52 Symposium on Mathematical Physics

"Channels, Maps and All That"

Toruń, June 14-17, 2021

Rate-operator quantum jumps: continuous-measurement interpretation and non-Markovian dynamics

Andrea Smirne

In collaboration with Jyrki Piilo, Dariusz Chruscinski, Kimmo Luoma & Matteo Caiaffa

More details in Phys. Rev. Lett. **124**, 190402 (2020)

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Outline

- Introduction and motivation: MCWF and beyond
- A novel quantum-jump method
- Extension to general non-Markovian dynamics
- Conclusion and outlook

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Open quantum systems: from foundations to applications

- The systems we want to control and manipulate are subjected to interaction with the surrounding environment, i.e., they are open systems
- Quantum properties (such as entanglement and coherences) are particularly fragile under such an interaction

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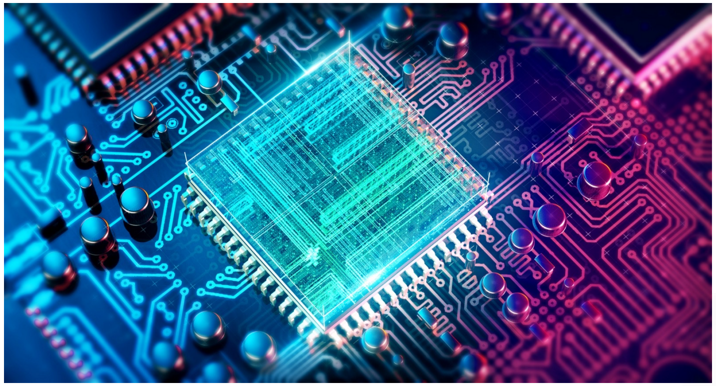
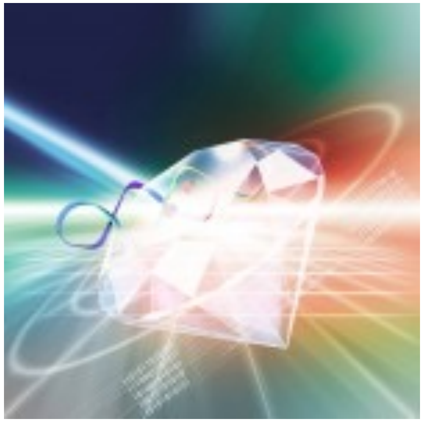
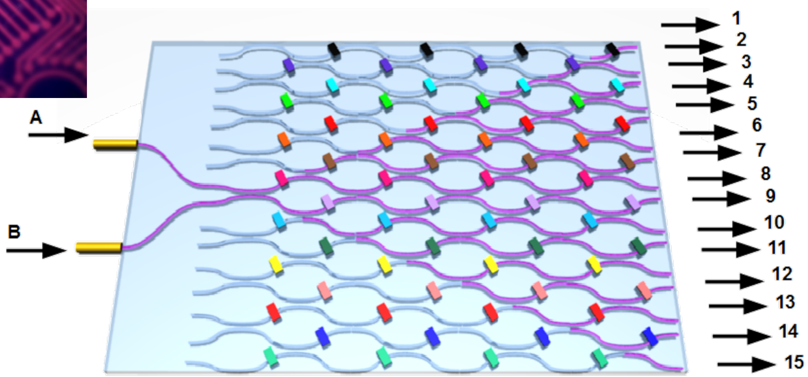
Quantum computation & communication



Quantum metrology and sensing



Quantum simulation



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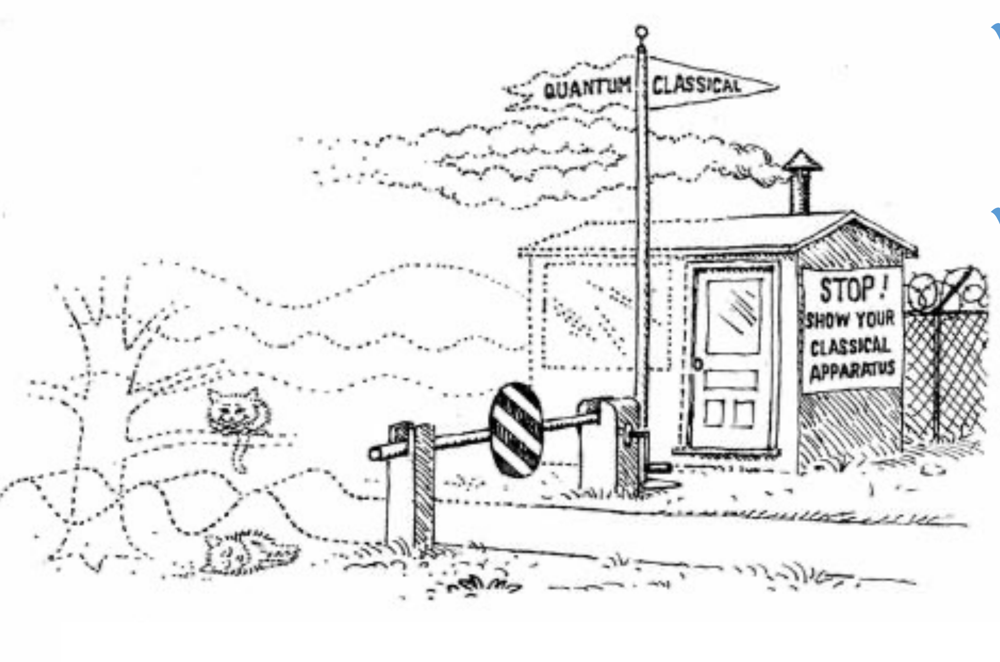
Foundations of Quantum Mechanics



Zurek, Physics Today (1991)

✓ Border between classical and quantum (decoherence theory)

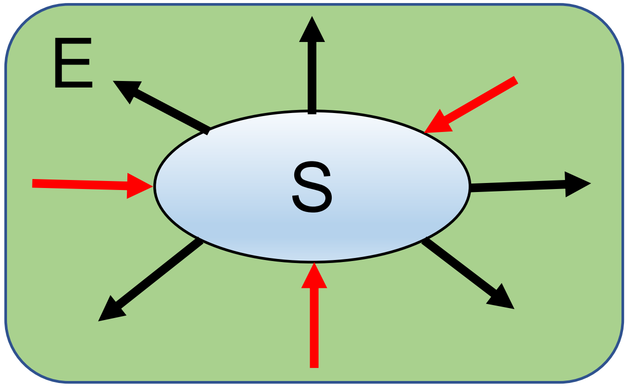
✓ A measurement is an interaction of the system with an "environment"

1st goal

To what extent can the environment be seen as a monitoring apparatus?

Non-Markovian quantum dynamics

- Memory effects: bidirectional system-environment information flow



$$\tau_E \sim \tau_R$$

Decay time of E excitations

Relaxation time of S

Interplay of S relaxation and E internal dynamics

⇨ non-trivial role of SE correlations!

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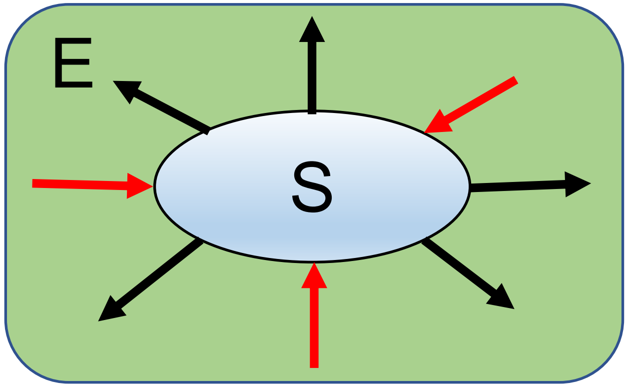

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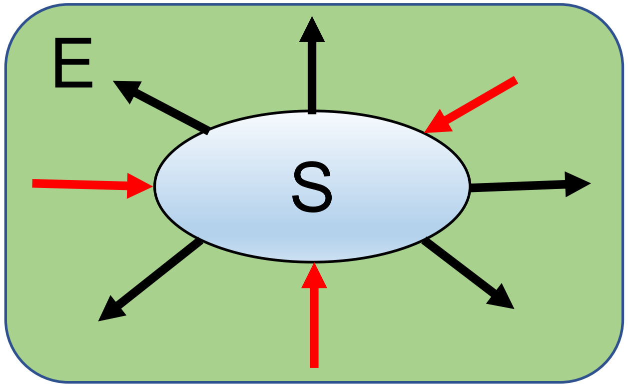
- Variety of analytical and numerical methods
  - Master equations
  - Path integrals
  - Perturbative expansions
  - Numerical ab-initio techniques
  - Stochastic methods

2nd goal

How can we treat memory effects in the description of the dynamics?

Non-Markovian quantum dynamics

- Memory effects: <u>bidirectional</u> system-environment <u>information flow</u>



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Stochastic unraveling – general idea

Starting point: time-local master equation for the system state $\rho(t)$

$$\frac{d}{dt}\rho(t) = -i[H_S(t), \rho(t)] + \sum_{\alpha=1}^{n^2-1} c_\alpha(t) \left(L_\alpha(t)\rho(t)L_\alpha(t)^\dagger - \frac{1}{2}\left\{ L_\alpha^\dagger(t) L_\alpha(t), \rho(t) \right\} \right)$$

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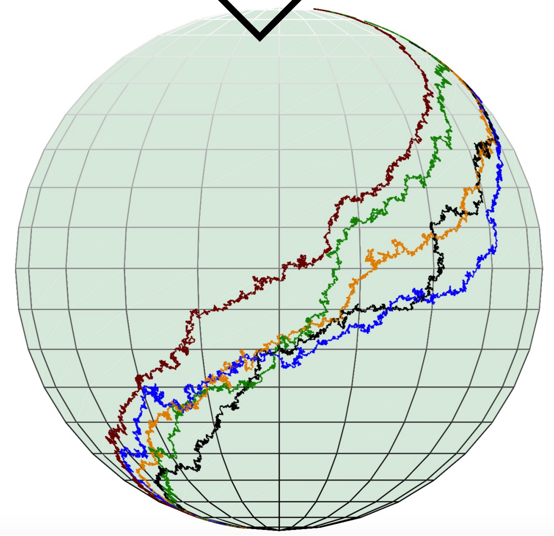
Infinitely many possible mappings



Stochastic trajectories on the set of pure states (fixed by a SDE)

Average of the trajectories

$$\rho(t) = \frac{1}{N}\sum_{i=1}^{N} |\psi_i(t)\rangle\langle\psi_i(t)|$$

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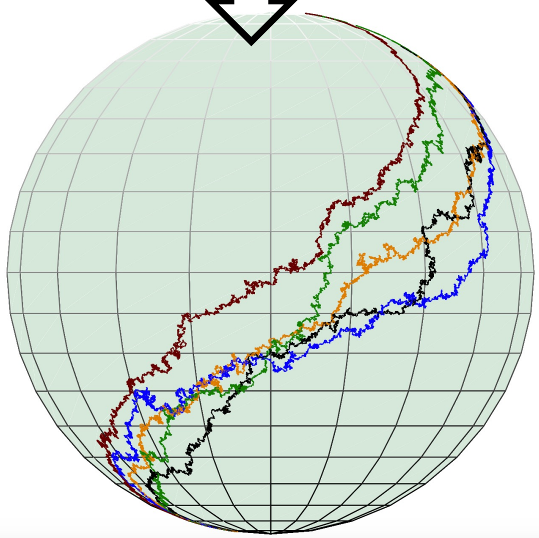

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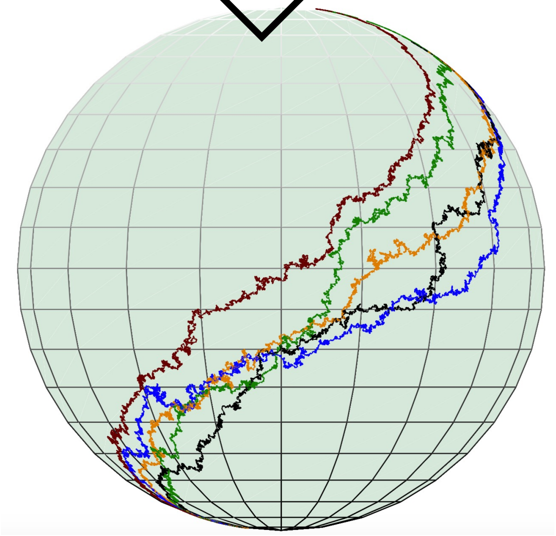

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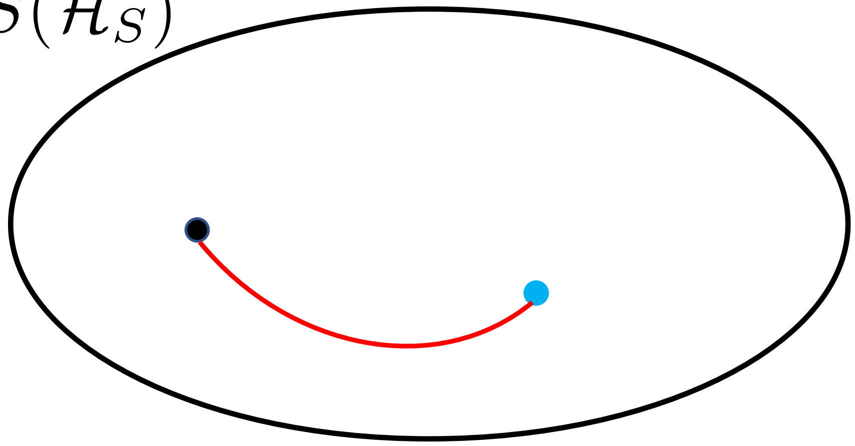
Monte Carlo wave function (MCWF) method

Carmichael *An Open System Approach to Quantum Optics* (1993)

$S(\mathcal{H}_S)$



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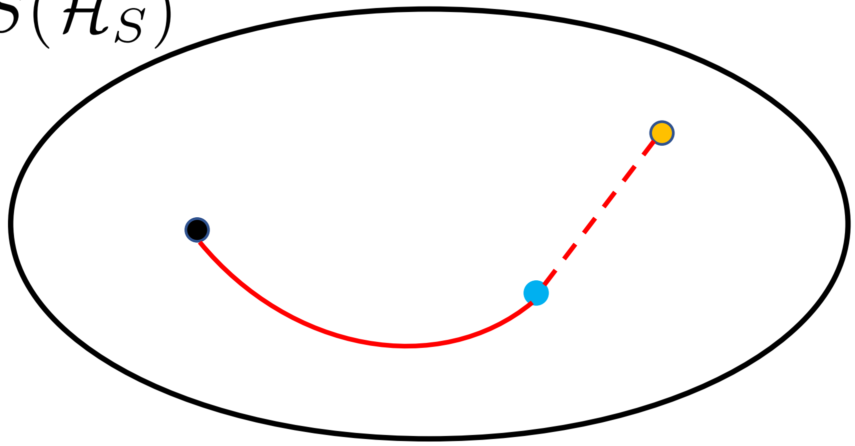

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rare events!

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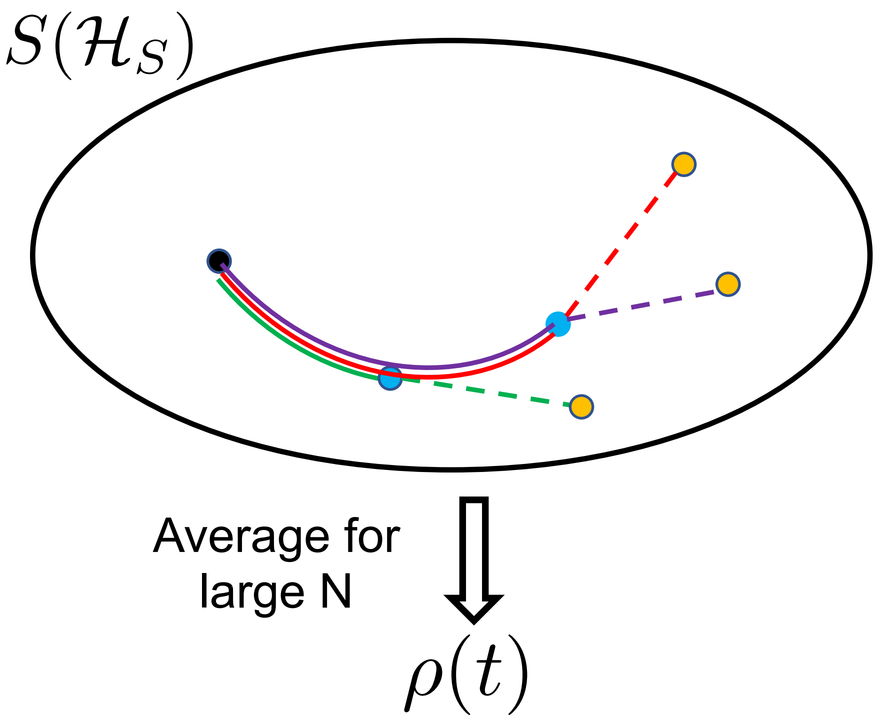

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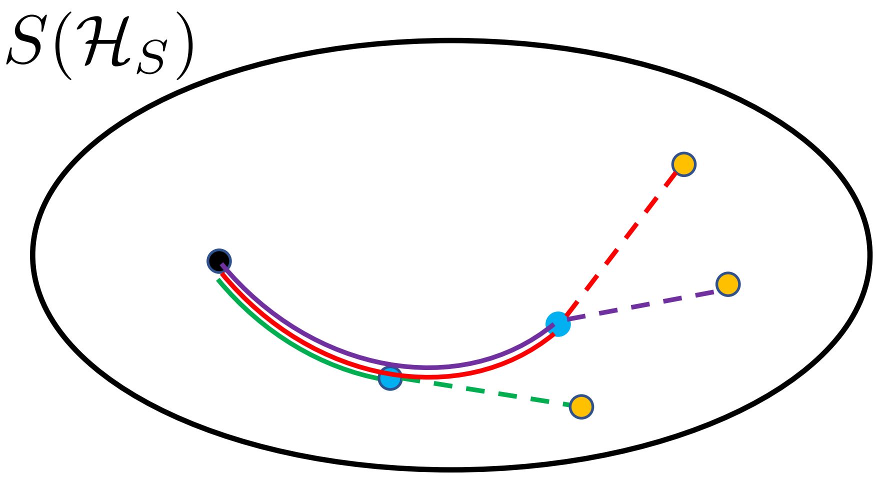

Average for large N

$\rho(t)$

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rare events!

- Quantum jumps observed experimentally in several platforms

Basche, Kummer & Brauchle Nature **373** (1995)   Jelezko et al APL **81** (2002)

Peil & Gabrielse PRL **83** (1999)   Gleyzes et al Nature **446** (2007)

MCWF: advantages and limits

✓ Pure states: the simulation cost scales linearly with the system dimension

Routinely used for complex, many body OQS   Daley Adv. Phys. **63** (2014)

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✓ Physical interpretation in terms of continuous measurements

$$|\psi(t)\rangle\langle\psi(t)| \mapsto \frac{L_\alpha|\psi(t)\rangle\langle\psi(t)|L_\alpha^\dagger}{\|L_\alpha|\psi(t)\rangle\|^2}$$

State transformation due to a measurement with outcome α

$$p_\alpha(t) = c_\alpha\|L_\alpha|\psi(t)\rangle\|^2 dt$$

Probability that the measurement gives outcome α

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The definition of the MCWF needs positive coefficients! $c_\alpha(t) \geq 0$

Positivity of the coefficients corresponds to CP-divisibility of the dynamics

Definition of Markovianity in   Rivas, Huelga & Plenio PRL **105** (2010)

Generalization of MCWF to negative coefficients   Piilo, Maniscalco, Harkonen, Suominen PRL **100** (2008)

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Can we extend the continuous-measurement interpretation?

Interplay of measurement backaction and backflow of information!

Diosi PRL **100** (2008) Wiseman & Gambetta PRL **101** (2008)

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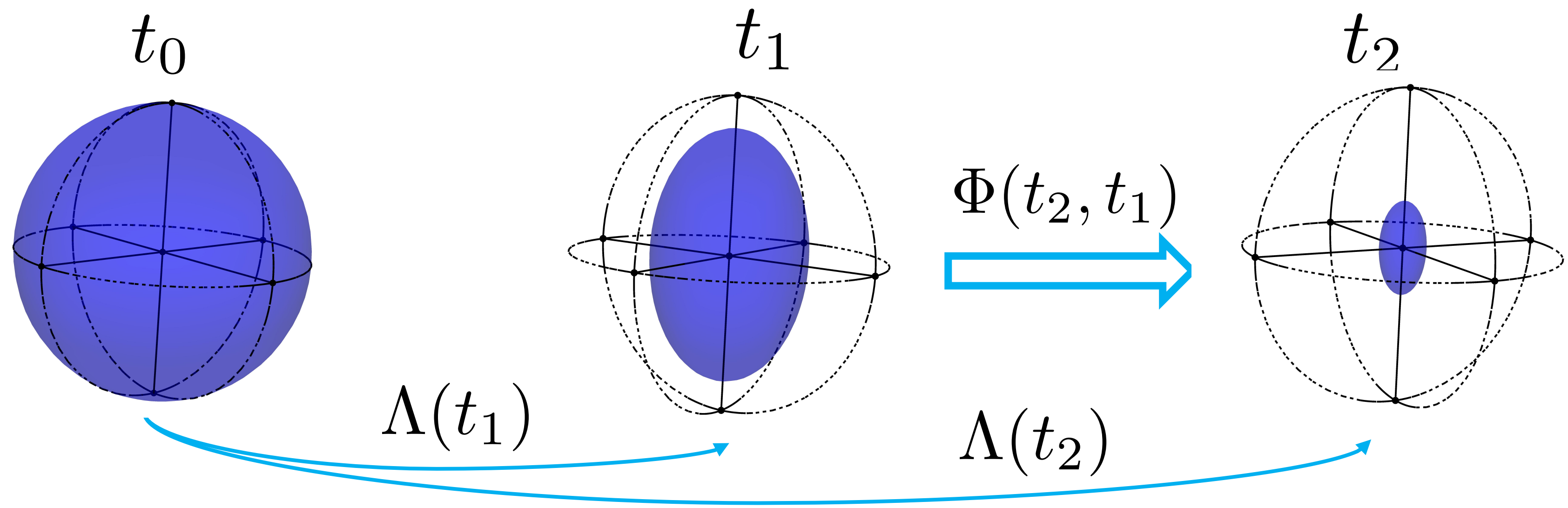
(C)P-divisible dynamics



The evolution of $\rho(t)$ can be split into (completely-)positive maps

$$\Lambda(t_2) = \Phi(t_2, t_1) \circ \Lambda(t_1)$$

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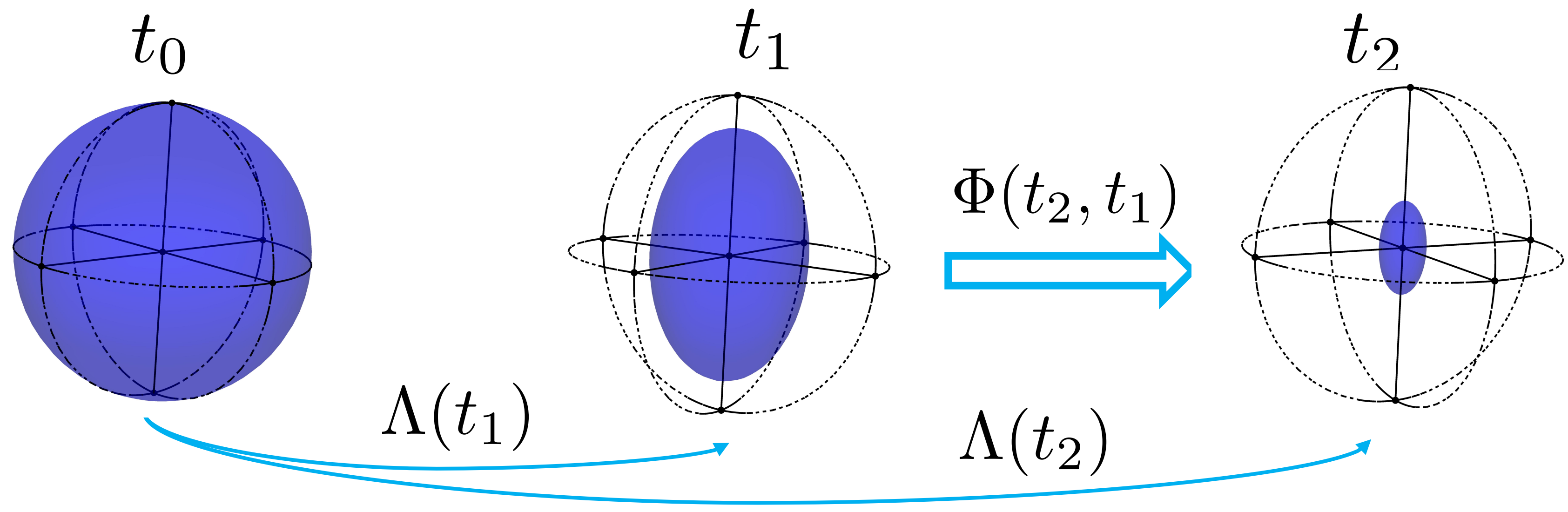

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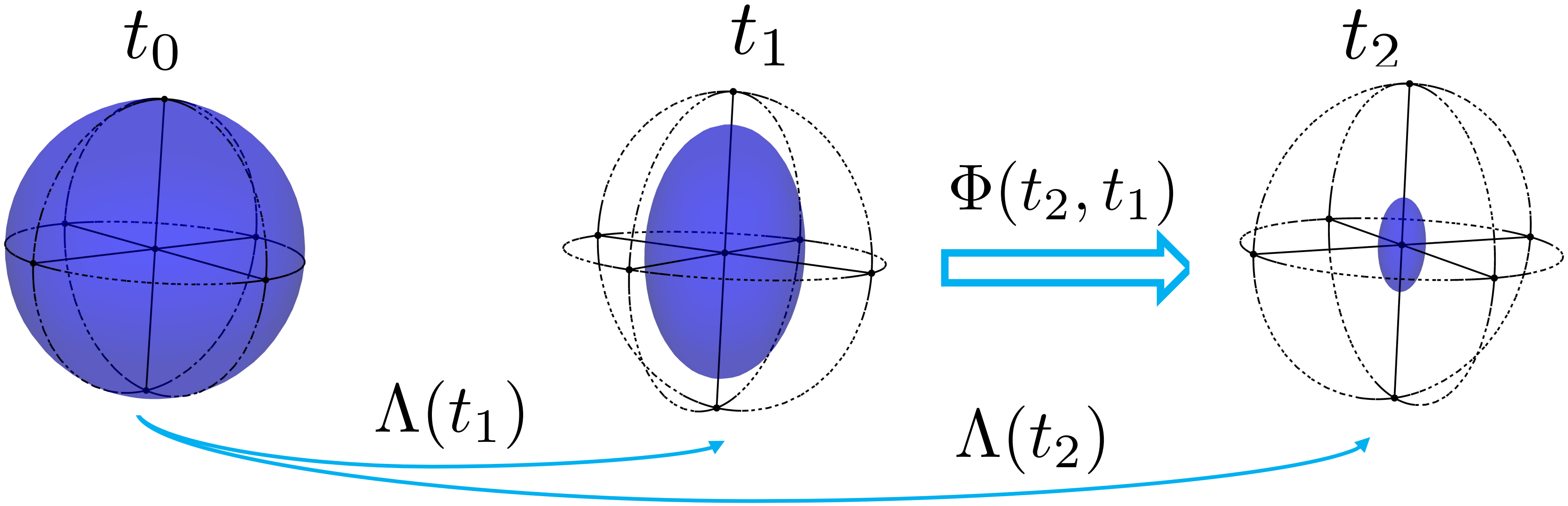

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$$\begin{pmatrix} \rho_{11} & \rho_{10} \\ \rho_{01} & \rho_{00} \end{pmatrix} \mapsto \begin{pmatrix} \rho_{11} e^{-\lambda t} & \rho_{10} e^{-\eta t} \\ \rho_{01} e^{-\eta t} & \rho_{00} e^{-\lambda t} \end{pmatrix} + (1 - e^{-\lambda t}) \frac{Id}{2}$$

P: any $\eta, \lambda \geq 0$

CP: $\eta \geq \lambda/2 \geq 0$!

Rate-operator quantum jumps

Rate Operator (RO)

Diosi PLA **114** (1986)

$$W^J_{\psi(t)} = \sum_{\alpha=1}^{n^2-1} c_\alpha(t)(L_\alpha(t) - \ell_{\psi(t),\alpha})|\psi(t)\rangle\langle\psi(t)|(L_\alpha(t) - \ell_{\psi(t),\alpha})^\dagger$$

$\ell_{\psi(t),\alpha} \rightarrow \langle\psi(t)|L_\alpha(t)|\psi(t)\rangle$

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Relying on

$$W^J_{\psi(t)} \geq 0 \quad \forall|\psi(t)\rangle \in \mathcal{H}_S \iff \Lambda(t) \ \text{P-divisible}$$

Kossakowski *On quantum statistical mechanics of non-Hamiltonian systems* RepMathPhys **3** (1972)

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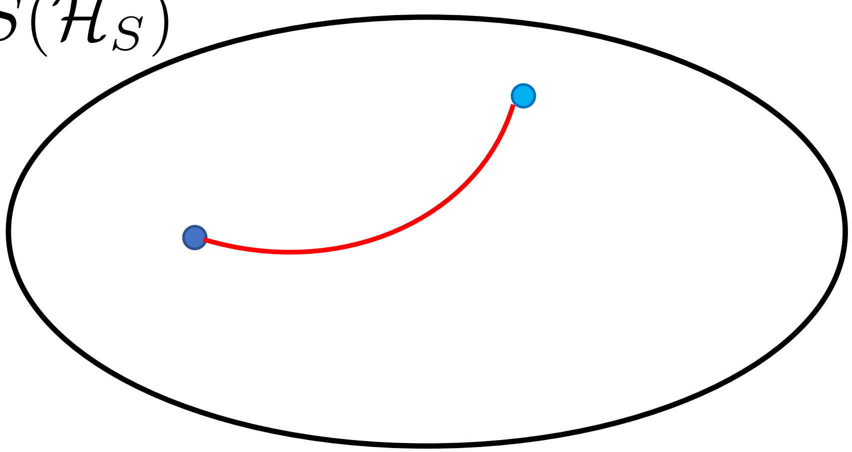

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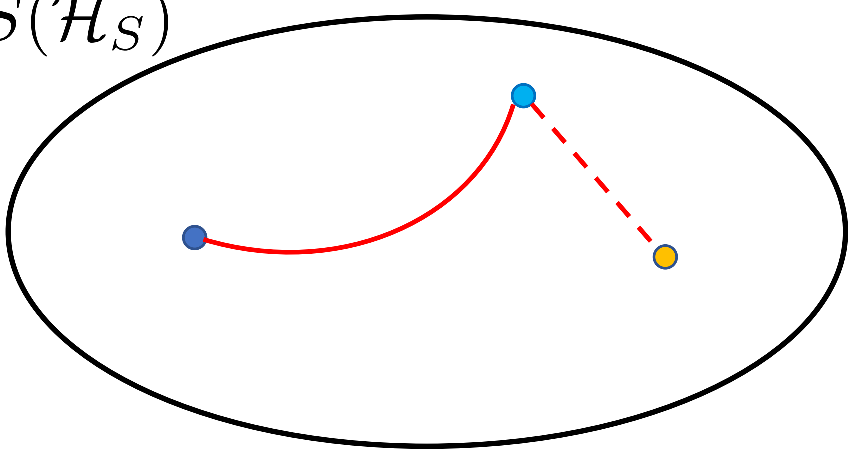

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- Interrupted by jumps

$$|\psi(t)\rangle \mapsto |\varphi_{\psi(t),j}\rangle \qquad p_j(t) = \lambda_{\psi(t),j}\,dt$$

$$W^J_{\psi(t)} = \sum_{j=1}^{n} \lambda_{\psi(t),j}|\varphi_{\psi(t),j}\rangle\langle\varphi_{\psi(t),j}|$$

Eigenvectors and **positive** eigenvalues

MCWF vs ROQJ

- Jumps and probabilities when the pre-jump state is $|\psi(t)\rangle$

	MCWF	ROQJ
Operators	$J_\alpha(t) = \sqrt{c_\alpha(t)} L_\alpha(t)$	$V_{\psi(t),j} = \sqrt{\lambda_{\psi(t),j}} \vert\varphi_{\psi(t),j}\rangle\langle\psi(t)\vert$
possible jumps between *t* and *t+dt*	$\vert\psi(t)\rangle \to \frac{J_\alpha(t)\vert\psi(t)\rangle}{\Vert J_\alpha(t)\vert\psi(t)\rangle\Vert}$	$\vert\psi(t)\rangle \to \frac{V_{\psi(t),j}\vert\psi(t)\rangle}{\Vert V_{\psi(t),j}\vert\psi(t)\rangle\Vert}$
probabilities	$p_\alpha(t) = \Vert J_\alpha(t)\vert\psi(t)\rangle\Vert^2 dt$	$p_j(t) = \Vert V_{\psi(t),j}\vert\psi(t)\rangle\Vert^2 dt$

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! The role of L_α and c_α in the MCWF is replaced by eigenvalues and eigenvectors of the RO ⇒ $p_j > 0$ also if some $c_\alpha < 0$

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deterministic evolution	$H_S - \frac{i}{2}\sum_\alpha c_\alpha L_\alpha^\dagger L_\alpha$	$H_S - \frac{i}{2}\sum_\alpha c_\alpha \left(L_\alpha^\dagger L_\alpha - 2\ell^*_{\psi(t),\alpha} L_\alpha + \lvert\ell_{\psi(t),\alpha}\rvert^2\right)$

! The role of L_α and c_α in the MCWF is replaced by eigenvalues and eigenvectors of the RO ⟹ $p_j > 0$ also if some $c_\alpha < 0$

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$$\sum_{j=1}^{n} p_j(t) \frac{V_{\psi(t),j}|\psi(t)\rangle\langle\psi(t)|V^\dagger_{\psi(t),j}}{\|V_{\psi(t),j}|\psi(t)\rangle\|^2} = \sum_{j=1}^{n} \lambda_{\psi(t),j}|\varphi_{\psi(t),j}\rangle\langle\varphi_{\psi(t),j}| = W^J_{\psi(t)} dt$$

MCWF vs ROQJ

- Jumps and probabilities when the pre-jump state is $|\psi(t)\rangle$

	MCWF	ROQJ
Operators	$J_\alpha(t) = \sqrt{c_\alpha(t)} L_\alpha(t)$	$V_{\psi(t),j} = \sqrt{\lambda_{\psi(t),j}} \lvert\varphi_{\psi(t),j}\rangle\langle\psi(t)\rvert$
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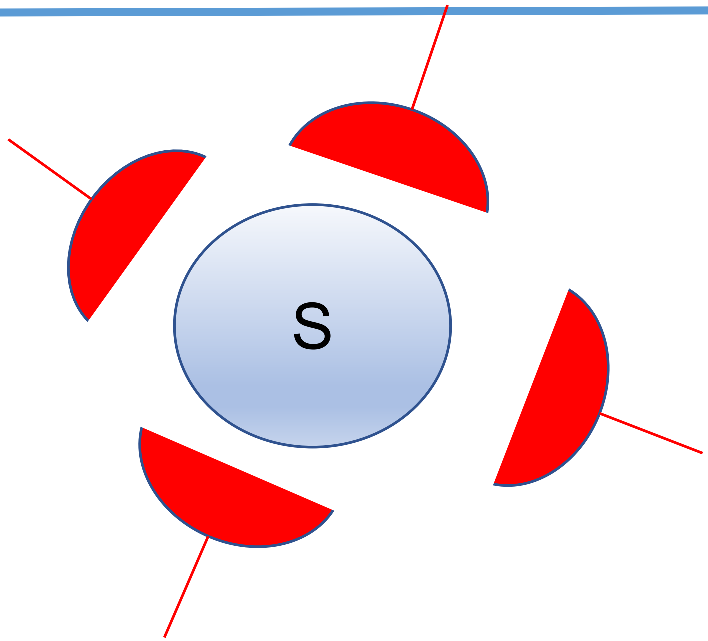
- Non-linearities from jump and deterministic parts cancel out

Continuous-measurement framework



The system is continuously monitored by *n* detectors: every dt two kinds of events

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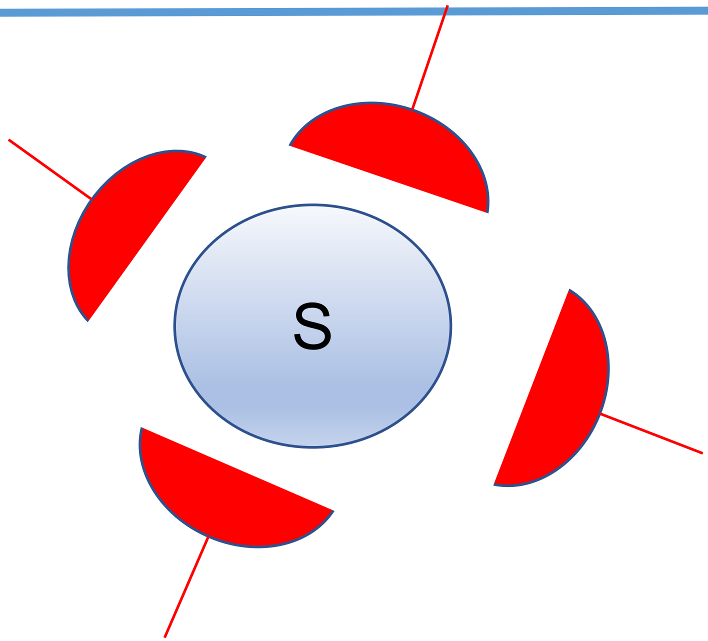

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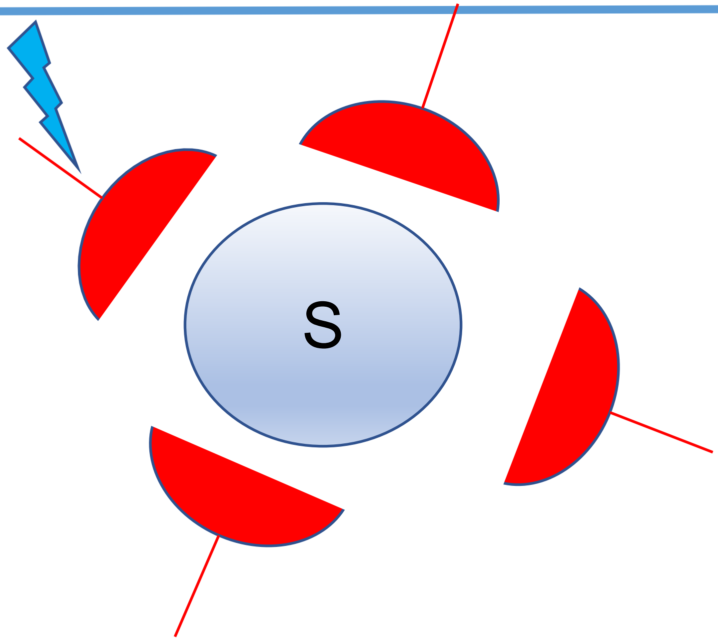

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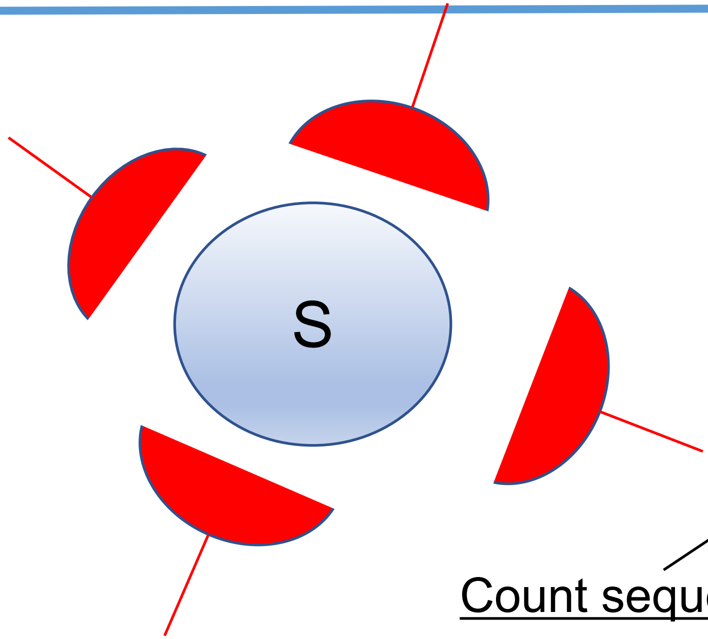

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<u>Count sequence</u>: Instants and types of the (non-null) counts

It fixes the system state $|\psi(\omega_t)\rangle$

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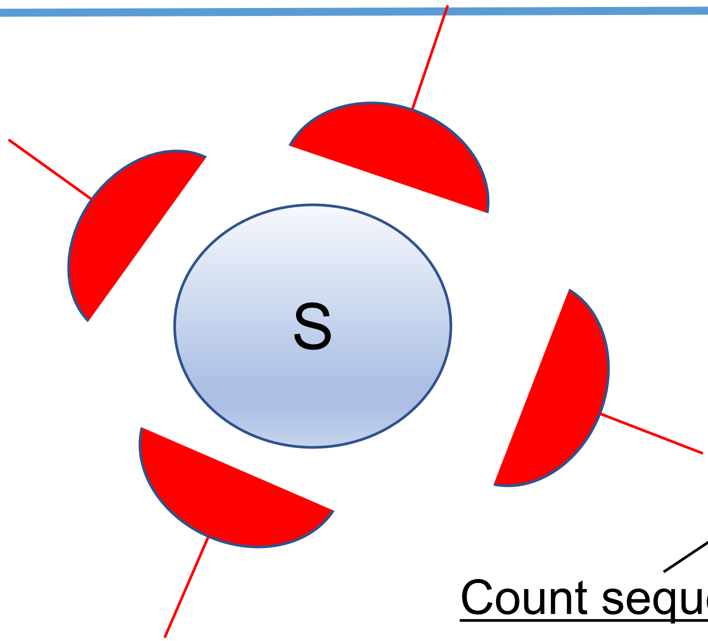

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It fixes the system state $|\psi(\omega_t)\rangle$

<u>Quantum instrument</u>: map from the set of outcomes to CP maps on $S(\mathcal{H}_S)$

$$\mathcal{I}_{\omega_t,\emptyset}\rho = F_{\omega_t,\emptyset}\rho F^{\dagger}_{\omega_t,\emptyset} \qquad F_{\omega_t,\emptyset} = \left(Id - iH^{eff}_{\omega_t}dt\right)|\psi(\omega_t)\rangle\langle\psi(\omega_t)|$$

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Trajectories of the continuous measurement

- State transformations and probabilities by postulates of quantum mechanics

$$\rho \mapsto \frac{\mathcal{I}_{\omega_t, j(\emptyset)} \rho}{\mathrm{Tr}\left\{\mathcal{I}_{\omega_t, j(\emptyset)} \rho\right\}} \qquad p_{j(\emptyset)}(t) = \mathrm{Tr}\left\{\mathcal{I}_{\omega_t, j(\emptyset)} \rho\right\}$$

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If we identify the count sequence with the sequence of jumps, we get the same trajectories and associated probabilities as the unraveling!!

Environment as a non-selective observer

- By construction, the average over the trajectories gives us

$$\frac{d}{dt}\rho(t) = -i[H_S(t), \rho(t)] + \sum_{\alpha=1}^{n^2-1} c_\alpha(t) \left(L_\alpha(t)\rho(t)L_\alpha(t)^\dagger - \frac{1}{2}\left\{L_\alpha^\dagger(t)L_\alpha(t), \rho(t)\right\} \right)$$

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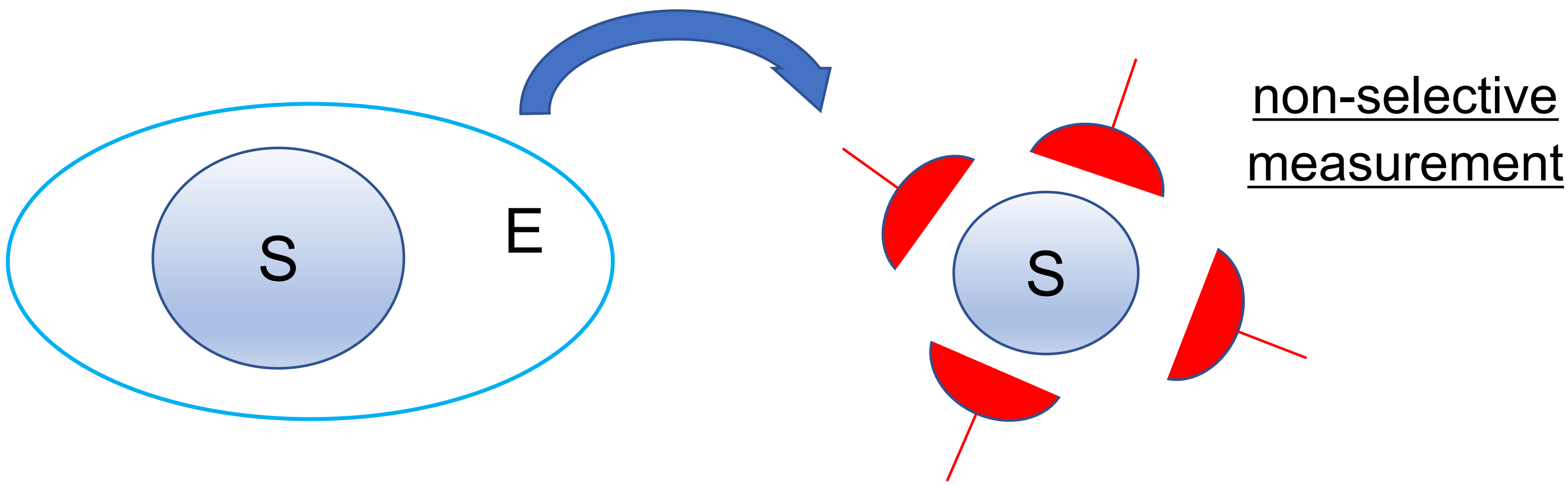

Consistent continuous-measurement picture Barchielli & Belavkin JPA **24** (1991)

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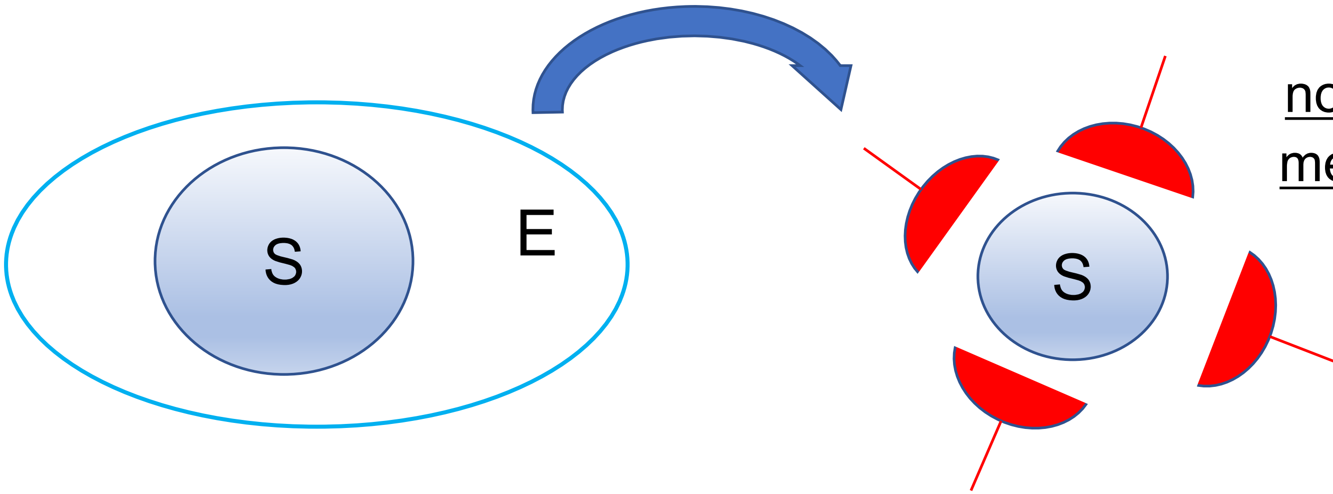

non-selective measurement

Consistent continuous-measurement picture Barchielli & Belavkin JPA **24** (1991)

! $\mathcal{I}_{\omega_t, j}(\emptyset)$ The instrument itself depends on the sequence of jumps
Dynamics' complete positivity not needed: positivity is enough

Case study 1: Qubit evolution

Hall, Cresser, Li & Andersson PRA **89** (2014)

$$\frac{d}{dt}\rho(t) = \frac{1}{2}\sum_{k=1}^{3}\gamma_k(t)\left[\sigma_k\rho(t)\sigma_k - \rho(t)\right]$$

$$\begin{aligned}\gamma_1(t) &= 1\\ \gamma_2(t) &= 1\\ \gamma_3(t) &= -\tanh(t)\end{aligned}$$

- The dynamics is completely positive and P-divisible
- One negative coefficient for any t>0

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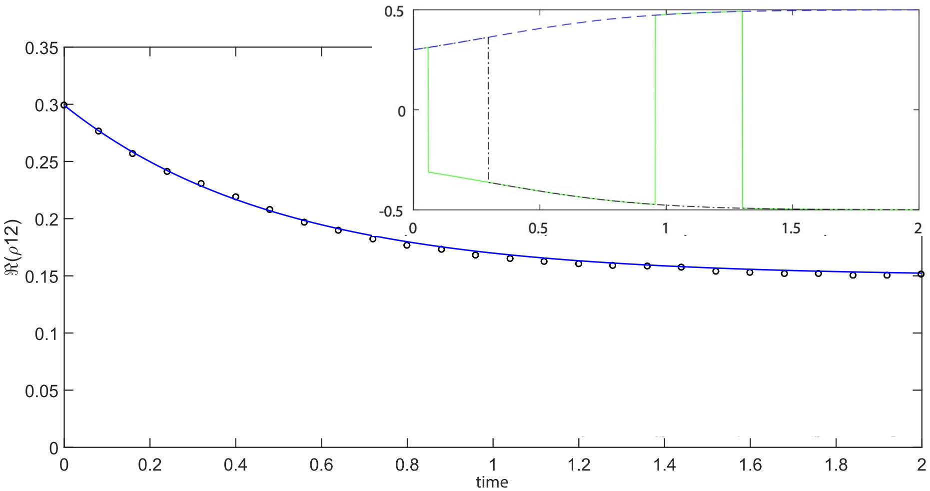

- Solid line: solution of the master equation
- Circles: average over 10^4 trajectories
- dt = 2 x 10^{-3}
- Inset: example of 3 trajectories
- Error bars within the circles

<u>Effective unraveling for all t>0; jumps associated with detectors' "clicks"</u>

Outline

- Introduction and motivation: MCWF and beyond
- A novel quantum-jump method
- Extension to general non-Markovian dynamics
- Conclusion and outlook

Beyond P-divisible dynamics

$$W^J_{\psi(t)} = \sum_{j^+} \lambda_{\psi(t),j^+} |\varphi_{\psi(t),j^+}\rangle\langle\varphi_{\psi(t),j^+}| - \sum_{j^-} |\lambda_{\psi(t),j^-}| \, |\varphi_{\psi(t),j^-}\rangle\langle\varphi_{\psi(t),j^-}|$$

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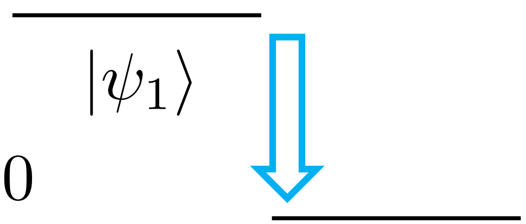

$$|\psi_1\rangle$$

$$\lambda_{\psi_1,j} > 0$$

$$|\psi_2\rangle = |\varphi_{\psi_1,j}\rangle$$

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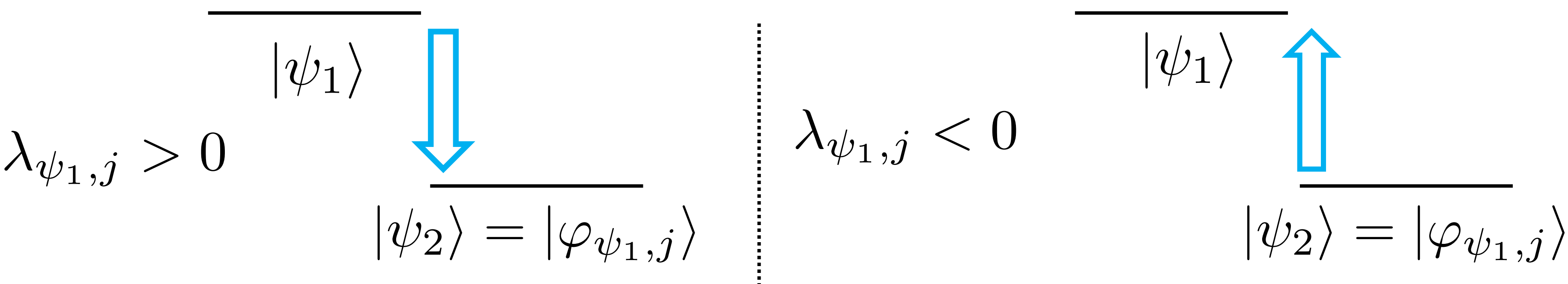

Case study 2: 7-coupled-site system

Dissipative network of coupled sites (transport phenomena, …)

- Coherent coupling among the sites $H_S = \sum_{i \neq j} \Omega_{i,j} |i\rangle\langle j|$ $0 \leqslant \Omega_{i,j} \leqslant 0.6$ uniformly random
- 49 decay channels: transitions+dephasing $c(t)|i\rangle\langle j|$ $c(t) < 0$ at some *t*

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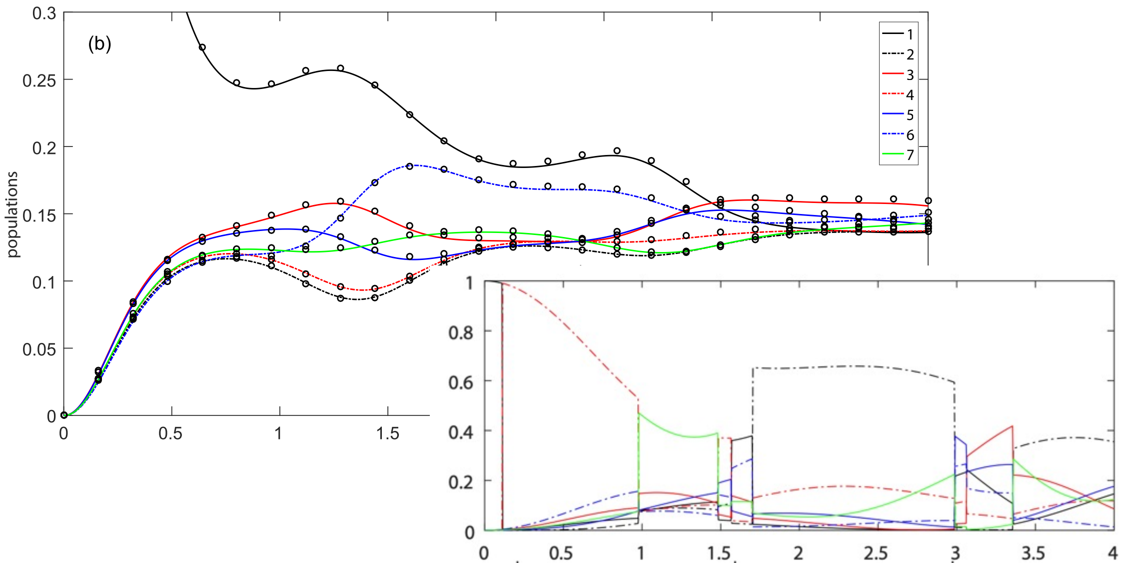

- Different site populations
- Trajectory of populations with reversed jumps
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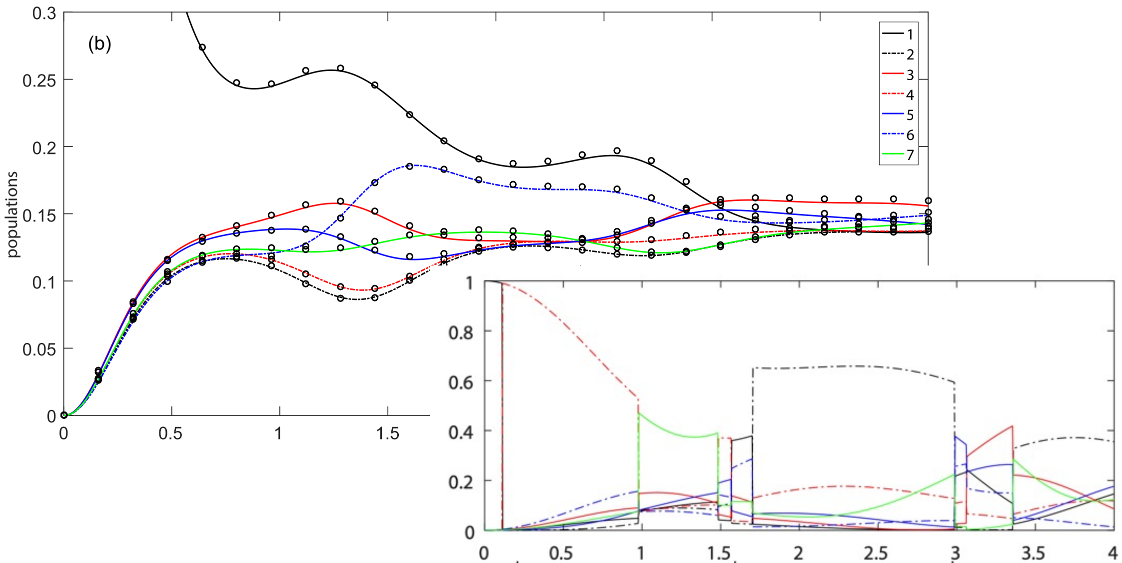

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Compared to existing extensions of MCWF:

- *n* instead of *n*^{2} jump operators
- it can be applied to a larger set of dynamics

Markovian, non-Markovian and in-between

- Non-Markovianity: memory effects influence the evolution

Markovian, non-Markovian and in-between

- Non-Markovianity: memory effects influence the evolution

Strength of memory effects

Qualitative scale!

- All coefficients are positive (CP-divisible dynamics)

  MCWF and standard measurement interpretation apply

! The jump probabilities at t depend on the previous sequence of jumps $p_\alpha(t) = c_\alpha \| L_\alpha |\psi(t)\rangle \|^2 dt$

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- Not P-divisible dynamics: reversed jumps

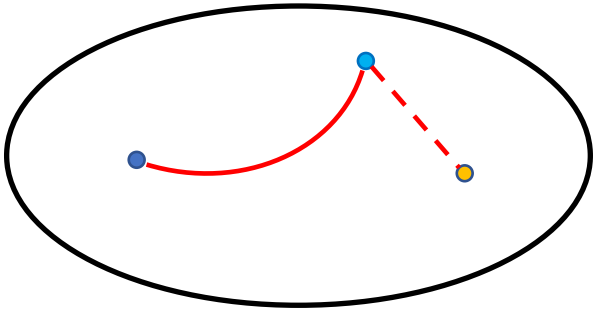
  The future on a trajectory depends on the <u>past of all trajectories</u>

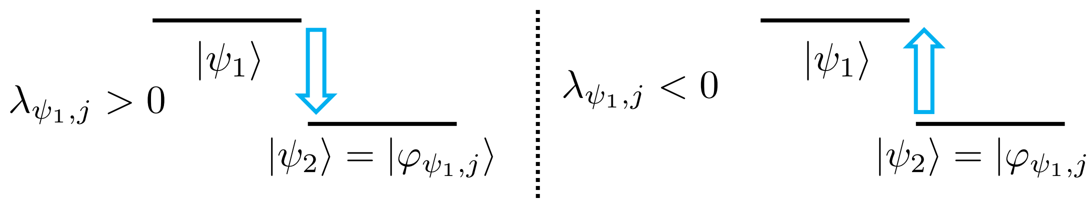
  No continuous measurement interpretation !!

Conclusion

✓ <u>Unified framework</u> to quantum-jump unraveling based on <u>rate operator</u>

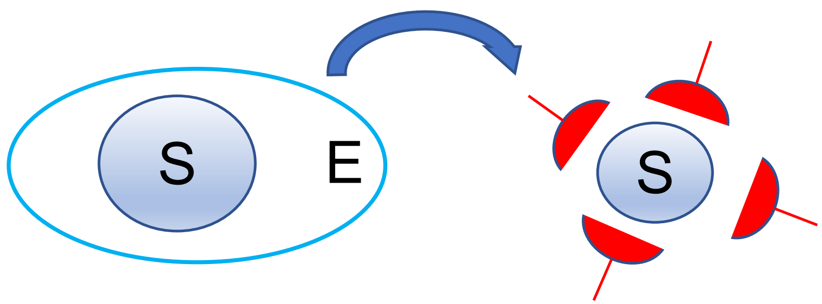
Phys. Rev. Lett. **124**, 190402 (2020)



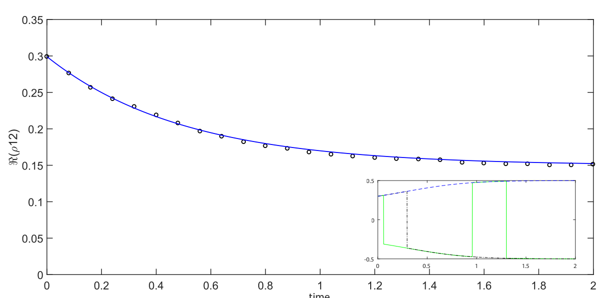


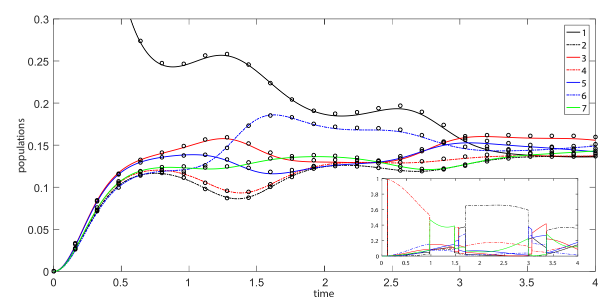
$|\psi(t)\rangle \mapsto |\varphi_{\psi(t),j}\rangle \qquad p_j(t) = \lambda_{\psi(t),j} dt$

✓ <u>Continuous-measurement interpretation</u>: positivity is enough! No need of CP



✓ Unraveling of <u>general dynamics</u>, for which other methods do not apply





Outlook

Exploit the definition of a family of rate operators arxiv:2009.11312

$$R_{\psi(t)} = \sum_{\alpha=1}^{N^2-1} c_\alpha(t) L_\alpha(t)(|\psi(t)\rangle\langle\psi(t)|) L_\alpha^\dagger(t)$$

$$H_{eff} = H - \frac{i}{2}\sum_{\alpha=1}^{n^2-1} c_\alpha L_\alpha^\dagger L_\alpha$$

Outlook

Exploit the definition of a family of rate operators arxiv:2009.11312

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- Connection with different properties: $R_{\psi(t)} > 0$ if the dynamics is dissipative

$$\mathcal{L}_t^\dagger(X^\dagger X) \geq \mathcal{L}_t^\dagger(X^\dagger)X + X^\dagger \mathcal{L}_t^\dagger(X) \quad \forall X \in \mathcal{B}(\mathcal{H}_S)$$

Outlook

Exploit the definition of a family of rate operators (arxiv:2009.11312)

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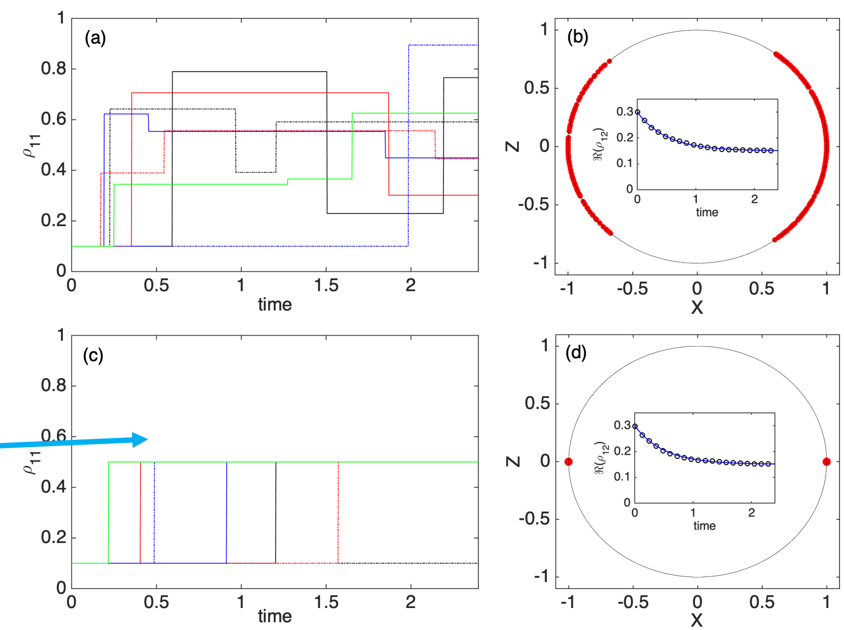
- Different measurement schemes for the same dynamics

Fixed basis after the jumps/measurements!



R1

R2

example trajectories

asymptotic states


Thank you

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