£2 worth of symmetry

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Abstract

The recently circulated English £2 coin symbolises the Industrial Age by 19 interlocking cog-wheels. This leads to a study of clockwise and anticlockwise motion leading to a selection rule for interlocking cog wheels. This leads to remarks on mirror symmetry, screws and handedness which in turn leads to a discussion of the properties of neutrinos and the 6th June 1998 announcement of the SuperKamiokande results on the masses of neutrinos

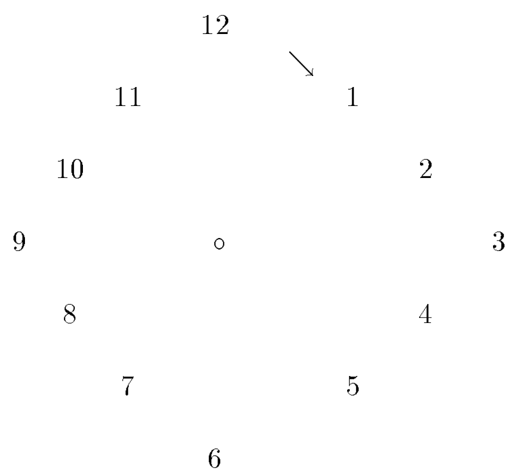
1. Introduction

There are two threads tying together this lecture. On 6th June 1998 I was flying from Rome to Warsawa and the stewardess handed me a copy of the International Herald Tribune of that day and to my considerable surprise I saw an article on the neutrino mass on the front page. The following week I flew to England and while there went to a bank and exchanged two £1 coins for a newly minted £2 coin.

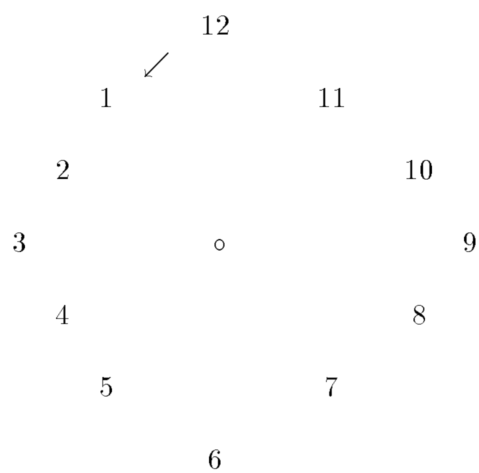
Much of the life of Isaac Newton was associated not with physics and mathematics but with the English Royal Mint. This association has recently been commemorated in England with the issue of a $2 coin for general circulation. The coin reputedly traces technological progress within four concentric circles. The innermost circle symbolises the Iron Age, the second circle the Industrial Age, the third circle the Electronic Age and the outermost circle the Internet Age. Around the edge of the coin are Newton's words "Standing on the shoulders of giants".

If we look closely at the coin we find that the Industrial Age is symbolised by 19 interlocking cog wheels. This leads us to investigate the properties of clockwise and anti-clockwise rotations leading to a selection rule for interlocking cog wheels. This takes us to studying mirror symmetry and the properties of left- and right-handed screws which form an analogue of the handedness properties of neutrinos and their parity non-conservation. After some discussion of the symmetry properties of neutrinos and the class of particles known as leptons we consider the 1998 neutrino oscillation experiments and their consequences for physics.

2 Clockwise and Anti-Clockwise Motion



Clockwise



Anti-Clockwise

The notion of clockwise and anticlockwise is undoubtedly ancient and associated with earlier notions of sunwise and anti-sunwise based upon the observation of the movement of shadows from left to right when, in the Northern hemisphere, one is facing North. The choice, and it was a choice, was dictated by the emergence of civilisation as we know it in the Northern Hemisphere. The choice could be regarded as an accident of history. Once the choice is made of clockwise motion then the choice is fixed for both hemispheres. This is a characteristic of physics, the attribution of negative charge to the electron is a free choice. Once having made the choice the sign of all other charges in the universe is fixed. Our freedom of choice brings constraints once we have exercised our freedom of choice there is no freedom left. We become slaves to our choice.

There are many examples of clockwise and anti-clockwise motion:-

3 Examples of Clockwise and Anti-Clockwise Motion

Clockwise Motion

1. Most Clocks
2. Northern Hemisphere Ocean Currents
3. Right-handed Screws
4. Tibetan Prayer Wheels

Anti-Clockwise Motion

1. Southern Hemisphere Ocean Currents
2. Left-handed Screws
3. Athletic Tracks
4. Grand Prix Motor Races
5. Horse Racing Tracks
6. American Base Ball
7. Muslims circling the Ka'aba seven times
8. Visiting the Stations of the Cross

4 Interlocking Cog-wheels and Motion

A single cog-wheel can be rotated both clockwise AND anti-clockwise. If two cog-wheels interlock one must turn clockwise and the other anti-clockwise and rotation is possible. If three cog-wheels interlock rotation is impossible whereas for four interlocking cog-wheels rotation is again possible. These observations lead us to a theorem concerning numbers of interlocking cog-wheels.

Theorem *Rotation is impossible for an odd number* (> 1) *of interlocking cog-wheels.*

Thus one could not construct a set of 19 interlocking cog-wheels as depicted on the English £2 coin and have motion!

Fortunately the Industrial Revolution involved EVEN numbers of interlocking cog-wheels otherwise the Industrial Revolution could not have happened.[1]

[1] At the end of the lecture Mr xxx suggested that if the cog-wheels were on a Möbius strip then motion would be possible for an *odd* number of cog-wheels whereas motion would not be possible for an *even* number of interlocking cog-wheels which is indeed the case!

They prided themselves on being practical men.
In the language of this defunct school of statesmen,
a practical man is a man who practises the blunders of his predecessors.
— Benjamin Disraeli, *Coningsby (1844)*

One is free to start to turn any one of an even number of interlocking cog-wheels in a clockwise or anti-clockwise. One has a freedom of choice but once that freedom is exercised the sense of rotation of all the other cog-wheels is determined.

Freedom brings constraints - With Freedom comes Slavery!

5 Selection Rules

Our cog-wheel theorem is an example of a *Selection Rule.* It tells us what is not possible NOT what is possible. Maybe with an even number of cog-wheels one or more will be stuck preventing motion. The establishment of selection rules is an important part of physics. Selection rules are associated with symmetry. If the symmetry exists then the selection rules hold. However, symmetry is always tentative and requires experimental verification. Even then one can never be sure that a more sensitive experiment may reveal a breakdown of the symmetry. The existence of symmetry forbids possible experiments - if the "impossible experiment" can be done then the symmetry is not an exact symmetry.

What never! Well hardly ever!
- Gilbert & Sullivan

6 Mirror Symmetry

If I place an object 1 metre in Front of a plane mirror its image appears 1 metre Behind the mirror. In general under reflection one has

$$\boxed{\mathbf{r} \rightarrow -\mathbf{r}}$$

View your Right Hand parallel to the plane of the mirror and it appears as a Left-Hand. Rotate your Right Hand clockwise and parallel to the plane of the mirror and it appears as a anti-clockwise rotating Left-Hand. However, rotate your Right Hand clockwise and perpendicular to the plane of the mirror and you see a Right Hand rotating clockwise.

What can be more like my hand or my ear than their reflections in a mirror? And still the hand in the mirror cannot be a substitute for my real hand
- Immanuel Kant

7 Right- and Left-Handed Screws

Carpenters and mechanics are familiar with two types of screws.

1. Right-Handed Screws that move Forward when turned in a Clockwise Direction.
2. Left-Handed Screws move that Forward when turned in an Anti-Clockwise Direction.

Under Reflection a Right-Handed Screw becomes a Left-Handed Screw and vice versa.

The contemporary scientific revolution has effected the dissolution of one of the most extensive superstitious beliefs of the age: the materialistic, clockwork universe of nineteenth-century physics. But perhaps all of this need not be considered on the old true/false scale of dualities and polarities. Perhaps it can be used merely to suspend temporarily our disbeliefs
— Sara Maitland *Women fly when men aren't watching*, Virago Press, London (1993)

8 Neutrinos

Neutrinos were hypothesized by Pauli in 1930 to overcome the so-called $\beta-$decay crisis. The assumed decay scheme

$$^A_ZX \Rightarrow ^A_{Z+1} Y + e^- \tag{1}$$

appeared to violate energy conservation and spin statistics. Pauli never published his hypothesis but made the suggestion in declining an invitation to a Ball. Pauli saw that both conservation principles could be satisfied if a very light neutral spin $s = \frac{1}{2}$ was added to the right-hand-side of Eq.(1). The energy conservation principle would be saved if energy was shared between the emitted electron and the neutral particle. It was assumed that the interaction of the neutral particle was very weak and would pass through any container, carrying with it energy. The spin statistics conservation principle was saved as now the spins on both sides of Eq.(1) would balance according to the quantum theory of angular momentum addition.

Positive evidence for the existence of the neutrino was not obtained until 1956 when in a heroic experiment Reines and Cowan observed interactions directly attributable to neutrinos emerging from a high-flux nuclear reactor.

You boil it in sawdust: you salt it in glue;
You condense it with locusts and tape
Still keeping one principal object in view
To preserve its symmetrical shape.
— Lewis Carroll *The Hunting of the Snark*

9 Some neutrino Facts

1. Many billions of neutrinos pass through you every second travelling from the sun. Most travel right through the earth.

2. Experimentally neutrinos and anti-neutrinos are distinct particles.

3. The sun emits $\sim 2 \times 10^{38}$ neutrinos/second.

4. The earth receives $> 4 \times 10^{10}$ neutrinos/sec/cm^2.

5. The human body emits about 3.4×10^8 neutrinos/day.

6. You receive an additional 5×10^9 neutrinos/sec from natural radioactivity in the earth.

7. You also receive from nuclear power plants all over the world 10 *to* 100×10^9 neutrinos/sec.

8. You receive just as many neutrinos in the night as in the day.

10 Neutrinos and SuperNova1987a

In 1987 those fortunate to live in the Southern Hemisphere were able to look towards the Small Magellanic Cloud and easily see a star close by in a part of the sky containing no bright stars. Indeed it was easier to find the Small Magellanic Cloud by first looking for the star, SuperNova1987a. This sight was not visible in the Northern Hemisphere as to see it photons would have had to pass right through the earth! However, there were two places in the Northern Hemisphere where SuperNova1987a was "seen" - in Ohio, USA and in Japan.

SuperNova1987a released vast quantities of neutrinos most passing straight through the earth. A few neutrinos were detected in Japan and Ohio, USA. These led to the conclusion that the mass of a neutrino could not be greater than $\sim 10eV$. But it could still be zero? This was a problem that physicists had pondered upon ever since 1930.

11 Neutrino Handedness

The mass of the neutrino was experimentally determined to be close to zero, possibly actually zero, like the photon. But only experiment can answer the question. A massless neutrino travelling at the speed of light can be viewed as a left-handed particle if its spin momentum is in the opposite direction to its direction of motion (Helicity = -1) or right-handed if the spin momentum is in the same direction as its motion (Helicity = +1). A left-handed neutrino (ν_L) is analogous to a left-handed screw and a right-handed neutrino (ν_R) to a right-handed screw. In 1957 neutrinos were found to be left-handed and anti-neutrinos right-handed. If the neutrinos are truly massless then they should be states of pure helicity but if they have mass they should travel at a speed less than that of light in which case an observer travelling faster than the particle would notice its handedness change upon passing it!

12 The $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{T}$ Symmetries

Central to much of fundamental physics are the three discrete symmetries designated as $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{T}$.

Charge Conjugation, $\mathcal{C}$

Turns a particle into its antiparticle.

$$e^- \xrightarrow{\mathcal{C}} e^+ \quad , \nu_L \xrightarrow{\mathcal{C}} \bar{\nu}_L$$

Parity, $\mathcal{P}$

Reflects a system through the origin. Right-handed coordinates go into left-handed coordinates.

$$\mathbf{r} \xrightarrow{\mathcal{P}} -\mathbf{r} \quad , \mathbf{s} \xrightarrow{\mathcal{P}} \mathbf{s} \quad , \nu_L \xrightarrow{\mathcal{P}} \nu_R$$

Time Reversal, $\mathcal{T}$

Reverses the direction of motion of particles.

$$t \xrightarrow{\mathcal{P}} -t$$

$\mathcal{CPT}$ Theorem

All interactions are invariant under combined $\mathcal{C}, \mathcal{P}$ and $\mathcal{T}$.

Parity for neutrinos was observed, in 1957, to be maximally violated in weak interactions, but neutrinos appeared to respect $\mathcal{CP}$ and hence timereversal symmetry $\mathcal{T}$. (In 1964 it was shown experimentally that the neutral mesons known as kaons indeed violated $\mathcal{CP}$ conservation).

How would you like to live in Looking-glass House, Kitty?
I wonder if they'd give you milk in there?
Perhaps Looking-glass milk isn't good to drink
- Lewis Carroll *Through the Looking Glass* (1872)

$$
\begin{array}{ccc}
\textit{exists} & & \textit{doesn't exist} \\
\nu_L & \xrightarrow{\mathcal{P}} & \nu_R \\
\Big\downarrow \mathcal{C} & & \Big\downarrow \mathcal{C} \\
\bar{\nu}_L & \xrightarrow{\mathcal{P}} & \bar{\nu}_R \\
\textit{doesn't exist} & & \textit{exists}
\end{array}
$$

13 Three Flavours of Neutrinos

Three seemingly distinct types (or *flavours*) of neutrinos exist each partnering a charged spin $\frac{1}{2}$ light particle. These particles form the *leptons* which engage in weak interactions. The different pairs of leptons are distinguished by their lepton numbers $L_e,\ L_\mu,\ L_\tau$.

$$
\begin{array}{c}
\\ L_e \\ \\ L_\mu \\ \\ L_\tau
\end{array}
\begin{pmatrix}
e^- & \bar{\nu}_e & e^+ & \nu_e & Mass \\
1 & -1 & -1 & 1 & 0.511MeV \\
\mu^- & \bar{\nu}_\mu & \mu^+ & \nu_\mu & \\
1 & -1 & -1 & 1 & 106MeV \\
\tau^- & \bar{\nu}_\tau & \tau^+ & \nu_\tau & \\
1 & -1 & -1 & 1 & 1777MeV
\end{pmatrix}
$$

1. **Are the Lepton Numbers $L_e,\ L_\mu,\ L_\tau$ Conserved?**
2. **Yes, IF masses of neutrinos are all zero OR they all are of exactly the same mass.**
3. **But are they? Once again only experiment can supply the answer.**

14 Cosmic Ray Production of Neutrinos

Cosmic rays from the depths of outer space bombard the earth with little variation over the earth's atmosphere or with respect to the time of day. They largely involve very energetic protons of an energy far exceeding that produced by any man made accelerator. Striking nuclei in the upper atmosphere they produce very short-lived π mesons which rapidly decay producing electronic (ν_e and muonic (ν_μ) neutrinos in a two-step process.

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

$$\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e$$

$$\pi^+ \rightarrow e^+ + \nu_\mu + \bar{\nu}_\mu + \nu_e$$

$$\pi^- \rightarrow e^- + \nu_\mu + \bar{\nu}_\mu + \bar{\nu}_e$$

This leads to:-

Prediction The ratio of muon neutrinos to electron neutrinos coming from cosmic rays is $\sim 2:1$.

Observation The ratio of muon neutrinos to electron neutrinos coming from cosmic rays is $\sim 1:1$

Why the difference?

Once you enter the world of science or mathematics or philosophy,
endless plains open around you.
The more you learn, the more fascinating the whole thing becomes.
— Colin Wilson, *Voyage to a Beginning*, 1968

15 Neutrino Oscillations

In 1967 Bruno Pontecorvo raised the question "Is the lepton number conserved?". Could it be that in addition to the usual weak interaction there is also an interaction taking place that mixes lepton numbers? In that case the neutrino masses would be different from zero and what we call, for example, muon neutrinos (ν_μ) and tauon neutrinos (ν_τ) produced in weak interactions are actually superpositions of neutrinos ν_1 and ν_2 of definite masses m_1 and m_2.

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} cos\Theta & sin\Theta \\ -sin\Theta & cos\Theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

In that case one could predict that neutrino oscillations could occur with $\nu_\mu \leftrightarrow \nu_\tau$. The probability of such transformations occurring can be computed as

$$\boxed{P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau) = sin^2(2\Theta) sin^2(1.27 \times \Delta m^2 \times \frac{L}{E_\nu})}$$

where

$$\Delta m^2 = |m_1^2 - m_2^2| [eV^2]$$

$$L = \text{Distance to Source}[km]$$

$$E_\nu = \text{Neutrino Energy}[GeV]$$

$$\Theta = \text{Mixing angle}$$

If one could observe such oscillations one would have unequivocal evidence for neutrinos having mass. One would be looking for a very small effect but an effect of tremendous significance in theories of fundamental physics.

An imperfection as perfections guest
Is greatest beauty
- John Clare *The Majic of Beauty (In* The Rural Muse)
(1835)

16 SuperKamiokande and the detection of neutrinos

The detection of cosmic ray produced ν_μ and ν_e neutrinos is favoured by the fact that their energies are in the GeV range and can be easily distinguished from solar neutrinos whose energy is in the MeV range which can also be confused with natural background neutrino sources. The principal cosmic ray produced neutrino detector is known as SuperKamiokande and is located $\sim$ 1km under Mount Ikena in the Japanese Alps.

SuperKamiokande contains 50,000 tonnes of ultra pure water which is "watched" by 11,146 photomultiplier tubes each of diameter 50cm. It has been collecting data since April 1996.

If a neutrino interacts with a quark in the water it produces a very energetic charged lepton travelling at a speed, in the water, greater than the speed of light in water. The interaction results in a cone of blue light (Cerenkov radiation). Measuring the pattern, timing and intensity allows physicists to distinguish between interactions arising from incoming muon or electron type neutrinos. It is not currently possible to detect $_\tau$ neutrinos so if a ν_μ neutrino is transformed into a ν_τ neutrino it escapes detection and fewer ν_μ than expected will be detected.

17 The Ups and Downs of Cosmic Ray Muon Neutrinos

SuperKamiokande can distinguish between downward neutrinos produced in the atmosphere at a height $\sim 15km$ and upward neutrinos that travel through the earth to the detector a distance of $\sim 13,000km$.

The downward neutrinos travel too short a distance to convert to ν_τ whereas the upward neutrinos travel a long distance and as SuperKamiokande does not detect ν_τs and hence fewer ν_μ are found in the upward neutrinos than in the downward ν_μ leading to a marked reduction in the ratio of $\nu_\mu : \nu_e$.

SuperKamiokande has unequivocally established the existence of neutrino oscillations involving the ν_μ neutrinos and most probably the ν_τ.

This is only possible if there is a non-zero mass difference between the oscillating neutrinos and hence the conclusion that the neutrinos necessarily have mass.

18 Consequences of the SuperKamiokande Results

1. The measured mass difference is $\sim 0.07eV$.
2. The lepton number L_μ is not conserved.
3. Probably none of the lepton numbers are conserved.
4. Neutrinos could make a significant contribution to the mass of the universe.
5. All known spin $\frac{1}{2}$ particles have mass.
6. We still do not know the absolute masses of any neutrinos.

19 Concluding Remarks

All symmetry based laws must be experimentally tested to establish their limits. "Exact" symmetries in physics are rare. Indeed the world would be very imperfect if every symmetry was perfect.

The SuperKamiokande experiment demonstrates the danger in physics of making symmetry assumptions and the need to subject them to continuous experimentation. Further results from SuperKamiokande and related experiments are to be expected. We are entering a new era of fundamental physics and the future is, as always, uncertain.

I hope in part that I have demonstrated that science should always be a source of excitement, wonderment and enjoyment.

20 Acknowledgments

I am grateful to the Polish KBN for supporting my work via the grants 18/P3/94/07 and 2P03/97/13. I take the opportunity to mark my appreciation of many wonderful Polish friends who have contributed so much to my time in Poland often in ways they probably do not realise themselves. Finally, I must thank the organisers of the Conference. It is always a pleasure to return to Rzeszów and to meet again many friends and to make new friends.