Eternal adiabaticity and KAM-stability

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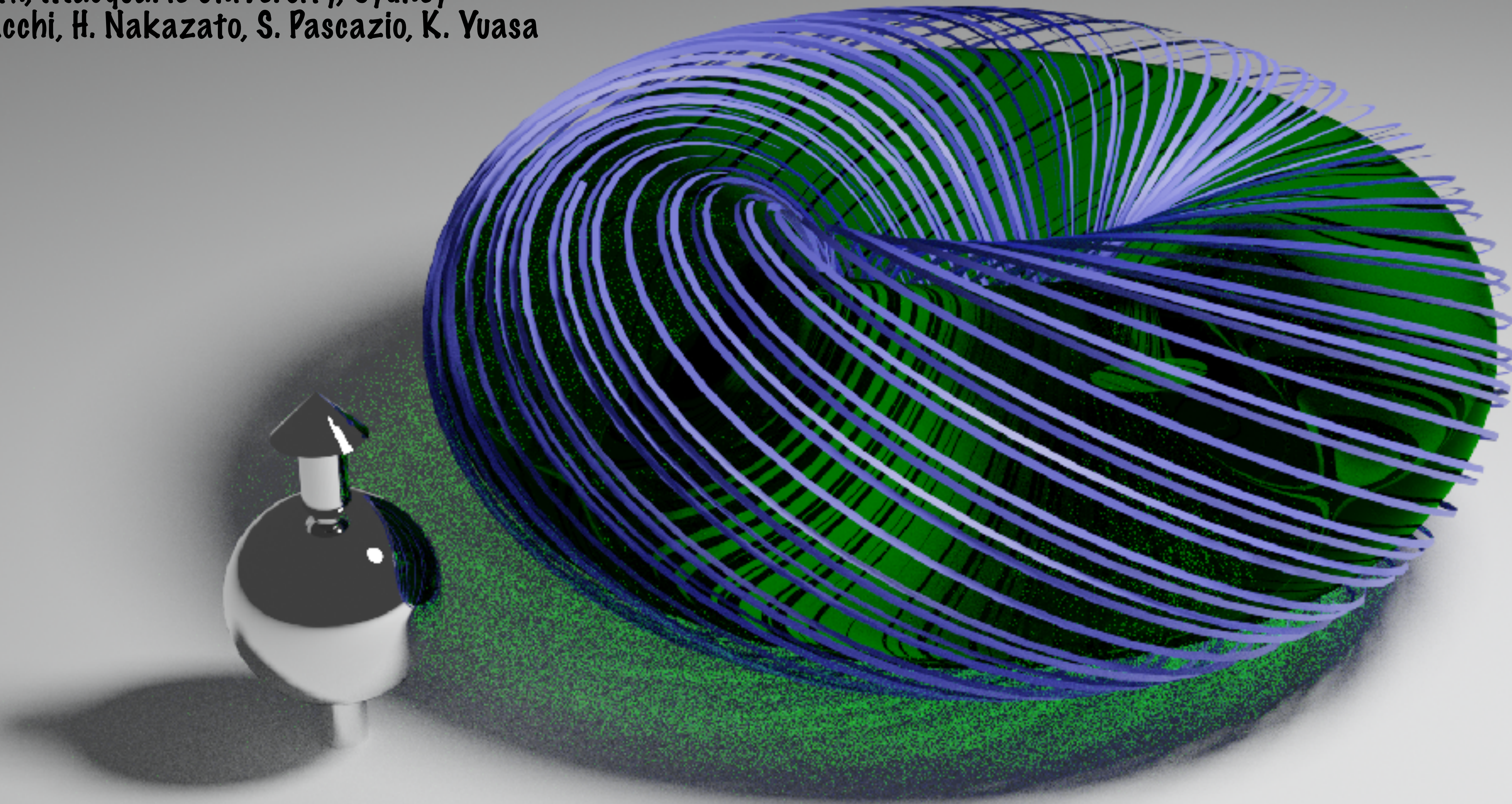

PRL + PRA '21

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Overview

- Develop a definition for "eternal adiabaticity" from a mathematical view
  - alternating between impossible and trivial
- Show that this is a useful property for stability of quantum dynamics
- Characterise which systems enjoy this property
- Focus on the **physical structure** of eternal adiabaticity in open quantum systems
  - Annoying counterexample which took us eternity to find
  - Present this in annoying detail

Axiomatic Approach I

Consider two matrices A, B

Is there a matrix C such that

C commutative " approximation " of B

(C not necessarily close to B)

$$[A, C] = 0 \quad \text{and} \quad \text{spectrum}(A + B) = \text{spectrum}(A + C)$$

Example:

$$C' = \left(-1 - \frac{1}{\sqrt{2}}\right)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$A = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad B = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\text{spectrum}(A + B) = \{1, -1\}$$

$$C = \left(1 - \frac{1}{\sqrt{2}}\right)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Diagonalisable case

WLG go in diagonal basis

$$A = \begin{pmatrix} a_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & a_n \end{pmatrix}$$

spectrum$(A + B) = \{\lambda_1, \ldots, \lambda_n\}$

$$C = \begin{pmatrix} \lambda_1 - a_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \lambda_n - a_d \end{pmatrix}$$

Fun fact for GKLS:

$$\mathscr{D}(\rho) = -i[Z, \rho]/2 + X\rho X - \rho$$

$$\mathscr{D} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & -i & 1 & 0 \\ 0 & 1 & i & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} - 1$$

$$0 = \det \begin{pmatrix} \lambda + i & -1 \\ -1 & \lambda - i \end{pmatrix} = \lambda^2 - i^2 - 1$$

Eigenvalues 0 but not the zero matrix -> non-diagonalisable

trivially possible

Non-diagonalisable case

Example:

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad C = \begin{pmatrix} c_1 & c_2 \\ c_3 & c_4 \end{pmatrix}$$

commuting is a too strong constraint for non-diagonalisable

$$[A, C] = \begin{pmatrix} c_3 & c_4 - c_1 \\ 0 & -c_3 \end{pmatrix} \overset{!}{=} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \qquad \Rightarrow \; C = \begin{pmatrix} c_1 & c_2 \\ 0 & c_1 \end{pmatrix}$$

$$\Rightarrow \textbf{spectrum}(A + C) = \{c_1, c_1\}$$

degenerate

Reminder: spectrum of triangular matrix are diagonal elements

$$B = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \Rightarrow \textbf{spectrum}(A + B) = \{1,0\}$$

generally impossible

Axiomatic Approach II

Consider two matrices $A = \sum_{\ell=1}^{d} a_\ell P_\ell + N_\ell$ and B

Spectral representation
Jordan Normal Form

Is there a matrix C such that $\forall \ell : \; [P_\ell, C] = 0$ and

spectrum$(A + B) =$ spectrum$(A + C)$

C block-respecting " approximation " of B

reduces to commutative in the diagonalisable case

Example: $A = \begin{pmatrix} a_1 & 1 & 0 \\ 0 & a_1 & 0 \\ 0 & 0 & a_2 \end{pmatrix}$ $P_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $P_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $N_1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $N_2 = 0$

spectrum$(A + B) = \{\lambda_1, \lambda_2, \lambda_3\}$

B can lift degeneracy

$$C = \begin{pmatrix} \lambda_1 - a_1 & 0 & 0 \\ 0 & \lambda_2 - a_1 & 0 \\ 0 & 0 & \lambda_3 - a_2 \end{pmatrix}$$

trivially possible

Remark:

spectrum$(A + B) =$ spectrum$(A + C)$ is equivalent to

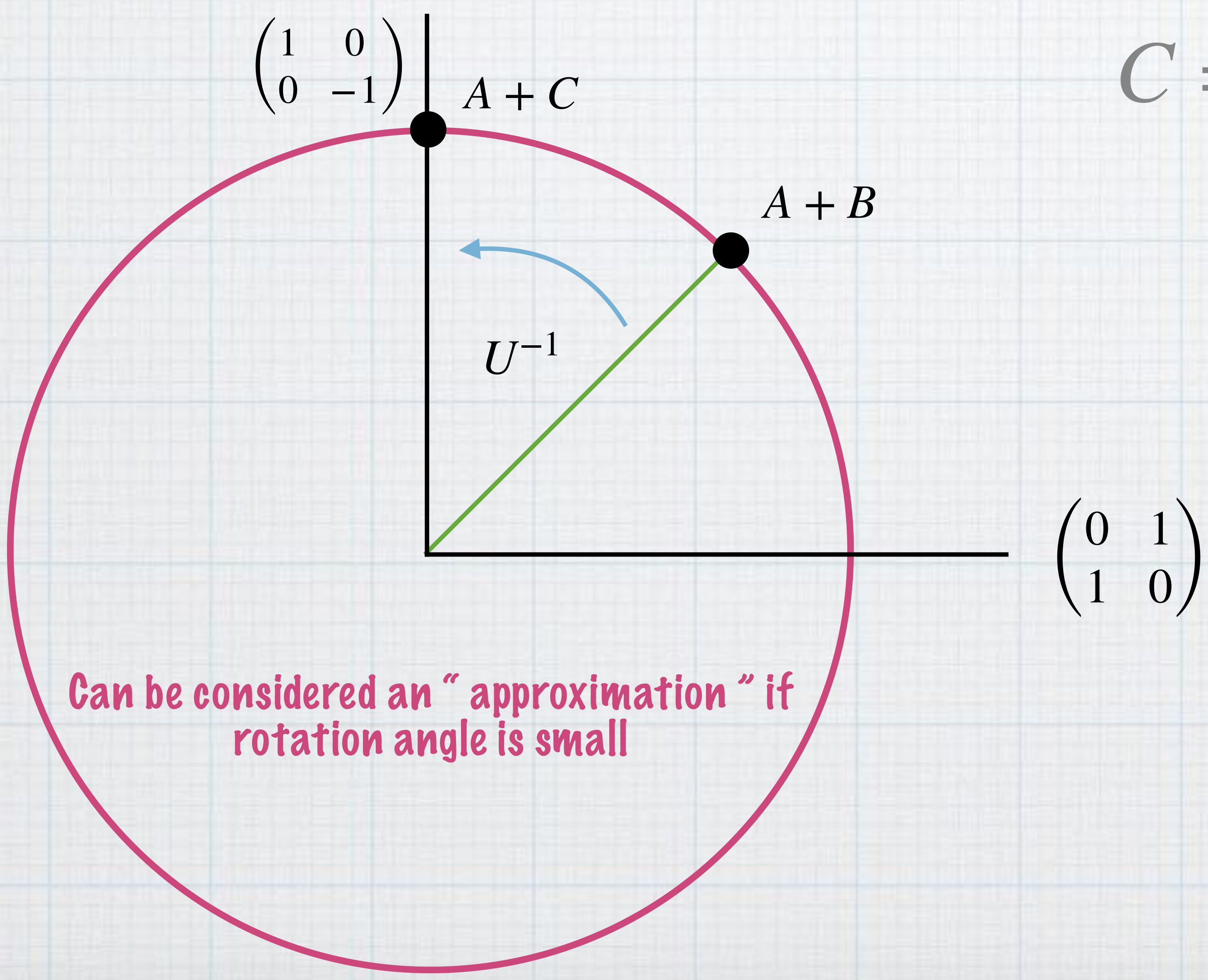
$\exists U = U(A, B)$ invertible: $U^{-1}(A + B)U = A + C$

adapted basis

Geometric View

$$A = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad B = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$C = \left(1 - \frac{1}{\sqrt{2}}\right)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$



consider

$$\|A\| \gg \|B\|$$

Axiomatic Approach III

Consider two matrices $A = \sum_{\ell=1}^{d} a_\ell P_\ell + N_\ell$ and B

If there is a matrix C such that

① $\forall \ell : \ [P_\ell, C] = 0$ and

② $\exists U = U(gA, B)$ invertible: $U^{-1}(gA + B)U = gA + C$

scaling parameter g

with

③ $\|U - 1\| = \mathcal{O}(g^{-1})$

then A, B called **eternally adiabatic**

$gA + C$ block-respecting isospectral approximation of $gA + B$

Dynamical View

Consider A, B eternally adiabatic

$$P_\ell e^{(gA+B)t} = P_\ell U e^{U^{-1}(gA+B)Ut} U^{-1}$$

$$\overset{②}{=} P_\ell U e^{(gA+C)t} U^{-1}$$

$$\overset{③}{=} P_\ell e^{(gA+C)t} + \mathcal{O}(g^{-1})$$

Independent of t

$$\overset{①}{=} e^{(gA+C)t} P_\ell + \mathcal{O}(g^{-1})$$

$$\overset{③}{=} U e^{(gA+C)t} U^{-1} P_\ell + \mathcal{O}(g^{-1})$$

$$\overset{②}{=} e^{(gA+B)t} P_\ell + \mathcal{O}(g^{-1})$$

If A, B generate dynamics, leakage between blocks is eternally suppressed

Quantum application

Consider H_0, V eternally adiabatic Hamiltonians

Let $\rho_G \propto \exp(-\beta H_0)$ be a Gibbs state of H_0.

Let the perturbed dynamics be $U_\epsilon(t) = \exp(-i(H_0 + \epsilon V)t)$

$\epsilon = \frac{1}{g}$

then

$$\text{supp}_{t\in\mathbb{R}} \|U_\epsilon(t)\rho_G U_\epsilon^\dagger(t) - \rho_G\| = \mathcal{O}(\epsilon)$$

trivially small for $t \ll \mathcal{O}(\epsilon^{-1})$

Proof: $\exp(-\beta H_0) = \sum_\ell e^{-\beta h_\ell} P_\ell$.

Eternal stability of Gibbs state under perturbed dynamics
-> KAM, PRL'21

Eternal Adiabaticity

When are two matrices A, B eternally adiabatic?

Theorem: if $e^{(gA+B)t} < M$ then A, B are eternally adiabatic.

any quantum dynamics (closed and open) in finite dimensions is eternally adiabatic!

Remarks:

- Intuition: fast oscillations and/or strong decay suppress transitions => Adiabatic Evolution
- Iterate to get better and better approximations (KAM)
- We give explicit non-perturbative bounds on $\|U - 1\|$
- Performance depends on the strength of B, number of distinct eigenvalues, on the amount of diagonalisability, and on the spectral gap of A
- We give explicit constructions of U, C: perturbative and non-perturbative

-> Eternal adiabaticity , PRA'21

Quantum application refined

Consider H_0, V arbitrary Hamiltonians on $|\mathscr{H}| < \infty$

Let $H_0 = \sum_{k=1}^{d} h_k P_k$ and $\eta = \min_{k \neq j} h_k - h_j$.

Let $\rho_G \propto \exp(-\beta H_0)$ be a Gibbs state of H_0.

Let the perturbed dynamics be $U_\epsilon(t) = \exp(-i(H_0 + \epsilon V)t)$

then $\text{supp}_{t \in \mathbb{R}} \| U_\epsilon(t) \rho_G U_\epsilon^\dagger(t) - \rho_G \| \leq 14\sqrt{d}\dfrac{\epsilon \|V\|}{\eta}$

$\text{supp}_{t \in \mathbb{R}} \| U_\epsilon(t) P_k U_\epsilon^\dagger(t) - P_k \| \leq 14\sqrt{d}\dfrac{\epsilon \|V\|}{\eta}$

Channels, Maps and all that

Eternal adiabaticity is a strong feature, allowing to bound adiabatic leakage, even without specifying C explicitly.

Nonetheless, sometimes we might want to think of C as an effective generator.

U is not unique - there is a much discussed gauge freedom

When A, B are Hermitian, we can choose U unitary, and C becomes Hermitian (des Cloizeaux/SW transformation).

When A, B are GKLS, what basis change U preserves this?

HS space is big!

Can C always be GKLS, eg. TP, HP, CCP?

[Kessler '12]

(Annoying) Example

$\mathcal{H} = \mathbb{C}^3$ qubit seems too simple

$$\dot{\rho} = -gi[H_0, \rho] + L_0 \rho L_0^\dagger - \frac{1}{2}\{L_0^\dagger L_0, \rho\}$$

$$A = -iH_0 \otimes 1 + i1 \otimes H_0^T \qquad B = L_0 \otimes \bar{L}_0 - \frac{1}{2} L_0^\dagger L_0 \otimes 1 - \frac{1}{2} 1 \otimes L_0^T \bar{L}_0$$

$$H_0 = \frac{1}{3}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix} \qquad L_0 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} = L_0^\dagger$$

interestingly, diagonalisable

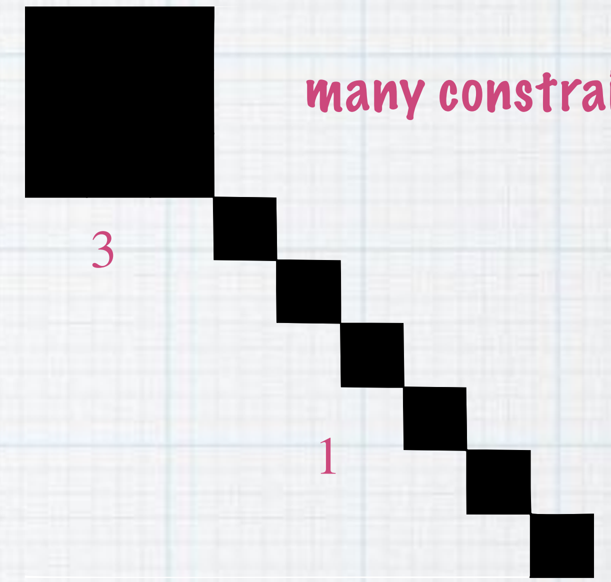
$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{i}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & i & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{i}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{2i}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2i}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

diagonal!

$$B = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

$$A = \begin{pmatrix} 0&0&0&0&0&0&0&0&0\\ 0&\frac{i}{3}&0&0&0&0&0&0&0\\ 0&0&i&0&0&0&0&0&0\\ 0&0&0&-\frac{i}{3}&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&\frac{2i}{3}&0&0&0\\ 0&0&0&0&0&0&-i&0&0\\ 0&0&0&0&0&0&0&-\frac{2i}{3}&0\\ 0&0&0&0&0&0&0&0&0 \end{pmatrix} \quad B = \begin{pmatrix} -1&0&0&0&0&0&0&0&1\\ 0&-\frac{1}{2}&0&0&0&0&0&0&0\\ 0&0&-1&0&0&0&1&0&0\\ 0&0&0&-\frac{1}{2}&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&-\frac{1}{2}&0&0&0\\ 0&0&1&0&0&0&-1&0&0\\ 0&0&0&0&0&0&0&-\frac{1}{2}&0\\ 1&0&0&0&0&0&0&0&-1 \end{pmatrix}$$

Example



many constraints

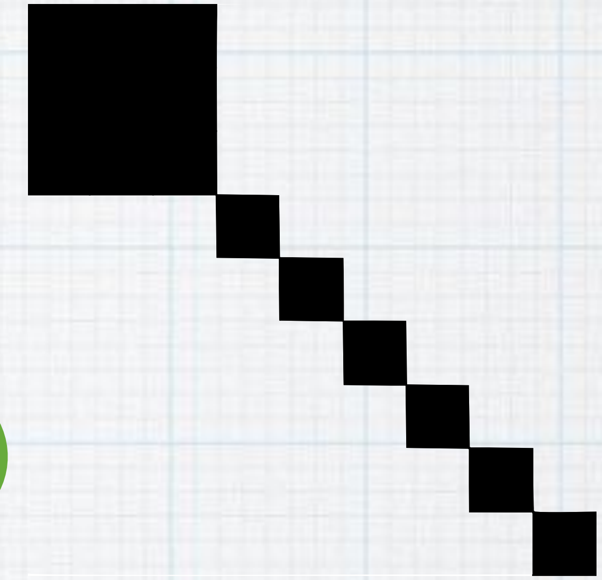
$$\textbf{spectrum}(A) = \left\{ 0^3, \pm\frac{i}{3}, \pm\frac{2i}{3}, \pm i \right\}$$

$igZ + X - 1$

$$gA + B = \begin{pmatrix} -1&0&0&0&0&0&0&0&1\\ 0&-\frac{1}{2}+\frac{gi}{3}&0&0&0&0&0&0&0\\ 0&0&gi-1&0&0&0&1&0&0\\ 0&0&0&-\frac{1}{2}-\frac{gi}{3}&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&-\frac{1}{2}+\frac{2gi}{3}&0&0&0\\ 0&0&1&0&0&0&-gi-1&0&0\\ 0&0&0&0&0&0&0&-\frac{1}{2}-\frac{2gi}{3}&0\\ 1&0&0&0&0&0&0&0&-1 \end{pmatrix}$$

$$\textbf{spectrum}(gA + B) = \left\{ 0^2, -2, -\frac{1}{2} \pm \frac{gi}{3}, -\frac{1}{2} \pm \frac{2gi}{3}, -1 \pm i\sqrt{g^2 - 1} \right\}$$

spectrum$(gA+B) = \left\{0^2, -2, -\frac{1}{2} \pm \frac{gi}{3}, -\frac{1}{2} \pm \frac{2gi}{3}, -1 \pm i\sqrt{g^2-1}\right\}$



Is there a commuting $gA + C$ with the same spectrum, HP, TP, CCP?

① ②

spoiler: no; assume there is and find contradiction

asking less: $gA + C$, not C CCP and not $\|U - 1\| = \mathcal{O}(g^{-1})$

TP: $(1 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 1)$ left eigenvector ev 0

TP: $(1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0)$ left eigenvector ev 0

HP: $r_k \in \mathbb{R}$

In eigenbasis of A, commuting obvious

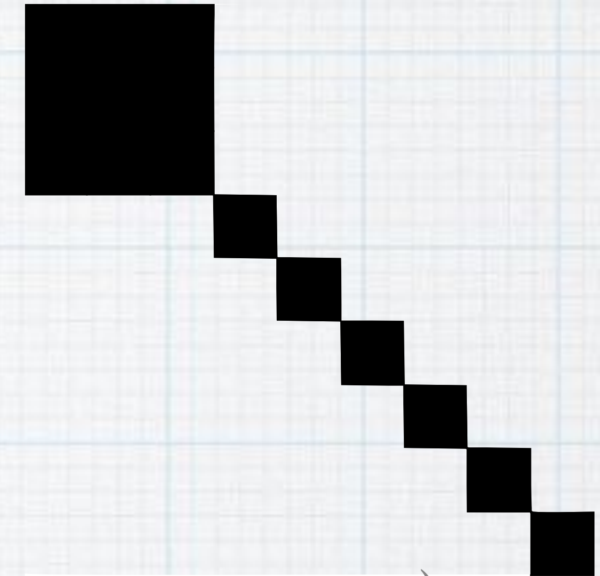
Most general commuting $gA + C$ HP, TP

Hermitian part of basis

complex pairs

$$gA + C = \begin{pmatrix} r_1 & r_2 & r_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ r_4 & r_5 & r_6 & 0 & 0 & 0 & 0 & 0 & 0 \\ -r_1 - r_4 & -r_2 - r_5 & -r_3 - r_6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & r_7 + ir_8 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & r_7 - ir_8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & r_9 + ir_{10} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & r_9 - ir_{10} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & r_{11} + ir_{12} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & r_{11} - ir_{12} \end{pmatrix}$$

$$\text{spectrum}(gA+B) = \left\{0^2, -2, -\frac{1}{2} \pm \frac{gi}{3}, -\frac{1}{2} \pm \frac{2gi}{3}, -1 \pm i\sqrt{g^2-1}\right\}$$



(TP and degeneracy)

Matching spectrum hard and may be non-unique

$$gA + C = \begin{pmatrix} r_1 & r_2 & r_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ r_4 & r_5 & r_6 & 0 & 0 & 0 & 0 & 0 & 0 \\ -r_1-r_4 & -r_2-r_5 & -r_3-r_6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & r_7+ir_8 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & r_7-ir_8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & r_9+ir_{10} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & r_9-ir_{10} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & r_{11}+ir_{12} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & r_{11}-ir_{12} \end{pmatrix}$$

Trace $(r_1 + r_5 - r_3 - r_6) = -2$

Trace $(r_7 + r_9 + r_{11}) = -2$

Imaginary: $\{\pm r_8, \pm r_{10}, \pm r_{12}\}$ **vs** $\left\{\pm\frac{g}{3}, \pm\frac{2g}{3}, \pm\sqrt{g^2-1}\right\}$

$g > 1$

Still 10 parameters to go...

$$\begin{pmatrix} r_1 & r_2 & r_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ r_4 & r_5 & r_1 - r_3 + r_5 + 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ -r_1 - r_4 & -r_2 - r_5 & -r_1 - r_5 - 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & r_7 + i r_8 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & r_7 - i r_8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & r_9 + i r_{10} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & r_9 - i r_{10} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -r_7 - r_9 - 2 + i r_{12} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -r_7 - r_9 - 2 - i r_{12} \end{pmatrix}$$

$$\{\pm r_8, \pm r_{10}, \pm r_{12}\} \; \{\pm \frac{g}{3}, \pm \frac{2g}{3}, \pm \sqrt{g^2 - 1}\}$$

CCP?

"reshuffle"

$$\mathbf{CJ}(gA + C) = \frac{1}{3} \begin{pmatrix} -r_1 - r_5 - 2 & 0 & 0 & 0 & r_7 - i r_8 & 0 & 0 & 0 & -r_7 - r_9 - 2 - i r_{12} \\ 0 & -r_2 - r_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -r_1 - r_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & r_1 - r_3 + r_5 + 2 & 0 & 0 & 0 & 0 & 0 \\ r_7 + i r_8 & 0 & 0 & 0 & r_5 & 0 & 0 & 0 & r_9 - i r_{10} \\ 0 & 0 & 0 & 0 & 0 & r_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & r_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & r_2 & 0 \\ -r_7 - r_9 - 2 + i r_{12} & 0 & 0 & 0 & r_9 + i r_{10} & 0 & 0 & 0 & r_1 \end{pmatrix}$$

Hermitian

$$\mathbf{spectrum}(P_\Omega^\perp \mathbf{CJ}(gA + B) P_\Omega^\perp) \supset$$

CCP : must be non-negative

$$\left\{ \pm c \sqrt{r_1^2 + (r_5 + 2r_7 - 2r_9 + 2)\, r_1 + r_5^2 + (r_8 + r_{10} - r_{12})^2 + 4(r_9 + 1) - 2r_5 (r_7 + 2r_9 + 1) + 4\left(r_7^2 + (r_9 + 2) r_7 + r_9^2\right)} \right\}$$

Mathematica

must be zero

$$r_1 = \frac{1}{2}\left(\pm\sqrt{-3r_5^2 + 12 r_7 r_5 + 12 r_9 r_5 + 12 r_5 - 12 r_7^2 - 4 r_8^2 - 12 r_9^2 - 4 r_{10}^2 - 4 r_{12}^2 - 24 r_7 - 24 r_7 r_9 - 24 r_9 - 8 r_8 r_{10} + 8 r_8 r_{12} + 8 r_{10} r_{12} - 12} - r_5 - 2r_7 + 2r_9 - 2\right)$$

real

must be non-negative
must be zero

$$r_5 = 2 + 2r_7 + 2r_9$$

$$r_{12} = r_8 + r_{10}$$

Checkmate!

$$P_\Omega^\perp = \begin{pmatrix} \frac{2}{3} & 0 & 0 & 0 & -\frac{1}{3} & 0 & 0 & 0 & -\frac{1}{3} \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & -\frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -\frac{1}{3} & 0 & 0 & 0 & -\frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \end{pmatrix}$$

$r_{12} = r_8 + r_{10}$

$\{\pm r_8, \pm r_{10}, \pm r_{12}\}$ $\{\pm \frac{g}{3}, \pm \frac{2g}{3}, \pm \sqrt{g^2 - 1}\}$

Imaginary part of the spectrum:

Cannot be matched

rationally dependent

$$\{\pm r_8, \pm r_{10}, \pm (r_8 + r_{10})\}$$

Should be:

rationally independent

$$\left\{\pm \frac{g}{3}, \pm \frac{2g}{3}, \pm \sqrt{g^2 - 1}\right\}$$

When A, B are Hermitian, C can always be chosen Hermitian

When A, B are GKLS, C cannot always be GKLS, eg. TP, HP, CCP

However, we can always construct C TP and HP. (Eternal adiabaticity , PRA'21)

Conclusions

- Defined **eternal adiabaticity** as a
  - block respecting
  - isospectral
  - approximation
- Allows us to obtain eternal stability against perturbations (Gibbs example)
- Any finite dimensional time independent quantum dynamics, noisy or noiseless, is eternally adiabatic
- The effective generator can always be TP and HP but not CCP

  ... violation is small $\mathcal{O}(g^{-1})$

  ... not all approximations are physical (Dyson...)

  ... still very annoying! (effective generators in low energy sector, adiabatic elimination, ...)
- Why not?
- Time-dependent version
- Infinite dimensions
- Thanks!

-> PRL'21 , PRA'21