_0\sqrt{1-\xi^2}\,t} + C_2 e^{-j\omega_0\sqrt{1-\xi^2}\,t}\right) +$$
$$+ e^{\xi\omega_0 t}\left(C_3 e^{j\omega_0\sqrt{1-\xi^2}\,t} + C_4 e^{-j\omega_0\sqrt{1-\xi^2}\,t}\right).$$

$$\xi < 1$$

$$x = e^{-\xi\omega_0 t}(C_1 \sin\omega_0\sqrt{1-\xi^2}\,t + C_2\cos\omega_0\sqrt{1-\xi^2}\,t) +$$
$$+ e^{\xi\omega_0 t}(C_3 \sin\omega_0\sqrt{1-\xi^2}\,t + C_4\cos\omega_0\sqrt{1-\xi^2}\,t).$$

Uklad2ksiM1.m

Wyniki obliczenia

```
xdot =

-omega0*ksi*exp(-omega0*ksi*t)*(C1*sin(omega0*(1-ksi^2)^(1/2)*t)+C2*cos(omega0*(1-ksi^2)^(1/2)*t))+exp(-
omega0*ksi*t)*(C1*cos(omega0*(1-ksi^2)^(1/2)*t)*omega0*(1-ksi^2)^(1/2)-C2*sin(omega0*(1-
ksi^2)^(1/2)*t)*omega0*(1-ksi^2)^(1/2))+omega0*ksi*exp(omega0*ksi*t)*(C3*sin(omega0*(1-
ksi^2)^(1/2)*t)+C4*cos(omega0*(1-ksi^2)^(1/2)*t))+exp(omega0*ksi*t)*(C3*cos(omega0*(1-
ksi^2)^(1/2)*t)*omega0*(1-ksi^2)^(1/2)-C4*sin(omega0*(1-ksi^2)^(1/2)*t)*omega0*(1-ksi^2)^(1/2))
```

```
xdotdot =

omega0^2*ksi^2*exp(-omega0*ksi*t)*(C1*sin(omega0*(1-ksi^2)^(1/2)*t)+C2*cos(omega0*(1-ksi^2)^(1/2)*t))-
2*omega0*ksi*exp(-omega0*ksi*t)*(C1*cos(omega0*(1-ksi^2)^(1/2)*t)*omega0*(1-ksi^2)^(1/2)-C2*sin(omega0*(1-
ksi^2)^(1/2)*t)*omega0*(1-ksi^2)^(1/2))+exp(-omega0*ksi*t)*(-C1*sin(omega0*(1-ksi^2)^(1/2)*t)*omega0^2*(1-
ksi^2)-C2*cos(omega0*(1-ksi^2)^(1/2)*t)*omega0^2*(1-
ksi^2))+omega0^2*ksi^2*exp(omega0*ksi*t)*(C3*sin(omega0*(1-ksi^2)^(1/2)*t)+C4*cos(omega0*(1-
ksi^2)^(1/2)*t))+2*omega0*ksi*exp(omega0*ksi*t)*(C3*cos(omega0*(1-ksi^2)^(1/2)*t)*omega0*(1-ksi^2)^(1/2)-
C4*sin(omega0*(1-ksi^2)^(1/2)*t)*omega0*(1-ksi^2)^(1/2))+exp(omega0*ksi*t)*(-C3*sin(omega0*(1-
ksi^2)^(1/2)*t)*omega0^2*(1-ksi^2)-C4*cos(omega0*(1-ksi^2)^(1/2)*t)*omega0^2*(1-ksi^2))
```

Wyznaczenie współczynników

$$C_1, C_2, C_3, C_4$$

$$x_1(0) = x(0) = 1, \quad x_2(0) = \dot{x}(0) = 1;$$

$$x_1(T) = x(T) = 0, \quad x_2(T) = \dot{x}(T) = 0;$$

$$\xi = 0{,}1; \ \omega_0 = 0{,}5\ s^{-1}; \ T = 5\ s.$$

```
syms ksi  omega0  C1  C2  C3  C4  t
% wprowadzenie x
x=exp(-ksi*omega0*t)*(C1*sin(omega0*sqrt(1-
ksi^2)*t)+C2*cos(omega0*sqrt(1-
ksi^2)*t))+exp(ksi*omega0*t)*(C3*sin(omega0*sqrt(1-
ksi^2)*t)+C4*cos(omega0*sqrt(1-ksi^2)*t));
xdot=diff(x,'t')
xdotdot= diff(xdot,'t')
```

Układ równań

```
>> t=0;
x0=exp(-ksi*omega0*t)*(C1*sin(omega0*sqrt(1-ksi^2)*t)+C2*cos(omega0*sqrt(1-ksi^2)*t))+exp(ksi*omega0*t)*(C3*sin(omega0*sqrt(1-ksi^2)*t)+C4*cos(omega0*sqrt(1-ksi^2)*t))
```

```
x0 = C2+C4
```

```
xdot0=-omega0*ksi*exp(-omega0*ksi*t)*(C1*sin(omega0*(1-ksi^2)^(1/2)*t)+C2*cos(omega0*(1-ksi^2)^(1/2)*t))+exp(-omega0*ksi*t)*(C1*cos(omega0*(1-ksi^2)^(1/2)*t)*omega0*(1-ksi^2)^(1/2)-C2*sin(omega0*(1-ksi^2)^(1/2)*t)*omega0*(1-ksi^2)^(1/2))+omega0*ksi*exp(omega0*ksi*t)*(C3*sin(omega0*(1-ksi^2)^(1/2)*t)+C4*cos(omega0*(1-ksi^2)^(1/2)*t))+exp(omega0*ksi*t)*(C3*cos(omega0*(1-ksi^2)^(1/2)*t)*omega0*(1-ksi^2)^(1/2)-C4*sin(omega0*(1-ksi^2)^(1/2)*t)*omega0*(1-ksi^2)^(1/2))
```

```
xdot0 = -omega0*ksi*C2+C1*omega0*(1-ksi^2)^(1/2)+omega0*ksi*C4+C3*omega0*(1-ksi^2)^(1/2)
```

```
>>syms  T
xT=exp(-ksi*omega0*T)*(C1*sin(omega0*sqrt(1-ksi^2)*T)+C2*cos(omega0*sqrt(1-ksi^2)*T))+exp(ksi*omega0*T)*(C3*sin(omega0*sqrt(1-ksi^2)*T)+C4*cos(omega0*sqrt(1-ksi^2)*T));
```

```
>>xdotT=-omega0*ksi*exp(-omega0*ksi*T)*(C1*sin(omega0*(1-ksi^2)^(1/2)*T)+C2*cos(omega0*(1-ksi^2)^(1/2)*T))+exp(-omega0*ksi*T)*(C1*cos(omega0*(1-ksi^2)^(1/2)*T)*omega0*(1-ksi^2)^(1/2)-C2*sin(omega0*(1-ksi^2)^(1/2)*T)*omega0*(1-ksi^2)^(1/2))+omega0*ksi*exp(omega0*ksi*T)*(C3*sin(omega0*(1-ksi^2)^(1/2)*T)+C4*cos(omega0*(1-ksi^2)^(1/2)*T))+exp(omega0*ksi*T)*(C3*cos(omega0*(1-ksi^2)^(1/2)*T)*omega0*(1-ksi^2)^(1/2)-C4*sin(omega0*(1-ksi^2)^(1/2)*T)*omega0*(1-ksi^2)^(1/2))
```

Rozwiązanie systemu równań

```
T=5;
ksi=0.1; omega0=0.5;
x0=1;  xdot0=1;  xT=0;  xdotT=0;
   eq1= C2+C4-x0;
   eq2=-omega0*ksi*C2+C1*omega0*(1-ksi^2)^(1/2)+omega0*ksi*C4+C3*omega0*(1-ksi^2)^(1/2)-xdot0;
   eq3=exp(-ksi*omega0*T)*(C1*sin(omega0*sqrt(1-ksi^2)*T)+C2*cos(omega0*sqrt(1-
ksi^2)*T))+exp(ksi*omega0*T)*(C3*sin(omega0*sqrt(1-ksi^2)*T)+C4*cos(omega0*sqrt(1-ksi^2)*T))-xT;
   eq4=-omega0*ksi*exp(-omega0*ksi*T)*(C1*sin(omega0*(1-ksi^2)^(1/2)*T)+C2*cos(omega0*(1-
ksi^2)^(1/2)*T))+exp(-omega0*ksi*T)*(C1*cos(omega0*(1-ksi^2)^(1/2)*T)*omega0*(1-ksi^2)^(1/2)-
C2*sin(omega0*(1-ksi^2)^(1/2)*T)*omega0*(1-ksi^2)^(1/2))+omega0*ksi*exp(omega0*ksi*T)*(C3*sin(omega0*(1-
ksi^2)^(1/2)*T)+C4*cos(omega0*(1-ksi^2)^(1/2)*T))+exp(omega0*ksi*T)*(C3*cos(omega0*(1-
ksi^2)^(1/2)*T)*omega0*(1-ksi^2)^(1/2)-C4*sin(omega0*(1-ksi^2)^(1/2)*T)*omega0*(1-ksi^2)^(1/2))- xdotT;
```

```
 [C1,C2,C3,C4]= solve(eq1,  eq2,  eq3, eq4)
C1 =
    6.7790
C2 =
    2.8872
C3 =
   -4.2890
C4 =
   -1.8872
```

Symulacja

```
T=5;
ksi=0.1; omega0=0.5; C1 =6.7790; C2 =2.8872; C3 =-4.2890;
C4 =-1.8872;
t=0:0.01:5;
   x=exp(-ksi*omega0*t).*(C1*sin(omega0*sqrt(1-ksi^2)*t)+C2*cos(omega0*sqrt(1-
ksi^2)*t))+exp(ksi*omega0*t).*(C3*sin(omega0*sqrt(1-ksi^2)*t)+C4*cos(omega0*sqrt(1-ksi^2)*t));
   xdot=-omega0*ksi*exp(-omega0*ksi*t).*(C1*sin(omega0*(1-ksi^2)^(1/2)*t)+C2*cos(omega0*(1-
ksi^2)^(1/2)*t))+exp(-omega0*ksi*t).*(C1*cos(omega0*(1-ksi^2)^(1/2)*t)*omega0*(1-ksi^2)^(1/2)-
C2*sin(omega0*(1-ksi^2)^(1/2)*t)*omega0*(1-ksi^2)^(1/2))+omega0*ksi*exp(omega0*ksi*t).*(C3*sin(omega0*(1-
ksi^2)^(1/2)*t)+C4*cos(omega0*(1-ksi^2)^(1/2)*t))+exp(omega0*ksi*t).*(C3*cos(omega0*(1-
ksi^2)^(1/2)*t)*omega0*(1-ksi^2)^(1/2)-C4*sin(omega0*(1-ksi^2)^(1/2)*t)*omega0*(1-ksi^2)^(1/2));
   xdotdot=omega0^2*ksi^2*exp(-omega0*ksi*t).*(C1*sin(omega0*(1-ksi^2)^(1/2)*t)+C2*cos(omega0*(1-
ksi^2)^(1/2)*t))-2*omega0*ksi*exp(-omega0*ksi*t).*(C1*cos(omega0*(1-ksi^2)^(1/2)*t)*omega0*(1-ksi^2)^(1/2)-
C2*sin(omega0*(1-ksi^2)^(1/2)*t)*omega0*(1-ksi^2)^(1/2))+exp(-omega0*ksi*t).*(-C1*sin(omega0*(1-
ksi^2)^(1/2)*t)*omega0^2*(1-ksi^2)-C2*cos(omega0*(1-ksi^2)^(1/2)*t)*omega0^2*(1-
ksi^2))+omega0^2*ksi^2*exp(omega0*ksi*t).*(C3*sin(omega0*(1-ksi^2)^(1/2)*t)+C4*cos(omega0*(1-
ksi^2)^(1/2)*t))+2*omega0*ksi*exp(omega0*ksi*t).*(C3*cos(omega0*(1-ksi^2)^(1/2)*t)*omega0*(1-ksi^2)^(1/2)-
C4*sin(omega0*(1-ksi^2)^(1/2)*t)*omega0*(1-ksi^2)^(1/2))+exp(omega0*ksi*t).*(-C3*sin(omega0*(1-
ksi^2)^(1/2)*t)*omega0^2*(1-ksi^2)-C4*cos(omega0*(1-ksi^2)^(1/2)*t)*omega0^2*(1-ksi^2));
```

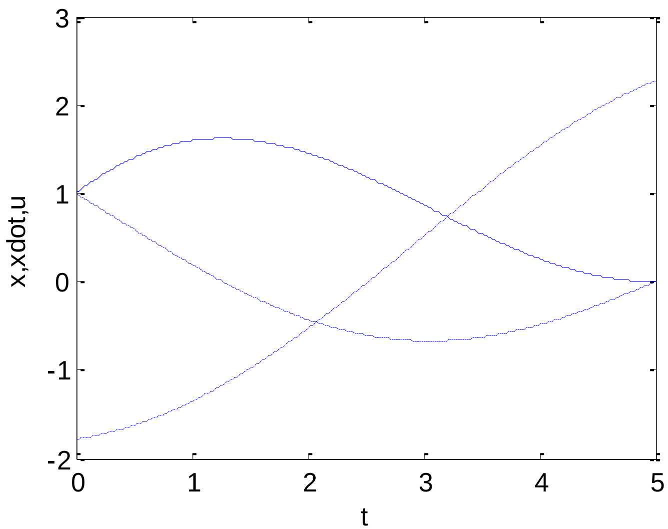
,

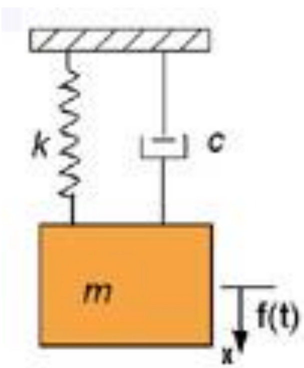
Wyznaczenie sterowania

$$u(t) = (\ddot{x} + 2\xi\omega_0\dot{x} + \omega_0^2 x)/\omega_0^2.$$

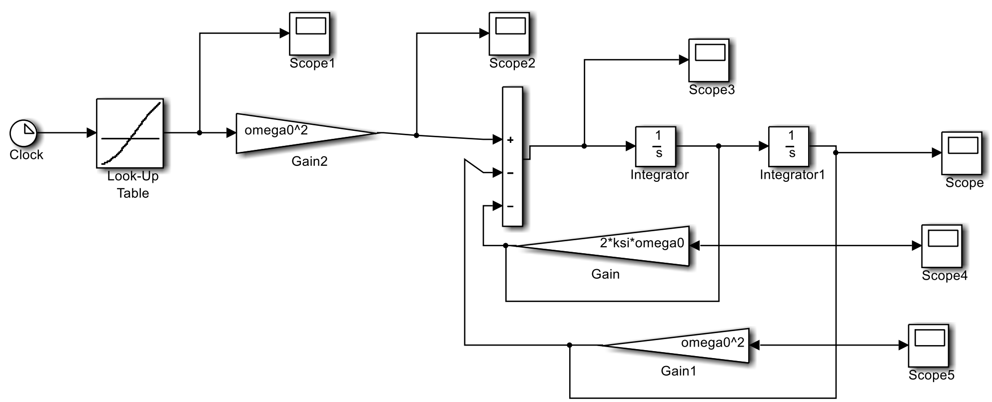
$\omega_0 = 0{,}5$ $\xi = 0{,}1$

```
u=(xdotdot+2*ksi*omega0*xdot+omega0^
2*x) /omega0^2;
plot(t,x)
hold on
plot(t,xdot, '--')
hold on
plot(t,u, '-.')
xlabel('t'), ylabel('x,xdot,u')
```





Symulacja



sprezynaKsiOmegaSterowanie.slx

Sterowanie optymalne