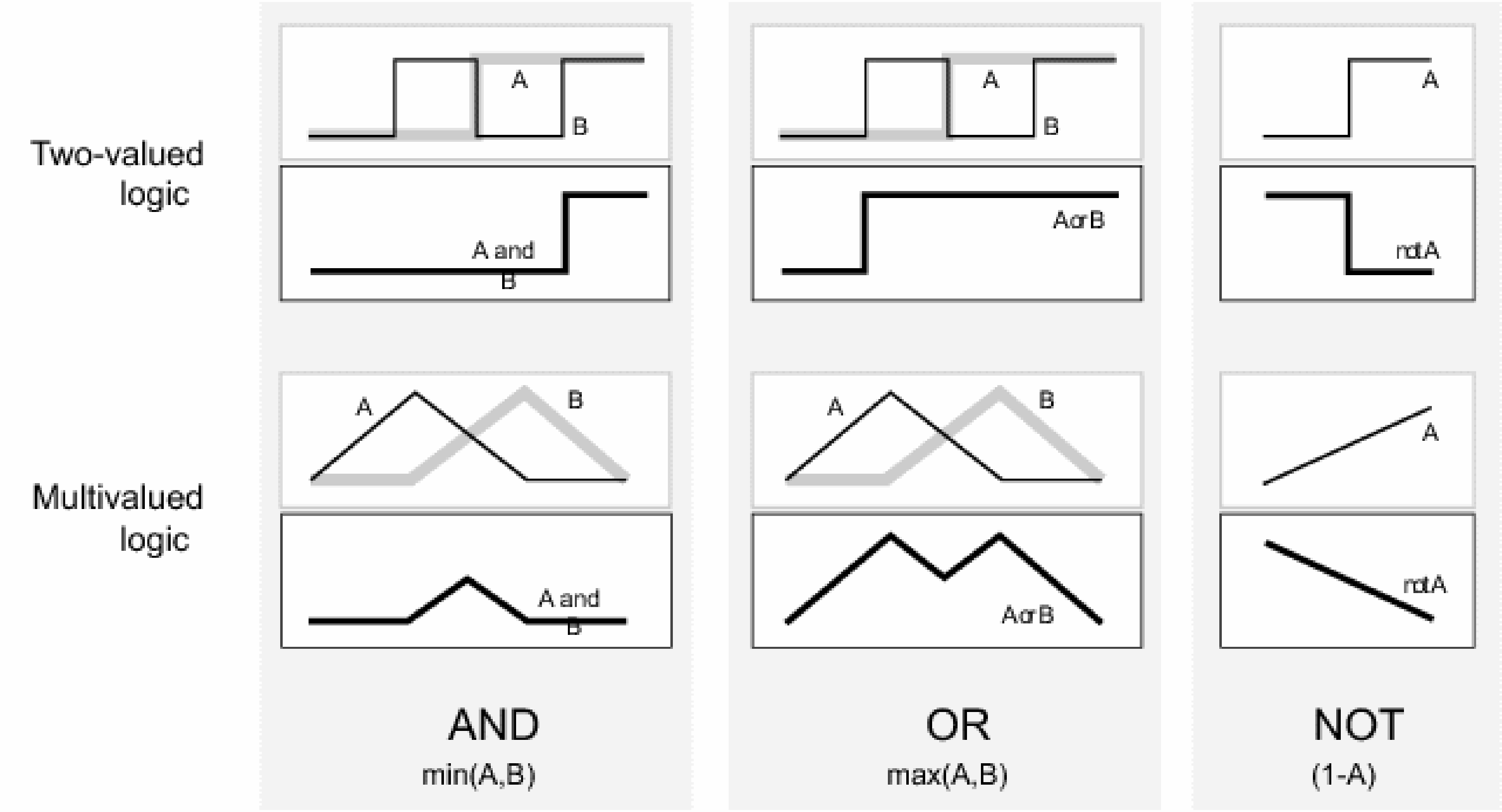
Logika rozmyta

Logika rozmyta, jedna z logik wielowartościowych, stanowi uogólnienie **klasycznej dwuwartościowej logiki**. Jest ściśle powiązana z teorią zbiorów rozmytych. W logice rozmytej między stanem **0 (fałsz)** a stanem **1 (prawda)** rozciąga się szereg wartości pośrednich, które określają stopień przynależności elementu do zbioru.



Dwuwartościowa logika

Macierz wszystkich funkcji dla alfabetu z ***k***

elementów ma $k^{(k^n)}$ wiersze.

(n – ilość zmiennych)

n=1 k=2

x	0	1	y
y_0	0	0	0
y_1	0	1	x
y_2	1	0	$\bar{x}$
y_3	1	1	1

Dwuwartościowa logika 2 zmiennych

x_1 x_2	0 0 1 1 0 1 0 1	
y_0	0 0 0 0	0
y_1	0 0 0 1	$x_1 x_2;\ x_1 \wedge x_2$ $(x_1 \,\&\, x_2;\ x_1 \cap x_2)$
y_2	0 0 1 0	$x_1 \leftarrow x_2$ $(x_1 \supset x_2;\ x_1 \setminus x_2)$
y_3	0 0 1 1	x_1
y_4	0 1 0 0	$x_2 \leftarrow x_1$ $(x_1 \not\subset x_2;\ x_2 \setminus x_1)$
y_5	0 1 0 1	x_2
y_6	0 1 1 0	$x_1 + x_2$ $(x_1 \bigtriangledown x_2;\ x_1 \oplus x_2)$
y_7	0 1 1 1	$x_1 \vee x_2$ $(x_1 + x_2;\ x_1 \cup x_2)$
y_8	1 0 0 0	$x_1 \downarrow x_2$ $(x_1 \overline{\vee} x_2;\ x_1 \circ x_2)$
y_9	1 0 0 1	$x_1 \sim x_2$ $(x_1 \equiv x_2;\ x_1 \leftrightarrow x_2)$
y_{10}	1 0 1 0	$\bar{x}_2$ $(x_2';\ \sim x_2;\ \neg x_2)$
y_{11}	1 0 1 1	$x_2 \rightarrow x_1$ $(x_1 \subset x_2;\ x_1 < x_2)$
y_{12}	1 1 0 0	$\bar{x}_1$ $(x_1';\ \sim x_1;\ \neg x_1)$
y_{13}	1 1 0 1	$x_1 \rightarrow x_2$ $(x_1 \supset x_2;\ x_1 > x_2)$
y_{14}	1 1 1 0	x_1 / x_2 $(x_1 \overline{\wedge} x_2;\ x_1 \overline{\&} x_2)$
y_{15}	1 1 1 1	1

k=2 n=2

$$k^{(k^n)}$$

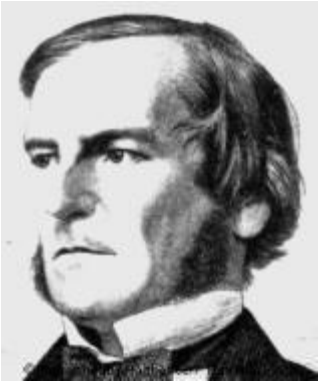
Wielowartościowa logika

$k = 3 \quad n = 2$

x_1	0	0	0	1	1	1	2	2	2
x_2	0	1	2	0	1	2	0	1	2
y_0	0	0	0	0	0	0	0	0	0
y_1	0	0	0	0	0	0	0	0	1
y_2	0	0	0	0	0	0	0	0	2
...	.	.	.	.	.	.	.	.	.
y_{2361}	0	1	0	0	1	2	2	0	1
...	.	.	.	.	.	.	.	.	.
y_{19682}	2	2	2	2	2	2	2	2	2

Operacji logiki dwuwartościowej

$a,b \in \{0,1\}$



matematyk angielski George Boole, 1815 -1864

Negacja – zaprzeczenie logiczne

Negacja	
a	NOT a
0	1
1	0

Alternatywa – suma logiczna

Alternatywa		
a	b	a OR b
0	0	0
0	1	1
1	0	1
1	1	1

Koniunkcja – iloczyn logiczny

Koniunkcja		
a	b	a AND b
0	0	0
0	1	0
1	0	0
1	1	1

Różnica symetryczna – suma modulo dwa

Różnica symetryczna		
a	b	a XOR b
0	0	0
0	1	1
1	0	1
1	1	0

Implikacja materialna

P	Q	$P \Rightarrow Q$
0	0	1
0	1	1
1	0	0
1	1	1

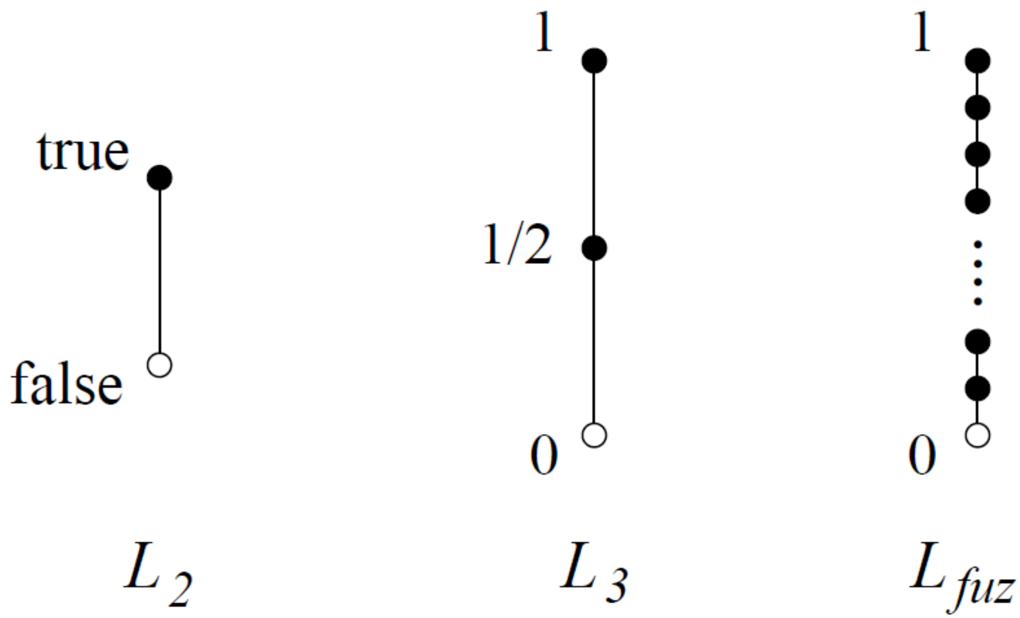
Implikacja materialna — zdanie logiczne powstałe przez połączenie dwóch zdań **P** (poprzednik implikacji) i **Q** (następnik implikacji) spójnikiem implikacji ⇒.

Sprawdzanie zdań

$$(p \vee q) \wedge r \Rightarrow p \vee (q \wedge r)$$

p	q	r	$p \vee q$	$\overbrace{(p \vee q) \wedge r}^{P}$	$q \wedge r$	$\overbrace{p \vee (q \wedge r)}^{L}$	$L \Rightarrow P$
0	0	0	0	0	0	0	1
0	0	1	0	0	0	0	1
0	1	0	1	0	0	0	1
0	1	1	1	1	1	1	1
1	0	0	1	0	0	1	0
1	0	1	1	1	0	1	1
1	1	0	1	0	0	1	0
1	1	1	1	1	1	1	1

Wyrażenie nie jest tautologią


true
false
1
1/2
0
1
0
L_2
L_3
L_{fuz}

Operacji logiki rozmytej

a,b∈[0,1]

Operacja	KD-logika	Ł-logika
⌉ a	1-a	1-a
a&b	min(a,b)	max(0,a+b-1)
a∨b	max(a,b)	min(1,a+b)
a→b	(1-a)∨b = max(1-a,b)	min(1,b+1-a)
a↔b	min(max(1-a,b),max(1-b,a))	max(0,min(1,b+1-a) +min(1,a+1-b)-1)

Formuła rozmyta

$$x, x_1, x_2 \longrightarrow [0,\ 1]$$

$$\neg x = 1 - x$$

$$x_1 \wedge x_2 = \min(\mathrm{x}_1, \mathrm{x}_2) \quad x_1 \vee x_2 = \max(x_1, x_2)$$

$$(\{0,\ 1\}, \vee, \wedge, \neg,) \longrightarrow ([0,\ 1], \vee, \wedge, \neg)$$

$$\forall x \in]0, 1[, \ x \wedge (\neg x) \neq 0, \ x \vee (\neg x) \neq 1$$

Zdanie logiczne rozmyte

Funkcja $[0, 1]^n \longrightarrow [0, 1]$

(i) $0, 1, x_i, i = 1, n$ zdanie

(ii) zdanie $f \longrightarrow \neg f$ zdanie

(iii) zdania $f \quad g \longrightarrow f \wedge g \quad f \vee g$ zdanie

Sprawdzanie zdań rozmytych

$$x_i \vee (\neg x_i) \geqslant \frac{1}{2}$$

$$\neg x = 1 - x$$

Zdanie sprzeczne

$$P(a) \leqslant \frac{1}{2} \quad \forall a \in [0,\ 1]^n$$

$$P \wedge (x_i \vee (\neg x_i)) = P = [P \wedge x_i] \vee [P \wedge (\neg x_i)]$$

Twierdzenie

Jeśli

$$f(a) = g(a) \ \forall a \in \{0, \frac{1}{2}, 1\}^n$$

to

$$f(a) = g(a) \ \forall a \in [0, 1]^n$$

$$\underset{\sim}{a} = \mu_{\underset{\sim}{\mathbf{A}}}(x),\ \underset{\sim}{b} = \mu_{\underset{\sim}{\mathbf{B}}}(x), \ldots;\ \underset{\sim}{a}, \underset{\sim}{b}, \ldots \in \mathbf{M} = [0, 1]$$

$$\underset{\sim}{a} \wedge \underset{\sim}{b} = \mathrm{MIN}\,(\underset{\sim}{a}, \underset{\sim}{b}),$$

$$\underset{\sim}{a} \vee \underset{\sim}{b} = \mathrm{MAX}\,(\underset{\sim}{a}, \underset{\sim}{b}),$$

$$\overline{\underset{\sim}{a}} = 1 - \underset{\sim}{a},$$

$$\underset{\sim}{a} \oplus \underset{\sim}{b} = (\overline{\underset{\sim}{a}} \wedge \underset{\sim}{b}) \vee (\underset{\sim}{a} \wedge \overline{\underset{\sim}{b}})$$

Właściwości

$$\underset{\sim}{a} \wedge 0 = 0,$$
$$\underset{\sim}{a} \vee 0 = \underset{\sim}{a},$$
$$\underset{\sim}{a} \wedge 1 = \underset{\sim}{a},$$
$$\underset{\sim}{a} \vee 1 = \underset{\sim}{1},$$
$$\overline{(\overline{\underset{\sim}{a}})} = \underset{\sim}{a},$$

$$\underset{\sim}{a} \wedge \underset{\sim}{b} = \underset{\sim}{b} \wedge \underset{\sim}{a},$$
$$\underset{\sim}{a} \vee \underset{\sim}{b} = \underset{\sim}{b} \vee \underset{\sim}{a},$$

$$(\underset{\sim}{a} \vee \underset{\sim}{b}) \wedge \underset{\sim}{c} = \underset{\sim}{a} \wedge (\underset{\sim}{b} \wedge \underset{\sim}{c}),$$
$$(\underset{\sim}{a} \vee \underset{\sim}{b}) \vee \underset{\sim}{c} = \underset{\sim}{a} \vee (\underset{\sim}{b} \vee \underset{\sim}{c}),$$

$$\underset{\sim}{a} \wedge \underset{\sim}{a} = \underset{\sim}{a},$$
$$\underset{\sim}{a} \vee \underset{\sim}{a} = \underset{\sim}{a},$$

$$\underset{\sim}{a} \wedge \overline{\underset{\sim}{a}} \neq 0,$$
$$\underset{\sim}{a} \vee \overline{\underset{\sim}{a}} \neq 1,$$

$$\underset{\sim}{a} \wedge (\underset{\sim}{b} \vee \underset{\sim}{c}) = (\underset{\sim}{a} \wedge \underset{\sim}{b}) \vee (\underset{\sim}{a} \wedge \underset{\sim}{c}),$$
$$\underset{\sim}{a} \vee (\underset{\sim}{b} \wedge \underset{\sim}{c}) = (\underset{\sim}{a} \vee \underset{\sim}{b}) \wedge (\underset{\sim}{d} \vee \underset{\sim}{c}),$$

$$\overline{\underset{\sim}{a} \wedge \underset{\sim}{b}} = \overline{\underset{\sim}{a}} \vee \overline{\underset{\sim}{b}},$$
$$\overline{\underset{\sim}{a} \vee \underset{\sim}{b}} = \overline{\underset{\sim}{a}} \wedge \overline{\underset{\sim}{b}}.$$

Twierdzenie

$$\underset{\sim}{a} \wedge (\underset{\sim}{b} \vee \underset{\sim}{c}) = (\underset{\sim}{a} \wedge \underset{\sim}{b}) \vee (\underset{\sim}{a} \wedge \underset{\sim}{c})$$

Dowód

$$0 \leqslant \underset{\sim}{a} \leqslant \underset{\sim}{b} \leqslant \underset{\sim}{c} \leqslant 1$$

$$\underset{\sim}{a} \wedge (\underset{\sim}{b} \vee \underset{\sim}{c}) = \mathrm{MIN}\,[\underset{\sim}{a},\ \mathrm{MAX}\,(\underset{\sim}{b},\ \underset{\sim}{c})\,] = \mathrm{MIN}\,(\underset{\sim}{a},\ \underset{\sim}{c}) = \underset{\sim}{a},$$

$$(\underset{\sim}{a} \wedge \underset{\sim}{b}) \vee (\underset{\sim}{a} \wedge \underset{\sim}{c}) = \mathrm{MAX}\,[\mathrm{MIN}\,(\underset{\sim}{a},\ \underset{\sim}{b}),\ \mathrm{MIN}\,(\underset{\sim}{a},\ \underset{\sim}{c})\,] =$$

$$= \mathrm{MAX}\,(\underset{\sim}{a},\ \underset{\sim}{a}) = \underset{\sim}{a}.$$

Funkcje logiczne zmiennych rozmytych

$$\underset{\sim}{f}(\underset{\sim}{a}, \underset{\sim}{b}, \ldots) \qquad 0 \leqslant \underset{\sim}{f} \leqslant 1$$

Uproszczenie

$$\underset{\sim}{a} \wedge \overline{\underset{\sim}{a}} \neq 0,$$
$$\underset{\sim}{a} \vee \overline{\underset{\sim}{a}} \neq 1,$$

$$\underset{\sim}{f}(\underset{\sim}{a}, \underset{\sim}{b}) = \underset{\sim}{a} \vee (\underset{\sim}{a} \wedge \underset{\sim}{b}) = (\underset{\sim}{a} \wedge 1) \vee (\underset{\sim}{a} \wedge \underset{\sim}{b}) = \underset{\sim}{a} \wedge (1 \vee \underset{\sim}{b})$$

$$= \underset{\sim}{a} \wedge 1$$

$$= \underset{\sim}{a}$$

$$\underset{\sim}{a} \wedge (\underset{\sim}{b} \vee \underset{\sim}{c}) = (\underset{\sim}{a} \wedge \underset{\sim}{b}) \vee (\underset{\sim}{a} \wedge \underset{\sim}{c})$$

Reguła pochłaniania

$$\underset{\sim}{a} \vee (\underset{\sim}{a} \wedge \underset{\sim}{b}) = \underset{\sim}{a}.$$

$$\underset{\sim}{a} \wedge (\underset{\sim}{a} \vee \underset{\sim}{b}) = \underset{\sim}{a}.$$

Przykład

$$\underset{\sim}{f}(\underset{\sim}{a}, \underset{\sim}{b}, \underset{\sim}{c}) = (\underset{\sim}{a} \wedge \underset{\sim}{b} \wedge \underset{\sim}{\bar{c}}) \vee [\underset{\sim}{\bar{a}} \wedge (\underset{\sim}{\bar{b}} \vee \underset{\sim}{c})] \vee \underset{\sim}{\bar{a}} \vee (\underset{\sim}{b} \wedge \underset{\sim}{\bar{c}}) =$$

$$= \underbrace{(\underset{\sim}{a} \wedge \underset{\sim}{b} \wedge \underset{\sim}{\bar{c}})}_{(1)} \vee \underbrace{(\underset{\sim}{\bar{a}} \wedge \underset{\sim}{\bar{b}})}_{(2)} \vee \underbrace{(\underset{\sim}{\bar{a}} \wedge \underset{\sim}{c})}_{(3)} \vee \underbrace{\underset{\sim}{\bar{a}}}_{(4)} \vee \underbrace{(\underset{\sim}{b} \wedge \underset{\sim}{\bar{c}})}_{(5)} = (\underset{\sim}{b} \wedge \underset{\sim}{\bar{c}}) \vee \underset{\sim}{\bar{a}}$$

Obliczenia

Jeśli $\underset{\sim}{x} \leqslant \underset{\sim}{y},$ to $\bar{\underset{\sim}{y}} \leqslant \bar{\underset{\sim}{x}}.$

$$\bar{\underset{\sim}{a}} = 1 - \underset{\sim}{a},$$

$\underset{\sim}{a} \leqslant \underset{\sim}{b} \leqslant \bar{\underset{\sim}{b}} \leqslant \bar{\underset{\sim}{a}},$ $\underset{\sim}{a} \leqslant \bar{\underset{\sim}{b}} \leqslant \underset{\sim}{b} \leqslant \bar{\underset{\sim}{a}},$ $\bar{\underset{\sim}{a}} \leqslant \underset{\sim}{b} \leqslant \bar{\underset{\sim}{b}} \leqslant \underset{\sim}{a},$ $\bar{\underset{\sim}{a}} \leqslant \bar{\underset{\sim}{b}} \leqslant \underset{\sim}{b} \leqslant \underset{\sim}{a},$

$\underset{\sim}{b} \leqslant \underset{\sim}{a} \leqslant \bar{\underset{\sim}{a}} \leqslant \bar{\underset{\sim}{b}},$ $\underset{\sim}{b} \leqslant \bar{\underset{\sim}{a}} \leqslant \underset{\sim}{a} \leqslant \bar{\underset{\sim}{b}},$ $\bar{\underset{\sim}{b}} \leqslant \underset{\sim}{a} \leqslant \bar{\underset{\sim}{a}} \leqslant \underset{\sim}{b},$ $\bar{\underset{\sim}{b}} \leqslant \bar{\underset{\sim}{a}} \leqslant \underset{\sim}{a} \leqslant \underset{\sim}{b}.$

$ab\bar{b}\bar{a},$	$ba\bar{a}\bar{b},$
$a\bar{b}b\bar{a},$	$b\bar{a}a\bar{b},$
$\bar{a}b\bar{b}a,$	$\bar{b}aab,$
$\bar{a}\bar{b}ba,$	$\bar{b}\bar{a}ab.$

$$\underset{\sim}{f}(\underset{\sim}{a}, \underset{\sim}{b}) = (\underset{\sim}{a} \wedge \underset{\sim}{\bar{a}}) \vee (\underset{\sim}{\bar{a}} \wedge \underset{\sim}{b} \wedge \underset{\sim}{\bar{b}});$$

	$\leqslant$	$\leqslant$	$\leqslant$	$\underset{\sim}{a} \wedge \underset{\sim}{\bar{a}}$	$\underset{\sim}{\bar{a}} \wedge \underset{\sim}{b} \wedge \underset{\sim}{\bar{b}}$	$(\underset{\sim}{a} \wedge \underset{\sim}{\bar{a}}) \vee (\underset{\sim}{\bar{a}} \wedge \underset{\sim}{b} \wedge \underset{\sim}{\bar{b}})$
$\underset{\sim}{a}$	$\underset{\sim}{b}$	$\underset{\sim}{\bar{b}}$	$\underset{\sim}{\bar{a}}$	$\underset{\sim}{a}$	$\underset{\sim}{b}$	$\underset{\sim}{b}$
$\underset{\sim}{a}$	$\underset{\sim}{\bar{b}}$	$\underset{\sim}{b}$	$\underset{\sim}{\bar{a}}$	$\underset{\sim}{a}$	$\underset{\sim}{\bar{b}}$	$\underset{\sim}{\bar{b}}$
$\underset{\sim}{\bar{a}}$	$\underset{\sim}{b}$	$\underset{\sim}{\bar{b}}$	$\underset{\sim}{a}$	$\underset{\sim}{\bar{a}}$	$\underset{\sim}{\bar{a}}$	$\underset{\sim}{\bar{a}}$
$\underset{\sim}{\bar{a}}$	$\underset{\sim}{\bar{b}}$	$\underset{\sim}{b}$	$\underset{\sim}{a}$	$\underset{\sim}{\bar{a}}$	$\underset{\sim}{\bar{a}}$	$\underset{\sim}{\bar{a}}$
$\underset{\sim}{b}$	$\underset{\sim}{a}$	$\underset{\sim}{\bar{a}}$	$\underset{\sim}{\bar{b}}$	$\underset{\sim}{a}$	$\underset{\sim}{b}$	$\underset{\sim}{a}$
$\underset{\sim}{b}$	$\underset{\sim}{\bar{a}}$	$\underset{\sim}{a}$	$\underset{\sim}{\bar{b}}$	$\underset{\sim}{\bar{a}}$	$\underset{\sim}{b}$	$\underset{\sim}{\bar{a}}$
$\underset{\sim}{\bar{b}}$	$\underset{\sim}{a}$	$\underset{\sim}{\bar{a}}$	$\underset{\sim}{b}$	$\underset{\sim}{a}$	$\underset{\sim}{\bar{b}}$	$\underset{\sim}{a}$
$\underset{\sim}{\bar{b}}$	$\underset{\sim}{\bar{a}}$	$\underset{\sim}{a}$	$\underset{\sim}{b}$	$\underset{\sim}{\bar{a}}$	$\underset{\sim}{\bar{b}}$	$\underset{\sim}{\bar{a}}$

Funkcje f1=f2 jeśli tablicy są jednakowe

Postać wielomianowa

$$f(a, b, c) = (\bar{a} \wedge \bar{b}) \vee (a \wedge b \wedge \bar{c}).$$

$$f(a, b, c) = (\bar{a} \vee a) \wedge (\bar{a} \vee b) \wedge (\bar{a} \wedge \bar{c}) \wedge (\bar{b} \vee a) \wedge (\bar{b} \vee b) \wedge (\bar{b} \vee \bar{c}).$$

$$f(a, b, c) = (a \vee b) \wedge c \wedge (\bar{a} \vee b \vee c) = (a \vee \bar{b}) \wedge c,$$

$$f(a, b, c) = (a \wedge c) \vee (\bar{b} \wedge c),$$

Maksymalny jednomian. Redukowany wielomian.

$$\underset{\sim}{f}(\underset{\sim}{a}, \underset{\sim}{b}) = (\underset{\sim}{a} \wedge \underset{\sim}{\bar{a}}) \vee (\underset{\sim}{a} \wedge \underset{\sim}{b}) \vee (\underset{\sim}{a} \wedge \underset{\sim}{\bar{b}})$$

$$\underset{\sim}{f}(\underset{\sim}{a}, \underset{\sim}{b}) = (\underset{\sim}{a} \wedge \underset{\sim}{b}) \vee (\underset{\sim}{a} \wedge \underset{\sim}{\bar{b}})$$

$$\underset{\sim}{f}(\underset{\sim}{a}, \underset{\sim}{b}, \underset{\sim}{c}) = (\underset{\sim}{\bar{a}} \wedge \underset{\sim}{b} \wedge \underset{\sim}{\bar{c}}) \vee (\underset{\sim}{\bar{b}} \wedge \underset{\sim}{c})$$

$$\underset{\sim}{f}(\underset{\sim}{a}, \underset{\sim}{b}, \underset{\sim}{c}) = (\underset{\sim}{\bar{a}} \vee \underset{\sim}{\bar{b}}) \vee (\underset{\sim}{\bar{a}} \vee \underset{\sim}{c}) \wedge (\underset{\sim}{b} \wedge \underset{\sim}{\bar{b}}) \wedge (\underset{\sim}{b} \vee \underset{\sim}{c}) \wedge (\underset{\sim}{c} \vee \underset{\sim}{\bar{c}}) \wedge (\underset{\sim}{\bar{b}} \vee \underset{\sim}{\bar{c}}).$$

Ilość funkcji $\underset{\sim}{f}(\underset{\sim}{a}, \underset{\sim}{b}, \ldots)$

$\underset{\sim}{a}$, $\bar{\underset{\sim}{a}}$, $\underset{\sim}{a} \wedge \bar{\underset{\sim}{a}}$, $\underset{\sim}{a} \vee \bar{\underset{\sim}{a}}$

$\underset{\sim}{a} \wedge \tilde{\underset{\sim}{a}} = \underset{\sim}{a}$ jeśli $\underset{\sim}{a} \leqslant \bar{\underset{\sim}{a}}$, $\underset{\sim}{a} \wedge \bar{\underset{\sim}{a}} = \bar{\underset{\sim}{a}}$, $\bar{\underset{\sim}{a}} \leqslant \underset{\sim}{a}$.

$\underset{\sim}{f}(\underset{\sim}{a}, \underset{\sim}{b})$

1	a	(1)
2	$\bar{a}$	(2)
3	b	(3)
4	$\bar{b}$	(4)
4		

$1 \wedge 2$	$a \wedge \bar{a}$	(5)
$1 \wedge 3$	$a \wedge b$	(6)
$1 \wedge 4$	$a \wedge \bar{b}$	(7)
$2 \wedge 3$	$\bar{a} \wedge b$	(8)
$2 \wedge 4$	$\bar{a} \wedge \bar{b}$	(9)
$3 \wedge 4$	$b \wedge \bar{b}$	(10)
6		

$1 \wedge 2 \wedge 3$	$a \wedge \bar{a} \wedge b$	(11)
$1 \wedge 2 \wedge 4$	$a \wedge \bar{a} \wedge \bar{b}$	(12)
$1 \wedge 3 \wedge 4$	$a \wedge b \wedge \bar{b}$	(13)
$2 \wedge 3 \wedge 4$	$\bar{a} \wedge b \wedge \bar{b}$	(14)
4		

$1 \wedge 2 \wedge 3 \wedge 4$	$a \wedge \bar{a} \wedge b \wedge \bar{b}$	(15)
1		

$1 \vee 2$	$a \vee \bar{a}$	(16)
$1 \vee 3$	$a \vee b$	(17)
$1 \vee 4$	$a \vee \bar{b}$	(18)
$2 \vee 3$	$\bar{a} \vee b$	(19)
$2 \vee 4$	$\bar{a} \vee \bar{b}$	(20)
$3 \vee 4$	$b \vee \bar{b}$	(21)

6

$1 \vee (2 \wedge 3)$	$a \vee (\bar{a} \wedge b)$	(22)
$1 \vee (2 \wedge 4)$	$a \vee (\bar{a} \wedge \bar{b})$	(23)
$1 \vee (3 \wedge 4)$	$a \vee (b \wedge \bar{b})$	(24)
$2 \vee (1 \wedge 3)$	$\bar{a} \vee (a \wedge b)$	(25)
$2 \vee (1 \wedge 4)$	$\bar{a} \vee (a \wedge \bar{b})$	(26)
$2 \vee (3 \wedge 4)$	$\bar{a} \vee (b \wedge \bar{b})$	(27)
$3 \vee (1 \wedge 2)$	$b \vee (a \wedge \bar{a})$	(28)
$3 \vee (1 \wedge 4)$	$b \vee (a \wedge \bar{b})$	(29)
$3 \vee (2 \wedge 4)$	$b \vee (\bar{a} \wedge \bar{b})$	(30)
$4 \vee (1 \wedge 2)$	$\bar{b} \vee (a \wedge \bar{a})$	(31)
$4 \vee (1 \wedge 3)$	$\bar{b} \vee (a \wedge b)$	(32)
$4 \vee (2 \wedge 3)$	$\bar{b} \vee (\bar{a} \wedge b)$	(33)

$$
\left.\begin{array}{l}
1 \vee 2 \vee 3 \\
1 \vee 2 \vee 4 \\
1 \vee 3 \vee 4 \\
2 \vee 3 \vee 4
\end{array}\right|_{4}
\begin{array}{ll}
a \vee \bar{a} \vee b & (71) \\
a \vee \bar{a} \vee \bar{b} & (72) \\
a \vee b \vee \bar{b} & (73) \\
\bar{a} \vee b \vee \bar{b} & (74)
\end{array}
$$

$$
\left.\begin{array}{l}
1 \vee 2 \vee (3 \wedge 4) \\
1 \vee 3 \vee (2 \wedge 4) \\
1 \vee 4 \vee (2 \wedge 3) \\
2 \vee 3 \vee (1 \wedge 4) \\
2 \vee 4 \vee (1 \wedge 3) \\
3 \vee 4 \vee (1 \wedge 2)
\end{array}\right|_{6}
\begin{array}{ll}
a \vee \bar{a} \vee (b \wedge \bar{b}) & (75) \\
a \vee b \vee (\bar{a} \wedge \bar{b}) & (76) \\
a \vee \bar{b} \vee (\bar{a} \wedge b) & (77) \\
\bar{a} \vee b \vee (a \wedge \bar{b}) & (78) \\
\bar{a} \vee \bar{b} \vee (a \wedge b) & (79) \\
b \vee \bar{b} \vee (a \wedge \bar{a}) & (80)
\end{array}
$$

$1 \vee (2 \wedge 3) \vee (2 \wedge 4) \vee \vee (3 \wedge 4)$	$a \wedge (\bar{a} \wedge b) \vee (\bar{a} \wedge \bar{b}) \vee \vee (b \wedge \bar{b})$
$2 \vee (1 \wedge 3) \vee (1 \wedge 4) \vee \vee (3 \wedge 4)$	$\bar{a} \wedge (a \wedge b) \vee (a \wedge \bar{b}) \vee \vee (b \wedge \bar{b})$
$3 \vee (1 \wedge 2) \vee (1 \wedge 4) \vee \vee (2 \wedge 4)$	$b \wedge (a \wedge \bar{a}) \vee (a \wedge \bar{b}) \vee \vee (\bar{a} \wedge \bar{b})$
$4 \vee (1 \wedge 2) \vee (1 \wedge 3) \vee \vee (2 \wedge 3)$	$\bar{b} \wedge (a \wedge \bar{a}) \vee (a \wedge b) \vee \vee (\bar{a} \wedge b)$

4

$(1 \wedge 2) \vee (1 \wedge 3) \vee (1 \wedge 4) \vee (2 \wedge 3) \vee (2 \wedge 4)$	$(a \wedge \bar{a}) \vee (a \wedge b) \vee (a \wedge \bar{b}) \vee (\bar{a} \wedge b) \vee (\bar{a} \wedge \bar{b})$
$(1 \wedge 2) \vee (1 \wedge 3) \vee (1 \wedge 4) \vee (2 \wedge 3) \vee (3 \wedge 4)$	$(a \wedge \bar{a}) \vee (a \wedge b) \vee (a \wedge \bar{b}) \vee (\bar{a} \wedge b) \vee (b \wedge \bar{b})$
$(1 \wedge 2) \vee (1 \wedge 3) \vee (1 \wedge 4) \vee (2 \wedge 4) \vee (3 \wedge 4)$	$(a \wedge \bar{a}) \vee (a \wedge b) \vee (a \wedge \bar{b}) \vee (\bar{a} \wedge \bar{b}) \vee (b \wedge \bar{b})$
$(1 \wedge 2) \vee (1 \wedge 3) \vee (2 \wedge 3) \vee (2 \wedge 4) \vee (3 \wedge 4)$	$(a \wedge \bar{a}) \vee (a \wedge b) \vee (\bar{a} \wedge b) \vee (\bar{a} \wedge \bar{b}) \vee (b \wedge \bar{b})$

$$
\left.\begin{array}{l}(1 \wedge 2) \vee (1 \wedge 3) \vee (1 \wedge 4) \vee \\ \vee (2 \wedge 3) \vee (2 \wedge 4) \vee (3 \wedge 4)\end{array}\right|_{1} \begin{array}{l}(a \wedge \bar{a}) \vee (a \wedge b) \vee (a \wedge \bar{b}) \vee \\ \vee (\bar{a} \wedge b) \vee (\bar{a} \wedge \bar{b}) \vee (b \wedge \bar{b})\end{array}
$$

15	1
55	2
64	3
25	4
6	5
1	6
166	

Analiza funkcji logicznych

$$\mathbf{M} = [0,1]$$

$$\mathbf{I}_1 = [\alpha_0 = 0,\ \alpha_1 [,\ \ \mathbf{I}_2 = [\alpha_1,\ \alpha_2 [,\ \ \ldots,\ \ \mathbf{I}_m = [\alpha_{m-1},\ \alpha_m = 1]$$

$f(\underset{\sim}{a}_1,\ \underset{\sim}{a}_2,\ \ldots,\ \underset{\sim}{a}_n),\quad \underset{\sim}{a}_i \in [0,1],\ i = 1,2,\ldots,n,$ należy do $\mathbf{I}_k$

Przykład

$$\underset{\sim}{f}(\underset{\sim}{a},\ \underset{\sim}{b},\ \underset{\sim}{c}) = (\overline{\underset{\sim}{a}} \wedge \overline{\underset{\sim}{b}}) \vee (\underset{\sim}{a} \wedge \underset{\sim}{b} \wedge \overline{\underset{\sim}{c}})$$

$$\underset{\sim}{f}(\underset{\sim}{a},\ \underset{\sim}{b},\ \underset{\sim}{c}) \in \mathbf{I}_k,$$

$$\alpha_{k-1} \leqslant \underset{\sim}{f}(\underset{\sim}{a},\ \underset{\sim}{b},\ \underset{\sim}{c}) < \alpha_k$$

Hipoteza 1

$$\underset{\sim}{f}(\underset{\sim}{a}, \underset{\sim}{b}, \underset{\sim}{c}) = (\overline{\underset{\sim}{a}} \wedge \overline{\underset{\sim}{b}}) \vee (\underset{\sim}{a} \wedge \underset{\sim}{b} \wedge \overline{\underset{\sim}{c}})$$

>=

$$\overline{\underset{\sim}{a}} \wedge \overline{\underset{\sim}{b}} \geqslant \underset{\sim}{a} \wedge \underset{\sim}{b} \wedge \overline{\underset{\sim}{c}}. \Rightarrow \alpha_{k-1} \leqslant \overline{\underset{\sim}{a}} \wedge \overline{\underset{\sim}{b}} < \alpha_k \Rightarrow$$

$$\Rightarrow \alpha_{k-1} \leqslant \mathrm{MIN}\,(\overline{\underset{\sim}{a}}, \overline{\underset{\sim}{b}}) < \alpha_k \Rightarrow \alpha_{k-1} \leqslant \mathrm{MIN}\,(1-\underset{\sim}{a},\ 1-\underset{\sim}{b}) < \alpha_k.$$

$1-\underset{\sim}{a} \geqslant \alpha_{k-1}$ i $1-\underset{\sim}{b} \geqslant \alpha_{k-1}$ i $1-\underset{\sim}{a} < \alpha_k$ lub $1-\underset{\sim}{b} < \alpha_k$.

Hipoteza 2

$$\underset{\sim}{f}\left(\underset{\sim}{a}, \underset{\sim}{b}, \underset{\sim}{c}\right) = \left(\overline{\underset{\sim}{a}} \wedge \overline{\underset{\sim}{b}}\right) \vee \left(\underset{\sim}{a} \wedge \underset{\sim}{b} \wedge \overline{\underset{\sim}{c}}\right)$$

<

$$\overline{\underset{\sim}{a}} \wedge \overline{\underset{\sim}{b}} < \underset{\sim}{a} \wedge \underset{\sim}{b} \wedge \overline{\underset{\sim}{c}}. \Rightarrow \alpha_{k-1} \leqslant \underset{\sim}{a} \wedge \underset{\sim}{b} \wedge \overline{\underset{\sim}{c}} < \alpha_k \Rightarrow$$

$$\Rightarrow \alpha_{k-1} \leqslant \mathrm{MIN}\left(\underset{\sim}{a}, \underset{\sim}{b}, \overline{\underset{\sim}{c}}\right) < \alpha_k \Rightarrow \alpha_{k-1} \leqslant \mathrm{MIN}\left(\underset{\sim}{a}, \underset{\sim}{b}, 1 - \underset{\sim}{c}\right) < \alpha_k$$

$\underset{\sim}{a} \geqslant \alpha_{k-1}$ i $\underset{\sim}{b} \geqslant \alpha_{k-1}$ i $1 - \underset{\sim}{c} \geqslant \alpha_{k-1}$ i

$\underset{\sim}{a} < \alpha_k$ lub $\underset{\sim}{b} < \alpha_k$ lub $\underset{\sim}{c} > 1 - \alpha_k$

Przykład1

$\underset{\sim}{a} = 0{,}55,\ \underset{\sim}{b} = 0{,}57,\ \underset{\sim}{c} = 0{,}80.$

$$\underset{\sim}{f}\left(\underset{\sim}{a},\ \underset{\sim}{b},\ \underset{\sim}{c}\right) = \left(\bar{\underset{\sim}{a}} \wedge \bar{\underset{\sim}{b}}\right) \vee \left(\underset{\sim}{a} \wedge \underset{\sim}{b} \wedge \bar{\underset{\sim}{c}}\right)$$

$$\begin{aligned}
\underset{\sim}{f}(\underset{\sim}{a},\ \underset{\sim}{b},\ \underset{\sim}{c}) &= \underset{\sim}{f}(0{,}55;\ 0{,}57;\ 0{,}80) = \\
&= (\bar{\underset{\sim}{a}} \wedge \bar{\underset{\sim}{b}}) \vee (\underset{\sim}{a} \wedge \underset{\sim}{b} \wedge \bar{\underset{\sim}{c}}) = \\
&= (0{,}45 \wedge 0{,}43) \vee (0{,}55 \wedge 0{,}57 \wedge 0{,}20) = \\
&= 0{,}43 \vee 0{,}20 = 0{,}43.
\end{aligned}$$

Przykład2

$$f(\underset{\sim}{a}, \underset{\sim}{b}, \underset{\sim}{c}) = (\underset{\sim}{a} \wedge \overline{\underset{\sim}{b}}) \vee (\overline{\underset{\sim}{a}} \wedge \underset{\sim}{c}) \;\dot{\vee}\; \overline{\underset{\sim}{c}};$$

$$[0;\ 0{,}2[,\ [0{,}2;\ 0{,}3[,\ [0{,}3;\ 1].$$

$[0;\ 0{,}2\ [$

Hipoteza 1

$$\underset{\sim}{a} \wedge \overline{\underset{\sim}{b}} > \overline{\underset{\sim}{a}} \wedge \underset{\sim}{c},\quad \underset{\sim}{a} \wedge \overline{\underset{\sim}{b}} > \overline{\underset{\sim}{c}}. \Rightarrow 0 \leqslant \underset{\sim}{a} \wedge \overline{\underset{\sim}{b}} < 0{,}2.$$

$$0 \leqslant \mathrm{MIN}\,(\underset{\sim}{a},\ 1 - \underset{\sim}{b}) < 0{,}2,$$

$\underset{\sim}{a} \geqslant 0$ i $\underset{\sim}{b} \leqslant 1$ $\mathscr{P}_1^{(1)}$

i

$\underset{\sim}{a} < 0{,}2$ lub $\underset{\sim}{b} > 0{,}8.$ $\mathscr{P}_2^{(1)}$

Przykład2 (cont.)

$$f(\underset{\sim}{a}, \underset{\sim}{b}, \underset{\sim}{c}) = (\underset{\sim}{a} \wedge \bar{\underset{\sim}{b}}) \vee (\bar{\underset{\sim}{a}} \wedge \underset{\sim}{c}) \wedge \bar{\underset{\sim}{c}};$$

$$[0;\ 0{,}2[,\ [0{,}2;\ 0{,}3[,\ [0{,}3;\ 1].$$

$[0;\ 0{,}2\ [$

Hipoteza 2

$$\bar{\underset{\sim}{a}} \wedge \underset{\sim}{c} > \underset{\sim}{a} \wedge \bar{\underset{\sim}{b}},\quad \bar{\underset{\sim}{a}} \wedge \underset{\sim}{c} > \bar{\underset{\sim}{c}}. \Rightarrow 0 \leqslant \bar{\underset{\sim}{a}} \wedge \underset{\sim}{c} < 0{,}2.$$

$\underset{\sim}{a} \leqslant 1$ i $\underset{\sim}{c} \geqslant 0$ $\quad \mathcal{P}_1^{(1)}$

i

$\underset{\sim}{a} > 0{,}8$ lub $\underset{\sim}{c} < 0{,}2.$ $\quad \mathcal{P}_2^{(1)}$

Przykład2 (cont.)

$$f(\underset{\sim}{a}, \underset{\sim}{b}, \underset{\sim}{c}) = (\underset{\sim}{a} \wedge \bar{\underset{\sim}{b}}) \vee (\bar{\underset{\sim}{a}} \wedge \underset{\sim}{c}) \wedge \bar{\underset{\sim}{c}};$$

$$[0;\ 0{,}2[,\ [0{,}2;\ 0{,}3[,\ [0{,}3;\ 1].$$

$$[0;\ 0{,}2\ [$$

Hipoteza 3

$$\bar{\underset{\sim}{c}} > \underset{\sim}{a} \wedge \bar{\underset{\sim}{b}},\quad \bar{\underset{\sim}{c}} > \bar{\underset{\sim}{a}} \wedge \underset{\sim}{c}. \Rightarrow 0 \leqslant \bar{\underset{\sim}{c}} < 0{,}2.$$

$$0 \leqslant 1 - \underset{\sim}{c} < 0{,}2,$$

$$0{,}8 < \underset{\sim}{c} \leqslant 1.$$

$$\mathscr{P}_2^{(1)} \quad \mathscr{P}_1^{(1)}$$

Przykład2 (cont.)

$$f(\underset{\sim}{a}, \underset{\sim}{b}, \underset{\sim}{c}) = (\underset{\sim}{a} \wedge \bar{\underset{\sim}{b}}) \vee (\bar{\underset{\sim}{a}} \wedge \underset{\sim}{c}) \wedge \bar{\underset{\sim}{c}};$$

$$[0;\ 0,2[,\ [0,2;\ 0,3[,\ [0,3;\ 1].$$

$[0,2;\ 0,3[$

Hipoteza 1

$$\underset{\sim}{a} \wedge \bar{\underset{\sim}{b}} > \bar{\underset{\sim}{a}} \wedge \underset{\sim}{c}, \quad \underset{\sim}{a} \wedge \bar{\underset{\sim}{b}} > \bar{\underset{\sim}{c}},$$

$$0,2 \leqslant \underset{\sim}{a} \wedge \bar{\underset{\sim}{b}} < 0,3,$$

$\underset{\sim}{a} \geqslant 0,2$ i $\underset{\sim}{b} \leqslant 0,8$ $\mathcal{P}_1^{(2)}$

i

$\underset{\sim}{a} < 0,3$ lub $\underset{\sim}{b} > 0,7.$ $\mathcal{P}_2^{(2)}$

Przykład2 (cont.)

$$f(\underset{\sim}{a}, \underset{\sim}{b}, \underset{\sim}{c}) = (\underset{\sim}{a} \wedge \bar{\underset{\sim}{b}}) \vee (\bar{\underset{\sim}{a}} \wedge \underset{\sim}{c}) \wedge \bar{\underset{\sim}{c}};$$

$$[0;\ 0{,}2[,\ [0{,}2;\ 0{,}3[,\ [0{,}3;\ 1].$$

$[0{,}2;\ 0{,}3[$

Hipoteza 2

$$\bar{\underset{\sim}{a}} \wedge \underset{\sim}{c} > \underset{\sim}{a} \wedge \bar{\underset{\sim}{b}},\quad \bar{\underset{\sim}{a}} \wedge \underset{\sim}{c} > \bar{\underset{\sim}{c}},$$

$$0{,}2 \leqslant \bar{\underset{\sim}{a}} \wedge \underset{\sim}{c} < 0{,}3,$$

$\underset{\sim}{a} \leqslant 0{,}8$ i $\underset{\sim}{c} \geqslant 0{,}2$ $\mathscr{P}_1^{(2)}$

i

$\underset{\sim}{a} > 0$ lub $\underset{\sim}{c} < 0{,}3.$ $\mathscr{P}_2^{(2)}$

Przykład2 (cont.)

$$f(\underset{\sim}{a}, \underset{\sim}{b}, \underset{\sim}{c}) = (\underset{\sim}{a} \wedge \bar{\underset{\sim}{b}}) \vee (\bar{\underset{\sim}{a}} \wedge \underset{\sim}{c}) \wedge \bar{\underset{\sim}{c}};$$

$[0;\ 0{,}2[,\ [0{,}2;\ 0{,}3[,\ [0{,}3;\ 1].$

$[0{,}2;\ 0{,}3[$

Hipoteza 3

$$\bar{\underset{\sim}{c}} > \underset{\sim}{a} \wedge \bar{\underset{\sim}{b}},\quad \bar{\underset{\sim}{c}} > \bar{\underset{\sim}{a}} \wedge \underset{\sim}{c},$$

$$0{,}2 \leqslant \bar{\underset{\sim}{c}} < 0{,}3$$

$\underset{\sim}{c} \leqslant 0{,}8$ i $\underset{\sim}{c} > 0{,}7.$

$\mathscr{P}_1^{(2)}$ $\mathscr{P}_2^{(2)}$

Przykład2 (cont.)

$$f(\underset{\sim}{a}, \underset{\sim}{b}, \underset{\sim}{c}) = (\underset{\sim}{a} \wedge \bar{\underset{\sim}{b}}) \vee (\bar{\underset{\sim}{a}} \wedge \underset{\sim}{c}) \wedge \bar{\underset{\sim}{c}};$$

$$[0;\ \ 0,2[,\ \ [0,2;\ 0,3[,\ \ [0,3;\ 1].$$

$[0,3;\ 1]$

Hipoteza 1

$$\underset{\sim}{a} \wedge \bar{\underset{\sim}{b}} > \bar{\underset{\sim}{a}} \wedge \underset{\sim}{c}, \qquad \underset{\sim}{a} \wedge \bar{\underset{\sim}{b}} > \bar{\underset{\sim}{c}}.$$

$$0,3 \leqslant \underset{\sim}{a} \wedge \bar{\underset{\sim}{b}} \leqslant 1,$$

i

$\underset{\sim}{a} \geqslant 0,3$ i $\underset{\sim}{b} \leqslant 0,7$ $\quad \mathscr{P}_1^{(3)}$

$\underset{\sim}{a} \leqslant 1$ lub $\underset{\sim}{b} \geqslant 0.$ $\quad \mathscr{P}_2^{(3)}$

Przykład2 (cont.)

$$f(\underset{\sim}{a}, \underset{\sim}{b}, \underset{\sim}{c}) = (\underset{\sim}{a} \wedge \bar{\underset{\sim}{b}}) \vee (\bar{\underset{\sim}{a}} \wedge \underset{\sim}{c}) \wedge \bar{\underset{\sim}{c}};$$

$$[0;\ 0{,}2[,\ [0{,}2;\ 0{,}3[,\ [0{,}3;\ 1].$$

$[0{,}3;\ 1]$

Hipoteza 2

$$\bar{\underset{\sim}{a}} \wedge \underset{\sim}{c} > \underset{\sim}{a} \wedge \bar{\underset{\sim}{b}}, \quad \bar{\underset{\sim}{a}} \wedge \underset{\sim}{c} > \bar{\underset{\sim}{c}},$$

$$0{,}3 \leqslant \bar{\underset{\sim}{a}} \wedge \underset{\sim}{c} \leqslant 1,$$

i

$\underset{\sim}{a} \leqslant 0{,}7$ i $\underset{\sim}{c} > 0{,}3$ $\mathscr{P}_1^{(3)}$

$\underset{\sim}{a} \geqslant 0$ lub $\underset{\sim}{c} \leqslant 1.$ $\mathscr{P}_2^{(3)}$

Przykład2 (cont.)

$$f(\underset{\sim}{a}, \underset{\sim}{b}, \underset{\sim}{c}) = (\underset{\sim}{a} \wedge \bar{\underset{\sim}{b}}) \vee (\bar{\underset{\sim}{a}} \wedge \underset{\sim}{c}) \wedge \bar{\underset{\sim}{c}};$$

$$[0;\ 0{,}2[,\ [0{,}2;\ 0{,}3[,\ [0{,}3;\ 1].$$

$[0{,}3;\ 1]$

Hipoteza 3

$$\bar{\underset{\sim}{c}} > a \wedge \bar{\underset{\sim}{b}}, \quad c > \bar{\underset{\sim}{a}} \wedge \underset{\sim}{c},$$

$$0{,}3 \leqslant \bar{\underset{\sim}{c}} \leqslant 1$$

$\underset{\sim}{c} \leqslant 0{,}7$ i $\underset{\sim}{c} \geqslant 0.$

$\mathscr{P}_1^{(3)}$ $\mathscr{P}_2^{(3)}$

Notacja

Δ i

∇ lub

$\overline{}$ nie

$$f(\underset{\sim}{a}, \underset{\sim}{b}, \ldots, \underset{\sim}{l})$$

$$[\alpha_{k-1}, \alpha_k]$$

$$\mathscr{P}_{\underset{\sim}{x}} = (x \mid \underset{\sim}{x} \geqslant \alpha_{k-1}),$$
$$\mathscr{P}_{\underset{\sim}{\bar{x}}} = (\underset{\sim}{x} \mid \underset{\sim}{x} \leqslant 1 - \alpha_{k-1}),$$
$$\mathscr{P}'_{\underset{\sim}{x}} = (\underset{\sim}{x} \mid \underset{\sim}{x} < \alpha_k),$$
$$\mathscr{P}'_{\underset{\sim}{\bar{x}}} = (\underset{\sim}{x} \mid \underset{\sim}{x} > 1 - \alpha_k).$$

$$\underset{\sim}{x} \wedge \underset{\sim}{y} \rightarrow \mathscr{P}_{\underset{\sim}{x}} \Delta \mathscr{P}_{\underset{\sim}{y}}$$

$$\underset{\sim}{\bar{a}} \wedge \underset{\sim}{b} \wedge \underset{\sim}{\bar{c}} \rightarrow \mathscr{P}_{\underset{\sim}{\bar{a}}} \Delta \mathscr{P}_{\underset{\sim}{b}} \Delta \mathscr{P}_{\underset{\sim}{\bar{c}}}$$

$$f(\underset{\sim}{a}, \underset{\sim}{b}, \underset{\sim}{c}) = (\underset{\sim}{\bar{a}} \wedge \underset{\sim}{b} \wedge \underset{\sim}{\bar{c}}) \vee (\underset{\sim}{\bar{b}} \wedge \underset{\sim}{c})$$

$$\Downarrow$$

$$\mathscr{P} = [\mathscr{P}_{\underset{\sim}{\bar{a}}} \Delta \mathscr{P}_{\underset{\sim}{b}} \Delta \mathscr{P}_{\underset{\sim}{\bar{c}}}) \nabla (\mathscr{P}_{\underset{\sim}{\bar{b}}} \Delta \mathscr{P}_{\underset{\sim}{c}})] \Delta [(\mathscr{P}'_{\underset{\sim}{\bar{a}}} \nabla \mathscr{P}'_{\underset{\sim}{b}} \nabla \mathscr{P}'_{\underset{\sim}{\bar{c}}}) \Delta (\mathscr{P}'_{\underset{\sim}{\bar{b}}} \nabla \mathscr{P}'_{\underset{\sim}{c}})]$$

Przykład3

$$\underset{\sim}{f}\,(\underset{\sim}{a},\underset{\sim}{b},\underset{\sim}{c}) = (\underset{\sim}{\bar a}\wedge\underset{\sim}{b}\wedge\underset{\sim}{\bar c})\vee(\underset{\sim}{\bar b}\wedge\underset{\sim}{c}) \qquad [\alpha_{k-1},\ \alpha_k\,[\,=[0{,}3;\ 0{,}8[$$

$$\mathscr{P} = [(\mathscr{P}_{\underset{\sim}{\bar a}}\Delta\mathscr{P}_{\underset{\sim}{b}}\Delta\mathscr{P}_{\underset{\sim}{\bar c}})\nabla(\mathscr{P}_{\underset{\sim}{\bar b}}\Delta\mathscr{P}_{\underset{\sim}{c}})]\,\Delta\,[(\mathscr{P}'_{\underset{\sim}{\bar a}}\nabla\mathscr{P}'_{\underset{\sim}{b}}\nabla\mathscr{P}'_{\underset{\sim}{\bar c}})\,\Delta\,(\mathscr{P}'_{\underset{\sim}{\bar b}}\nabla\mathscr{P}'_{\underset{\sim}{c}})]$$

$$\mathscr{P}_{\underset{\sim}{x}}\Delta\mathscr{P}_{\underset{\sim}{y}} \longrightarrow \mathscr{P}_{\underset{\sim}{x}}\mathscr{P}_{\underset{\sim}{y}}$$

$$\mathscr{P} = (\mathscr{P}_{\underset{\sim}{\bar a}}\mathscr{P}_{\underset{\sim}{b}}\mathscr{P}_{\underset{\sim}{\bar c}}\nabla\mathscr{P}_{\underset{\sim}{\bar b}}\mathscr{P}_{\underset{\sim}{c}})(\mathscr{P}'_{\underset{\sim}{\bar a}}\nabla\mathscr{P}'_{\underset{\sim}{b}}\nabla\mathscr{P}'_{\underset{\sim}{\bar c}})(\mathscr{P}'_{\underset{\sim}{\bar b}}\nabla\mathscr{P}'_{\underset{\sim}{c}}) =$$

$$= (\mathscr{P}_{\underset{\sim}{\bar a}}\mathscr{P}_{\underset{\sim}{b}}\mathscr{P}_{\underset{\sim}{\bar c}}\nabla\mathscr{P}_{\underset{\sim}{\bar b}}\mathscr{P}_{\underset{\sim}{c}})(\mathscr{P}'_{\underset{\sim}{\bar a}}\mathscr{P}'_{\underset{\sim}{\bar b}}\nabla\mathscr{P}'_{\underset{\sim}{\bar a}}\mathscr{P}'_{\underset{\sim}{c}}\nabla\mathscr{P}'_{\underset{\sim}{b}}\mathscr{P}'_{\underset{\sim}{\bar b}}\nabla\mathscr{P}'_{\underset{\sim}{b}}\mathscr{P}'_{\underset{\sim}{c}}\nabla$$

$$\nabla\mathscr{P}'_{\underset{\sim}{\bar b}}\mathscr{P}_{\underset{\sim}{\bar c}}\nabla\mathscr{P}'_{\underset{\sim}{c}}\mathscr{P}'_{\underset{\sim}{\bar c}}) = \mathscr{P}_{\underset{\sim}{\bar a}}\mathscr{P}'_{\underset{\sim}{\bar a}}\mathscr{P}_{\underset{\sim}{b}}\mathscr{P}'_{\underset{\sim}{\bar b}}\mathscr{P}_{\underset{\sim}{\bar c}}\nabla\mathscr{P}_{\underset{\sim}{\bar a}}\mathscr{P}'_{\underset{\sim}{\bar a}}\mathscr{P}_{\underset{\sim}{b}}\mathscr{P}_{\underset{\sim}{\bar c}}\mathscr{P}'_{\underset{\sim}{c}}\nabla$$

$$\nabla\mathscr{P}_{\underset{\sim}{\bar a}}\mathscr{P}_{\underset{\sim}{b}}\mathscr{P}'_{\underset{\sim}{b}}\mathscr{P}'_{\underset{\sim}{\bar b}}\mathscr{P}_{\underset{\sim}{\bar c}}\nabla\mathscr{P}_{\underset{\sim}{\bar a}}\mathscr{P}_{\underset{\sim}{b}}\mathscr{P}'_{\underset{\sim}{b}}\mathscr{P}_{\underset{\sim}{\bar c}}\mathscr{P}'_{\underset{\sim}{c}}\nabla\mathscr{P}_{\underset{\sim}{\bar a}}\mathscr{P}_{\underset{\sim}{b}}\mathscr{P}'_{\underset{\sim}{\bar b}}\mathscr{P}_{\underset{\sim}{\bar c}}\mathscr{P}'_{\underset{\sim}{\bar c}}\nabla$$

$$\nabla\mathscr{P}_{\underset{\sim}{\bar a}}\mathscr{P}_{\underset{\sim}{b}}\mathscr{P}_{\underset{\sim}{\bar c}}\mathscr{P}'_{\underset{\sim}{c}}\mathscr{P}'_{\underset{\sim}{\bar c}}\nabla\mathscr{P}'_{\underset{\sim}{\bar a}}\mathscr{P}_{\underset{\sim}{\bar b}}\mathscr{P}'_{\underset{\sim}{\bar b}}\mathscr{P}_{\underset{\sim}{c}}\nabla\mathscr{P}'_{\underset{\sim}{\bar a}}\mathscr{P}_{\underset{\sim}{\bar b}}\mathscr{P}_{\underset{\sim}{c}}\mathscr{P}'_{\underset{\sim}{c}}\nabla\underline{\mathscr{P}_{\underset{\sim}{\bar b}}\mathscr{P}'_{\underset{\sim}{b}}\mathscr{P}'_{\underset{\sim}{\bar b}}}\mathscr{P}_{\underset{\sim}{c}}\nabla$$

$$\nabla\mathscr{P}_{\underset{\sim}{\bar b}}\mathscr{P}'_{\underset{\sim}{b}}\mathscr{P}_{\underset{\sim}{c}}\mathscr{P}'_{\underset{\sim}{c}}\nabla\mathscr{P}_{\underset{\sim}{\bar b}}\mathscr{P}'_{\underset{\sim}{\bar b}}\mathscr{P}_{\underset{\sim}{c}}\mathscr{P}'_{\underset{\sim}{c}}\nabla\mathscr{P}_{\underset{\sim}{\bar b}}\mathscr{P}_{\underset{\sim}{c}}\mathscr{P}'_{\underset{\sim}{c}}\mathscr{P}'_{\underset{\sim}{\bar c}}\,.$$

Przykład3 (cont.)

$$0{,}3 \leqslant (\underset{\sim}{\bar{a}} \wedge \underset{\sim}{b} \wedge \underset{\sim}{\bar{c}}) \vee (\underset{\sim}{\bar{b}} \wedge \underset{\sim}{c}) < \underset{\sim}{0{,}8}$$

$$\mathscr{P}_{\underset{\sim}{\bar{b}}}\ \mathscr{P}'_{\underset{\sim}{b}}\ \mathscr{P}_{\underset{\sim}{\bar{b}}}\ \mathscr{P}_{\underset{\sim}{c}}$$

$$[\alpha_{k-1}, \alpha_k [= [0{,}3;\ 0{,}8[$$

$$\mathscr{P}_{\underset{\sim}{\bar{b}}} : \underset{\sim}{b} \leqslant (1 - 0{,}3),$$

$$\mathscr{P}'_{\underset{\sim}{b}} : \underset{\sim}{b} < 0{,}8,$$

$$\mathscr{P}'_{\underset{\sim}{\bar{b}}} : \underset{\sim}{b} > 0{,}2,$$

$$\mathscr{P}_{\underset{\sim}{c}} : \underset{\sim}{c} \geqslant 0{,}3.$$

$$\mathscr{P}_{\underset{\sim}{x}} = (\underset{\sim}{x} \mid \underset{\sim}{x} \geqslant \alpha_{k-1}),$$

$$\mathscr{P}_{\underset{\sim}{\bar{x}}} = (\underset{\sim}{x} \mid \underset{\sim}{x} \leqslant 1 - \alpha_{k-1}),$$

$$\mathscr{P}'_{\underset{\sim}{x}} = (\underset{\sim}{x} \mid \underset{\sim}{x} < \alpha_k),$$

$$\mathscr{P}'_{\underset{\sim}{\bar{x}}} = (\underset{\sim}{x} \mid \underset{\sim}{x} > 1 - \alpha_k).$$

$$0{,}2 < \underset{\sim}{b} \leqslant 0{,}7$$

$$\underset{\sim}{c} \geqslant 0{,}3$$

Funkcja dyskretna

$$\mathbf{M} = \{0;\ 0{,}1;\ 0{,}2;\ 0{,}3;\ 0{,}4;\ 0{,}5;\ \ 0{,}6;\ 0{,}7;\ 0{,}8;\ 0{,}9;\ 1\}$$

$$\underset{\sim}{f}(\underset{\sim}{a},\ \underset{\sim}{b}) = \bar{\underset{\sim}{a}} \wedge \underset{\sim}{b},$$

$$\underset{\sim}{a} \in \{0{,}2;\ \ 0{,}3;\ \ 0{,}4;\ \ 0{,}5\}$$

$$\underset{\sim}{b} \in \{0;\ \ 0{,}1\} \cup \{0{,}7;\ \ 0{,}8;\ \ 0{,}9\}$$

$\bar{\underset{\sim}{a}} \wedge \underset{\sim}{b}$ ($\underset{\sim}{a}$ \ $\underset{\sim}{b}$)	0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1
0	0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1
.1	0	.1	.2	.3	.4	.5	.6	.7	.8	.9	.9
.2	0	.1	.2	.3	.4	.5	.6	.7	.8	.8	.8
3.	0	.1	.2	.3	.4	.5	.6	.7	.7	.7	.7
.4	0	.1	.2	.3	.4	.5	.6	.6	.6	.6	.6
.5	0	.1	.2	.3	.4	.5	5	.5	.5	.5	.5
.6	0	.1	.2	.3	.4	.4	.4	.4	.4	.4	.4
.7	0	.1	.2	.3	.3	.3	.3	.3	.3	.3	.3
.8	0	.1	.2	.2	.2	.2	.2	.2	.2	.2	.2
.9	0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1
1	0	0	0	0	0	0	0	0	0	0	0

$$\bar{\underset{\sim}{a}} \wedge \underset{\sim}{b} \in \{0;\ \ \ 0{,}1\} \cup \{0{,}5;\ \ 0{,}6;\ \ 0{,}7;\ \ 0{,}8\}$$

Przykład4

$$\underset{\sim}{f}(\underset{\sim}{a}, \underset{\sim}{b}, \underset{\sim}{c}) = (\underset{\sim}{a} \wedge \bar{\underset{\sim}{b}} \wedge \underset{\sim}{c}) \vee \bar{\underset{\sim}{c}},$$

$$\underset{\sim}{a} \in \{0,3; \quad 0,4; \quad 0,5\};$$

$$\underset{\sim}{b} \in \{0,1; \ 0,2;\} \cup \{0,6\};$$

$$\underset{\sim}{c} \in \{0; \ 0,1\} \cup \{0,7; \ 0,8; \ 0,9; \ 1\}.$$

$$\underset{\sim}{d} = \underset{\sim}{a} \wedge \bar{\underset{\sim}{b}}$$

$$\underset{\sim}{d} = \underset{\sim}{a} \wedge \bar{\underset{\sim}{b}} \in \{0,3; \quad 0,4; \quad 0,5\}$$

$\underset{\sim}{d}=\underset{\sim}{a}\wedge\bar{\underset{\sim}{b}}$ ($\underset{\sim}{a}$ \ $\underset{\sim}{b}$)	0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1
0	0	0	0	0	0	0	0	0	0	0	0
.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	0
.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.1	0
.3	.3	.3	.3	.3	.3	.3	.3	.3	.2	.1	0
.4	.4	.4	.4	.4	.4	.4	.4	.3	.2	.1	0
.5	.5	.5	.5	.5	.5	.5	.4	.3	.2	.1	0
.6	.6	.6	.6	.6	.6	.5	.4	.3	.2	.1	0
.7	.7	.7	.7	.7	.6	.5	.4	.3	.2	.1	0
.8	.8	.8	.8	.7	.6	.5	.4	.3	.2	.1	0
.9	.9	.9	.8	.7	.6	.5	.4	.3	.2	.1	0
1	1	.9	.8	.7	.6	.5	.4	.3	.2	.1	0

Przykład4 (Cont.)

$$\underset{\sim}{e} = \underset{\sim}{d} \wedge \underset{\sim}{c} = \underset{\sim}{a} \wedge \overline{\underset{\sim}{b}} \wedge \underset{\sim}{c}$$

$$\underset{\sim}{e} = \underset{\sim}{d} \wedge \underset{\sim}{c} = \underset{\sim}{a} \wedge \overline{\underset{\sim}{b}} \wedge \underset{\sim}{c} \in \{0;\ 0,1\} \cup \{0,3;\ 0,4;\ 0,5\}$$

$\underset{\sim}{f}=\underset{\sim}{e} \vee \overline{\underset{\sim}{c}}$	0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1
0	1	.9	.8	.7	.6	.5	.4	.3	.2	.1	0
.1	1	.9	.8	.7	.6	.5	.4	.3	.2	.1	.1
.2	1	.9	.8	.7	.6	.5	.4	.3	.2	.2	.2
.3	1	.9	.8	.7	.6	.5	.4	.3	.3	.3	.3
.4	1	.9	.8	.7	.6	.5	.4	.4	.4	.4	.4
.5	1	.9	.8	.7	.6	.5	.5	.5	.5	.5	.5
.6	1	.9	.8	.7	.6	.6	.6	.6	.6	.6	.6
.7	1	.9	.8	.7	.7	.7	.7	.7	.7	.7	.7
.8	1	.9	.8	.8	.8	.8	.8	.8	.8	.8	.8
.9	1	.9	.9	.9	.9	.9	.9	.9	.9	.9	.9
1	1	1	1	1	1	1	1	1	1	1	1

$\underset{\sim}{e}=\underset{\sim}{d} \wedge \underset{\sim}{c}$ ($\underset{\sim}{d}$ \ $\underset{\sim}{c}$)	0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1
0	0	0	0	0	0	0	0	0	0	0	0
.1	0	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1
.2	0	.1	.2	.2	.2	.2	.2	.2	.2	.2	.2
.3	0	.1	.2	.3	.3	.3	.3	.3	.3	.3	.3
.4	0	.1	.2	.3	.4	.4	.4	.4	.4	.4	.4
.5	0	.1	.2	.3	.4	.5	.5	.5	.5	.5	.5
.6	0	.1	.2	.3	.4	.5	.6	.6	.6	.6	.6
.7	0	.1	.2	.3	.4	.5	.6	.7	.7	.7	.7
.8	0	.1	.2	.3	.4	.5	.6	.7	.8	.8	.8
.9	0	.1	.2	.3	.4	.5	.6	.7	.8	.9	.9
1	0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1

$$\underset{\sim}{f} = \underset{\sim}{e} \vee \overline{\underset{\sim}{c}} = (\underset{\sim}{d} \wedge \underset{\sim}{c}) \vee \overline{\underset{\sim}{c}} = (\underset{\sim}{a} \wedge \overline{\underset{\sim}{b}} \wedge \underset{\sim}{c}) \vee \overline{\underset{\sim}{c}}$$

$$\underset{\sim}{f}(\underset{\sim}{a},\ \underset{\sim}{b},\ \underset{\sim}{c}) \in \{0;\ 0,1;\ 0,2;\ 0,3;\ 0,4;\ 0,5\} \cup \{0,9;\ 1\}$$