

Przedostatnią całkę obliczamy według zadania 15.8:

$$\int \frac{2x dx}{(x^2+1)^2} = \frac{-1}{x^2+1}.$$

Ostatnią całkę obliczamy według zadania 16.22:

$$\int \frac{dx}{(x^2+1)^2} = \frac{1}{2} \cdot \frac{x}{x^2+1} + \frac{1}{2} \operatorname{arctg} x.$$

Ostatecznie więc po przekształceniu otrzymujemy

$$I = \ln \left| \frac{x-1}{x} \right| + \frac{1}{(x-1)^2} + \frac{1}{2} \cdot \frac{x+2}{x^2+1} + \frac{1}{2} \operatorname{arctg} x + C.$$

Zadania

Obliczyć całki (zad. 16.26 - 16.98):

16.26. $\int (2x+1)^3 dx.$

16.28. $\int \frac{3x-4}{x^2-x-6} dx.$

16.30. $\int \frac{x+13}{x^2-4x-5} dx.$

16.32. $\int \frac{6x-13}{x^2-\frac{7}{2}x+\frac{3}{2}} dx.$

16.34. $\int \frac{5x+11}{x^2+3x-10} dx.$

16.36. $\int \frac{dx}{x^2+2x-1}.$

16.38. $\int \frac{5+x}{10x+x^2} dx.$

16.40. $\int \frac{dx}{-5+6x-x^2}.$

16.42. $\int \frac{dx}{2x-3x^2}.$

16.44. $\int \frac{2x-1}{x^2-6x+9} dx.$

16.27. $\int \frac{dx}{(3x-2)^4}.$

16.29. $\int \frac{2x-3}{x^2-3x+3} dx.$

16.31. $\int \frac{2x+6}{2x^2+3x+1} dx.$

16.33. $\int \frac{4x-5}{2x^2-5x+3} dx.$

16.35. $\int \frac{\frac{5}{6}x-16}{x^2+3x-18} dx.$

16.37. $\int \frac{dx}{6x^2-13x+6}.$

16.39. $\int \frac{7x}{4+5x^2} dx.$

16.41. $\int \frac{dx}{1+x-x^2}.$

16.43. $\int \frac{3x+2}{x^2-x-2} dx.$

16.45. $\int \frac{x-1}{4x^2-4x+1} dx.$

$$16.46. \int \frac{2x-13}{(x-5)^2} dx.$$

$$16.48. \int \frac{dx}{2x^2-2x+5}.$$

$$16.50. \int \frac{dx}{13-6x+x^2}.$$

$$16.52. \int \frac{x+1}{x^2-x+1} dx.$$

$$16.54. \int \frac{2x-1}{x^2-2x+5} dx.$$

$$16.56. \int \frac{2x-20}{x^2-8x+25} dx.$$

$$16.58. \int \frac{x+6}{x^2-3} dx.$$

$$16.60. \int \frac{6x}{x^2+4x+13} dx.$$

$$16.62. \int \frac{4x-5}{x^2-6x+10} dx.$$

$$16.64. \int \frac{x^2}{5x^2+12} dx.$$

$$16.66. \int \frac{7x^2+7x-176}{x^3-9x^2+6x+56} dx.$$

$$16.68. \int \frac{3x^2-5x+2}{x^3-2x^2+3x-6} dx.$$

$$16.70. \int \frac{x^3+2x-6}{x^2-x-2} dx.$$

$$16.72. \int \frac{x^4}{x^2+1} dx.$$

$$16.74. \int \frac{2x^4-10x^3+21x^2-20x+5}{x^2-3x+2} dx.$$

$$16.75. \int \frac{x^2+5x+41}{(x+3)(x-1)(x-\frac{1}{2})} dx.$$

$$16.47. \int \frac{3x+1}{(x+2)^2} dx.$$

$$16.49. \int \frac{dx}{3x^2+2x+1}.$$

$$16.51. \int \frac{3dx}{9x^2-6x+2}.$$

$$16.53. \int \frac{4x-1}{2x^2-2x+1} dx.$$

$$16.55. \int \frac{2x-10}{x^2-2x+10} dx.$$

$$16.57. \int \frac{3x+4}{x^2+4x+8} dx.$$

$$16.59. \int \frac{x+6}{x^2+3} dx.$$

$$16.61. \int \frac{10x-44}{x^2-4x+20} dx.$$

$$16.63. \int \frac{5x}{2+3x} dx.$$

$$16.65. \int \frac{2x^2+7x+20}{x^2+6x+25} dx.$$

$$16.67. \int \frac{x^3-4x^2+1}{(x-2)^4} dx.$$

$$16.69. \int \frac{2x+1}{(x^2+1)^2} dx.$$

$$16.71. \int \frac{2x^3-19x^2+58x-42}{x^2-8x+16} dx.$$

$$16.73. \int \frac{72x^6}{3x^2+2} dx.$$

$$16.76. \int \frac{17x^2-x-26}{(x^2-1)(x^2-4)} dx$$

$$16.77. \int \frac{2x}{(x^2+1)(x^2+3)} dx.$$

$$16.79. \int \frac{4x^3-2x^2+6x-13}{x^4+3x^2-4} dx.$$

$$16.81. \int \frac{6x^3+4x+1}{x^4+x^2} dx.$$

$$16.83. \int \frac{dx}{x^3+x^2+x}.$$

$$16.85. \int \frac{5x^3+3x^2+12x-12}{x^4-16} dx.$$

$$16.87. \int \frac{4x^3+9x^2+4x+1}{x^4+3x^3+3x^2+x} dx.$$

$$16.89. \int \frac{dx}{(x^2+x+1)^2}.$$

$$16.91. \int \frac{dx}{(x^2+4x+8)^3}.$$

$$16.93. \int \frac{dx}{x^4+64}.$$

$$16.95. \int \frac{dx}{x^4+6x^2+25}.$$

$$16.97. \int \frac{x^3-2x^2+5x-8}{x^4+8x^2+16} dx.$$

$$16.78. \int \frac{10x^3+110x+400}{(x^2-4x+29)(x^2-2x+5)} dx.$$

$$16.80. \int \frac{10x^3+40x^2+40x+6}{x^4+6x^3+11x^2+6x} dx.$$

$$16.82. \int \frac{dx}{x^3-a^2x}.$$

$$16.84. \int \frac{dx}{x^4+x^2+1}.$$

$$16.86. \int \frac{15x^2+66x+21}{(x-1)(x^2+4x+29)} dx.$$

$$16.88. \int \frac{dx}{x^3(x-1)^2(x+1)}.$$

$$16.90. \int \frac{3x^2-17x+21}{(x-2)^3} dx.$$

$$16.92. \int \frac{x^3-2x^2+7x+4}{(x-1)^2(x+1)^2} dx.$$

$$16.94. \int \frac{5x^3-11x^2+5x+4}{(x-1)^4} dx.$$

$$16.96. \int \frac{9x^4-3x^3-23x^2+30x-1}{(x-1)^4(x+3)} dx.$$

$$16.98. \int \frac{3x^2+x-2}{(x-1)^3(x^2+1)} dx.$$