Logika rozmyta

Zadanie 1. Sprawdzić formule logiczne

Czy podane formule logiczne są tautologiami w różnych systemach logicznych

Definicja. Mówimy, że formuła f jest tautologią rachunku zdań, gdy jest zdaniem prawdziwym dla dowolnych argumentów zdań

$$\sim(p \wedge q) \leftrightarrow [(\sim p) \vee (\sim q)]$$

$$\sim(p \wedge q) \rightarrow (\sim p \vee \sim q)$$

$$((p \wedge q) \rightarrow r) \rightarrow (p \rightarrow (q \rightarrow r))$$

Operacja	KD-logika	Ł-logika
~ a	1-a	1-a
a&b	min(a,b)	max(0,a+b-1)
a∨b	max(a,b)	min(1,a+b)
a→b	(1-a)∨b = max(1-a,b)	min(1,b+1-a)
a↔b	min(max(1-a,b),max(1-b,a))	max(0,min(1,b+1-a) +min(1,a+1-b)-1)

Przykłady tautologii

$p \vee \neg p$ (prawo wyłączonego środka)

$p \Leftrightarrow p$

$\neg(p \wedge \neg p)$ (prawo sprzeczności, w klasycznej logice wyrażano je mówiąc: ``nie może być tak, że prawdą jest równocześnie zdanie p i jego negacja'')

$p \Leftrightarrow \neg(\neg p)$ (prawo podwójnej negacji)

$[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow (p \Rightarrow r)$ (prawo przechodniości implikacji)

$(p \Rightarrow q) \Leftrightarrow (\neg q \Rightarrow \neg p)$ (prawo transpozycji, zwane też prawem kontrapozycji)

$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$ i $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$ (prawa de Morgana)

$\neg(p \Rightarrow q) \Leftrightarrow p \wedge \neg q$

$(p \Leftrightarrow q) \Leftrightarrow (p \Rightarrow q) \wedge (q \Rightarrow p)$

Zadanie 2. Obliczyć wartość logiczną formuł

$$A = (p \wedge q) \vee (q \wedge (p \vee q))$$

$$\mathrm{B} = p \rightarrow (p \rightarrow (p \rightarrow q))$$

dla

$$p \in \{0,2; 0,3; 0,4; 0,5\}$$

$$q \in \{0; 0,1\} \cup \{0,7; 0,8; 0,9\}$$

Zadanie 3. Zbadać, czy rozumowanie jest dedukcyjne

Wnioskowanie (przykłady)

1. Jeśli Jan jest ojcem oskarżonego, to Jan może uchylić się od zeznań. Nieprawda, że Jan może uchylić się od zeznań. Zatem, nieprawda że Jan jest ojcem oskarżonego. Wnioskowanie
2. Jeśli Jan nie będzie schlebiał Piotrowi, to straci posadę. Jeśli Jan straci posadę, to popadnie w kłopoty finansowe. Jeśli Jan będzie schlebiał Piotrowi, to straci dobrą opinię. Zatem, Jan popadnie w kłopoty finansowe lub straci dobrą opinię.

Dedukcją nazywa się wnioskowanie oparte na jakieś regule niezawodnej, czyli takie, w którym **wniosek wynika logicznie z przesłanek**.

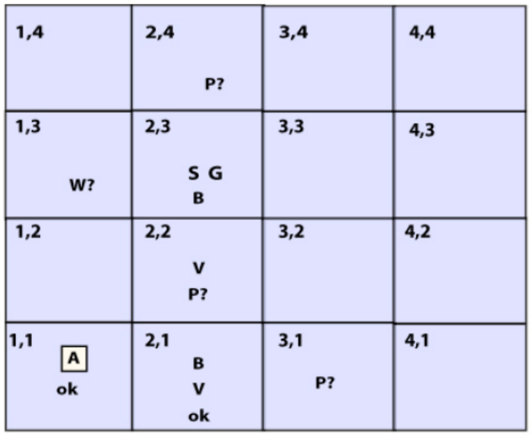
Reguła $\frac{A_1, \ldots, A_n}{B}$ jest niezawodna na gruncie KRP wtw implikacja

$$(A_1 \wedge \ldots \wedge A_n) \rightarrow B$$

jest tautologią KRP. W przeciwnym przypadku jest ona zawodna.

Klasyczny Rachunek Predykatów

Zadanie 4. Świat Wumpusa



Actuators:

- Left turn,
- Right turn
- Move forward
- Grab
- Release
- Shoot.

The Wumpus world Properties:

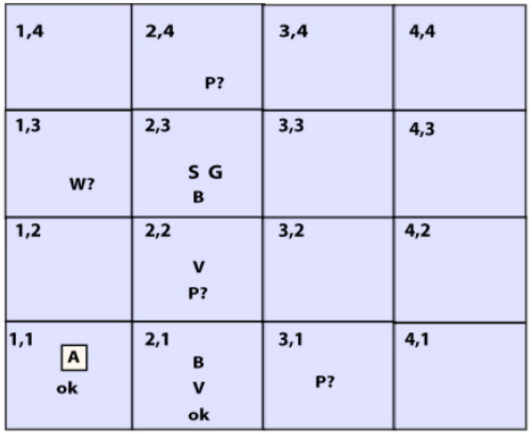
- **Partially observable:** The Wumpus world is partially observable because the agent can only perceive the close environment such as an adjacent room.
- **Deterministic:** It is deterministic, as the result and outcome of the world are already known.
- **Sequential:** The order is important, so it is sequential.
- **Static:** It is static as Wumpus and Pits are not moving.
- **Discrete:** The environment is discrete.
- **One agent:** The environment is a single agent as we have one agent only and Wumpus is not considered as an agent.

Atomic proposition variable for Wumpus world:

- Let $\mathbf{P}_{i,j}$ be true if there is a Pit in the room [i, j].
- Let $\mathbf{B}_{i,j}$ be true if agent perceives breeze in [i, j], (dead or alive).
- Let $\mathbf{W}_{i,j}$ be true if there is wumpus in the square[i, j].
- Let $\mathbf{S}_{i,j}$ be true if agent perceives stench in the square [i, j].
- Let $\mathbf{V}_{i,j}$ be true if that square[i, j] is visited.
- Let $\mathbf{G}_{i,j}$ be true if there is gold (and glitter) in the square [i, j].
- Let $\mathbf{OK}_{i,j}$ be true if the room is safe.

https://thiagodnf.github.io/wumpus-world-simulator/

https://www.javatpoint.com/the-wumpus-world-in-artificial-intelligence



Performance measure:

- +1000 reward points if the agent comes out of the cave with the gold.
- -1000 points penalty for being eaten by the Wumpus or falling into the pit.
- -1 for each action, and -10 for using an arrow.
- The game ends if either agent dies or came out of the cave.

Environment:

- A 4*4 grid of rooms.
- The agent initially in room square [1, 1], facing toward the right.
- Location of Wumpus and gold are chosen randomly except the first square [1,1].
- Each square of the cave can be a pit with probability 0.2 except the first square.

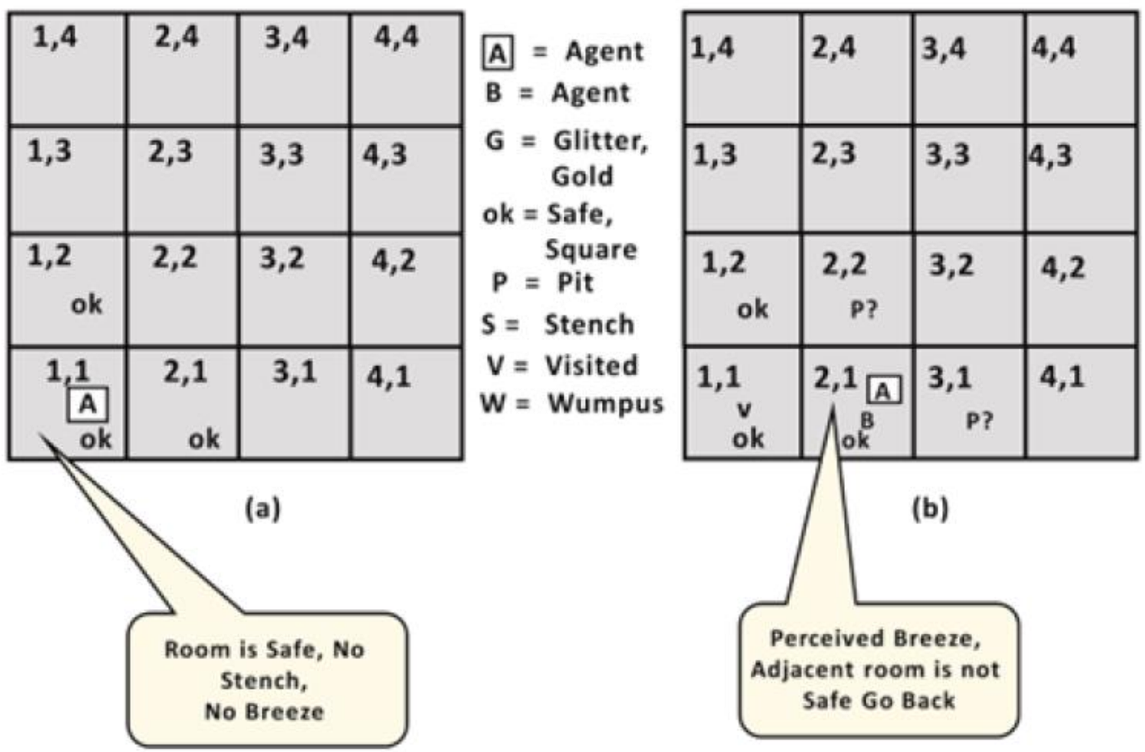
Sensors:

- The agent will perceive the **stench** if he is in the room adjacent to the Wumpus. (Not diagonally).
- The agent will perceive **breeze** if he is in the room directly adjacent to the Pit.
- The agent will perceive the **glitter** in the room where the gold is present.
- The agent will perceive the **bump** if he walks into a wall.
- When the Wumpus is shot, it emits a horrible **scream** which can be perceived anywhere in the cave.
- These percepts can be represented as five element list, in which we will have different indicators for each sensor.
- Example if agent perceives stench, breeze, but no glitter, no bump, and no scream then it can be represented as:
  [Stench, Breeze, None, None, None].

Agent's First step:

Initially, the agent is in the first room or on the square [1,1], and we already know that this room is safe for the agent, so to represent on the below diagram (a) that room is safe we will add symbol OK. Symbol A is used to represent agent, symbol B for the breeze, G for Glitter or gold, V for the visited room, P for pits, W for Wumpus.

At Room [1,1] agent does not feel any breeze or any Stench which means the adjacent squares are also OK.



Agent's second Step:

Now agent needs to move forward, so it will either move to [1, 2], or [2,1]. Let's suppose agent moves to the room [2, 1], at this room agent perceives some breeze which means Pit is around this room. The pit can be in [3, 1], or [2,2], so we will add symbol P? to say that, is this Pit room?

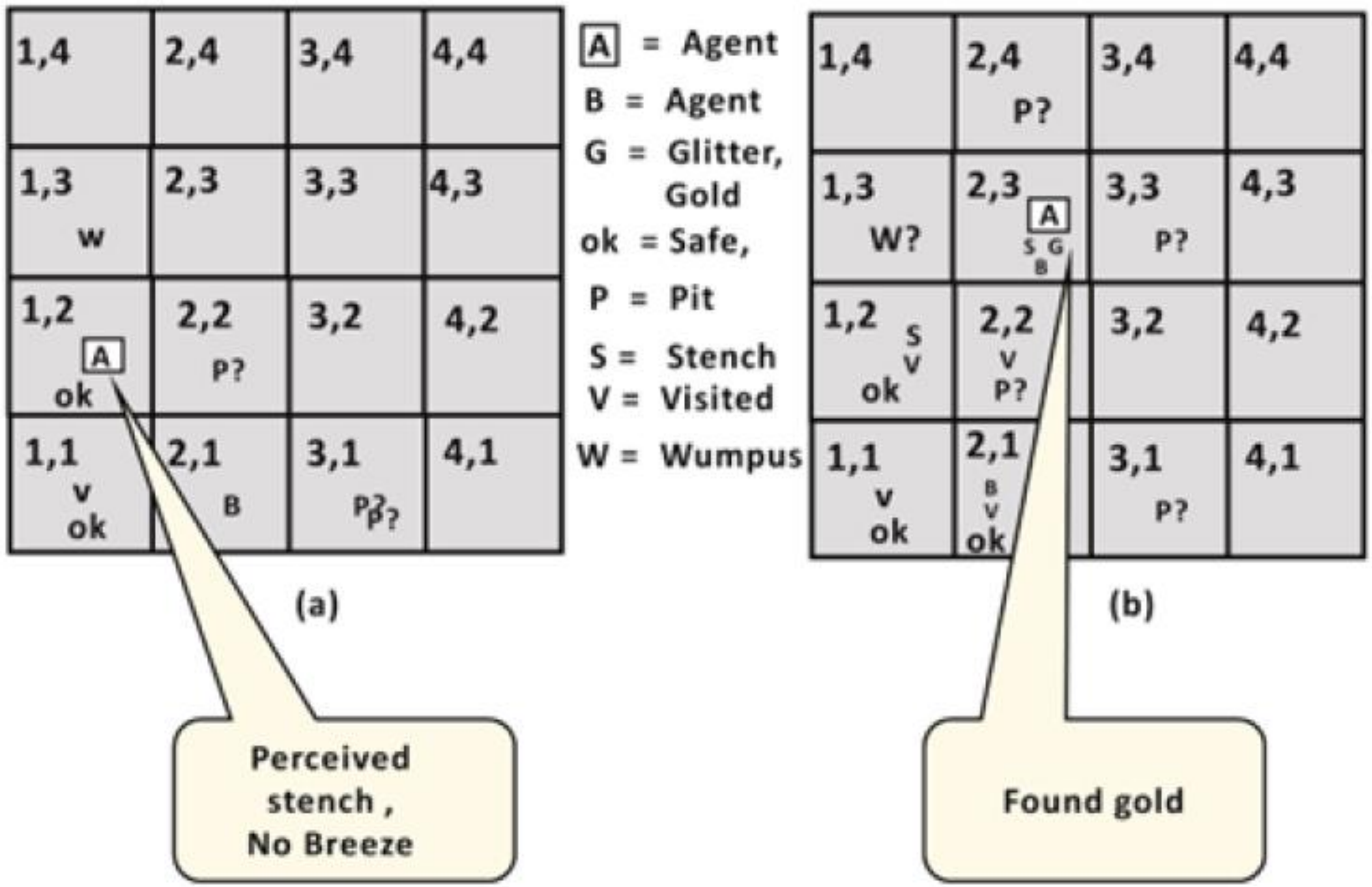
Now agent will stop and think and will not make any harmful move. The agent will go back to the [1, 1] room. The room [1,1], and [2,1] are visited by the agent, so we will use symbol V to represent the visited squares.

Agent's third step:

At the third step, now agent will move to the room [1,2] which is OK. In the room [1,2] agent perceives a stench which means there must be a Wumpus nearby. But Wumpus cannot be in the room [1,1] as by rules of the game, and also not in [2,2] (Agent had not detected any stench when he was at [2,1]). Therefore agent infers that Wumpus is in the room [1,3], and in current state, there is no breeze which means in [2,2] there is no Pit and no Wumpus. So it is safe, and we will mark it OK, and the agent moves further in [2,2].

Agent's fourth step:

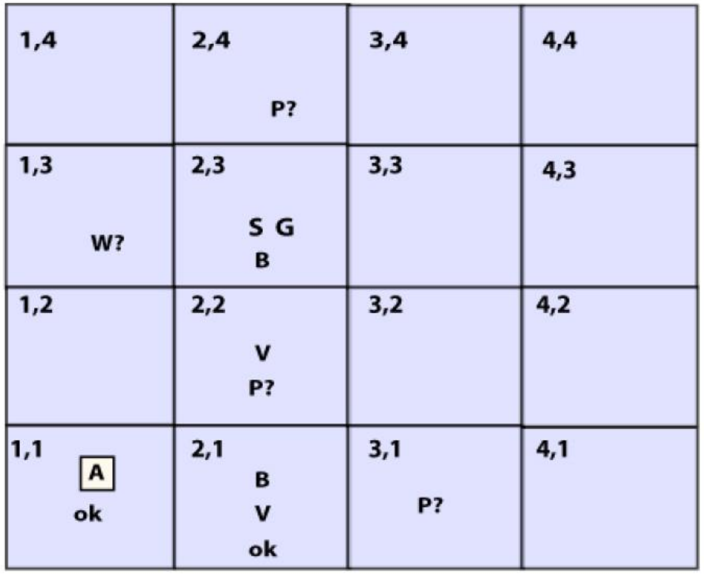
At room [2,2], here no stench and no breezes present so let's suppose agent decides to move to [2,3]. At room [2,3] agent perceives glitter, so it should grab the gold and climb out of the cave.


1,4
2,4
3,4
4,4
1,3
w
2,3
3,3
4,3
1,2
A
ok
2,2
P?
3,2
4,2
1,1
v
ok
2,1
B
3,1
P?
4,1
A = Agent
B = Agent
G = Glitter, Gold
ok = Safe,
P = Pit
S = Stench
V = Visited
W = Wumpus
1,4
2,4
P?
3,4
4,4
1,3
W?
2,3
A
S G
B
3,3
P?
4,3
1,2
S
V
ok
2,2
V
P?
3,2
4,2
1,1
v
ok
2,1
B
V
ok
3,1
P?
4,1
Perceived stench , No Breeze
Found gold
(a)
(b)

Knowledge-base for Wumpus world

As in the previous topic we have learned about the wumpus world and how a knowledge-based agent evolves the world. Now in this topic, we will create a knowledge base for the wumpus world, and will derive some proves for the Wumpus-world using propositional logic.

The agent starts visiting from first square [1, 1], and we already know that this room is safe for the agent. To build a knowledge base for wumpus world, we will use some rules and atomic propositions. We need symbol [i, j] for each location in the wumpus world, where i is for the location of rows, and j for column location.



Atomic proposition variable for Wumpus world

- Let $\mathbf{P}_{i,j}$ be true if there is a Pit in the room [i, j].
- Let $\mathbf{B}_{i,j}$ be true if agent perceives breeze in [i, j], (dead or alive).
- Let $\mathbf{W}_{i,j}$ be true if there is wumpus in the square[i, j].
- Let $\mathbf{S}_{i,j}$ be true if agent perceives stench in the square [i, j].
- Let $\mathbf{V}_{i,j}$ be true if that square[i, j] is visited.
- Let $\mathbf{G}_{i,j}$ be true if there is gold (and glitter) in the square [i, j].
- Let $\mathbf{OK}_{i,j}$ be true if the room is safe.

> Note: For a 4 * 4 square board, there will be 7*4*4= 122 propositional variables.

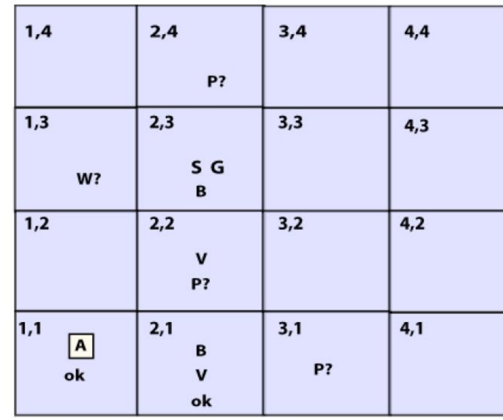
Some Propositional Rules for the wumpus world

(R1) $\neg S_{11} \rightarrow \neg W_{11} \wedge \neg W_{12} \wedge \neg W_{21}$

(R2) $\neg S_{21} \rightarrow \neg W_{11} \wedge \neg W_{21} \wedge \neg W_{22} \wedge \neg W_{31}$

(R3) $\neg S_{12} \rightarrow \neg W_{11} \wedge \neg W_{12} \wedge \neg W_{22} \wedge \neg W_{13}$

(R4) $S_{12} \rightarrow W_{13} \vee W_{12} \vee W_{22} \vee W_{11}$

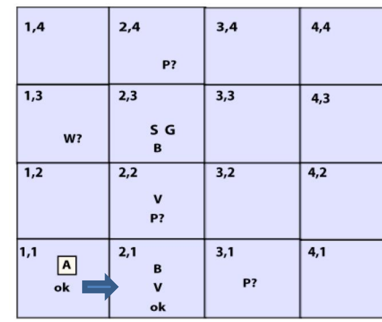


- Let $\mathbf{P}_{i,j}$ be true if there is a Pit in the room [i, j].
- Let $\mathbf{B}_{i,j}$ be true if agent perceives breeze in [i, j], (dead or alive).
- Let $\mathbf{W}_{i,j}$ be true if there is wumpus in the square[i, j].
- Let $\mathbf{S}_{i,j}$ be true if agent perceives stench in the square [i, j].
- Let $\mathbf{V}_{i,j}$ be true if that square[i, j] is visited.
- Let $\mathbf{G}_{i,j}$ be true if there is gold (and glitter) in the square [i, j].
- Let $\mathbf{OK}_{i,j}$ be true if the room is safe.

Representation of Knowledgebase for Wumpus world

Following is the Simple KB for wumpus world when an agent moves from room [1, 1], to room [2,1]:

$\neg W_{11}$	$\neg S_{11}$	$\neg P_{11}$	$\neg B_{11}$	$\neg G_{11}$	V_{11}	OK_{11}
$\neg W_{12}$	----	$\neg P_{12}$	-----	----	$\neg V_{12}$	OK_{12}
$\neg W_{21}$	$\neg S_{21}$	$\neg P_{21}$	B_{21}	$\neg G_{21}$	V_{21}	OK_{21}



Here in the first row, we have mentioned propositional variables for room[1,1], which is showing that room does not have wumpus($\neg W_{11}$), no stench ($\neg \mathbf{S_{11}}$), no Pit($\neg \mathbf{P_{11}}$), no breeze($\neg \mathbf{B_{11}}$), no gold ($\neg \mathbf{G_{11}}$), visited ($\mathbf{V_{11}}$), and the room is Safe($\mathbf{OK_{11}}$).

In the second row, we have mentioned propositional variables for room [1,2], which is showing that there is no wumpus, stench and breeze are unknown as an agent has not visited room [1,2], no Pit, not visited yet, and the room is safe.

In the third row we have mentioned propositional variable for room[2,1], which is showing that there is no wumpus($\neg$ W21), no stench ($\neg \mathbf{S_{21}}$), no Pit ($\neg \mathbf{P_{21}}$), Perceives breeze($\mathbf{B_{21}}$), no glitter($\neg \mathbf{G_{21}}$), visited ($\mathbf{V_{21}}$), and room is safe ($\mathbf{OK_{21}}$).

Prove that Wumpus is in the room (1, 3)

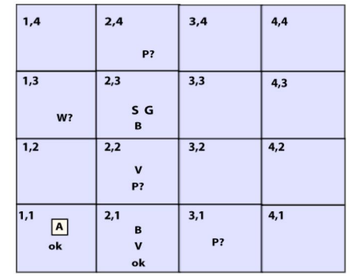
(R1) $\neg S_{11} \rightarrow \neg W_{11} \wedge \neg W_{12} \wedge \neg W_{21}$

(R2) $\neg S_{21} \rightarrow \neg W_{11} \wedge \neg W_{21} \wedge \neg W_{22} \wedge \neg W_{31}$

(R3) $\neg S_{12} \rightarrow \neg W_{11} \wedge \neg W_{12} \wedge \neg W_{22} \wedge \neg W_{13}$

(R4) $S_{12} \rightarrow W_{13} \vee W_{12} \vee W_{22} \vee W_{11}$

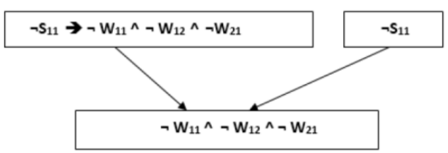
- Let $P_{i,j}$ be true if there is a Pit in the room [i, j].
- Let $B_{i,j}$ be true if agent perceives breeze in [i, j], (dead or alive).
- Let $W_{i,j}$ be true if there is wumpus in the square[i, j].
- Let $S_{i,j}$ be true if agent perceives stench in the square [i, j].
- Let $V_{i,j}$ be true if that square[i, j] is visited.
- Let $G_{i,j}$ be true if there is gold (and glitter) in the square [i, j].
- Let $OK_{i,j}$ be true if the room is safe.



We can prove that wumpus is in the room (1, 3) using propositional rules which we have derived for the wumpus world and using inference rule.

- **Apply Modus Ponens with ¬S11 and R1:**

We will firstly apply MP rule with R1 which is $\neg S_{11} \rightarrow \neg W_{11} \wedge \neg W_{12} \wedge \neg W_{21}$, and $\neg \mathbf{S_{11}}$ which will give the output $\neg W_{11} \wedge W_{12} \wedge W_{12}$.

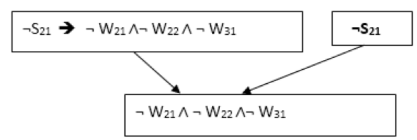


- **Apply And-Elimination Rule:**

After applying And-elimination rule to $\neg W_{11} \wedge \neg W_{12} \wedge \neg W_{21}$, we will get three statements: $\neg \mathbf{W_{11}}$, $\neg \mathbf{W_{12}}$, and $\neg \mathbf{W_{21}}$.

- **Apply Modus Ponens to $\neg S_{21}$, and R2:**

Now we will apply Modus Ponens to $\neg S_{21}$ and R2 which is $\neg S_{21} \rightarrow \neg W_{21} \wedge \neg W_{22} \wedge \neg W_{31}$, which will give the Output as $\neg \mathbf{W_{21}} \wedge \neg \mathbf{W_{22}} \wedge \neg \mathbf{W_{31}}$

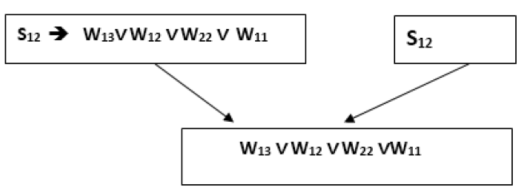


- **Apply And -Elimination rule:**

Now again apply And-elimination rule to $\neg \mathbf{W_{21}} \wedge \neg \mathbf{W_{22}} \wedge \neg \mathbf{W_{31}}$, We will get three statements: $\neg \mathbf{W_{21}}$, $\neg \mathbf{W_{22}}$, and $\neg \mathbf{W_{31}}$.

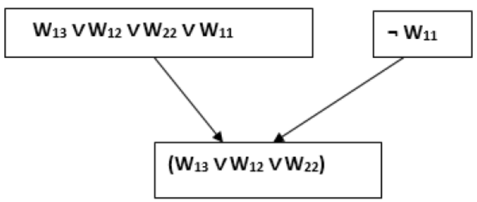
- **Apply MP to $\mathbf{S_{12}}$ and R4:**

Apply Modus Ponens to $\mathbf{S_{12}}$ and $\mathbf{R_4}$ which is $\mathbf{S_{12}} \rightarrow \mathbf{W_{13}} \vee \mathbf{W_{12}} \vee \mathbf{W_{22}} \vee \mathbf{W_{11}}$, we will get the output as $\mathbf{W_{13}} \vee \mathbf{W_{12}} \vee \mathbf{W_{22}} \vee \mathbf{W_{11}}$.



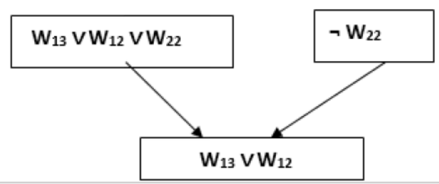
- **Apply Unit resolution on $W_{13} \vee W_{12} \vee W_{22} \vee W_{11}$ and $\neg W_{11}$:**

After applying Unit resolution formula on $W_{13} \vee W_{12} \vee W_{22} \vee W_{11}$ and $\neg W_{11}$ we will get $W_{13} \vee W_{12} \vee W_{22}$.



- **Apply Unit resolution on $W_{13} \vee W_{12} \vee W_{22}$ and $\neg W_{22}$:**

After applying Unit resolution on $\mathbf{W_{13} \vee W_{12} \vee W_{22}}$, and $\mathbf{\neg W_{22}}$, we will get $\mathbf{W_{13} \vee W_{12}}$ as output.



- **Apply Unit Resolution on $W_{13} \vee W_{12}$ and $\neg W_{12}$:**

After Applying Unit resolution on $\mathbf{W_{13} \vee W_{12}}$ **and** $\mathbf{\neg W_{12}}$, we will get $\mathbf{W_{13}}$ as an output, hence it is proved that the Wumpus is in the room [1, 3].