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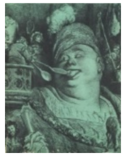

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Storage Capacity and Learning Capability of Quantum Neural Networks

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PhD ICFO:

ICFO – Quantum Optics Theory

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I dedicate this lectures to the memory of
Peter Wittek

Outline: Storage Capacity of Quantum Neural Networks

1. Classical Neural Networks
- 1.1 Real and artificial neurons
- 1.2 Perceptrons
- 1.3 Attractor neural networks (ANN)
- 1.4 Storage capacity

2. Learning
- 2.1 Learning = adopting connections, interactions, thresholds
- 2.2 Learning rules
- 2.3 Learning from examples
- 2.4 Gardner's program

3. Storage Capacity of Quantum Neural Networks
- 3.1 Some history
- 3.2 Quantum Attractor Neural Networks (QANN) – mathematical preliminaries
- 3.3 Our theorem 1: For n qubit-neurons (dimentision of Hilbert space $N= 2^n$), a quantum ANN can store up to $p= N = 2^n$ "patterns".
- 3.4 Our theorem 2: Relative volume of QANN, storing p patterns, decreases with (small) p as $\exp(-p^2/(N^4-N^2))$ and shrinks strictly to zero for $p>N=2^n$.

4. Outlook: Experiments?

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1. Classical Neural Networks

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• 1.1 Real neurons and McCullogh-Pitts neuron

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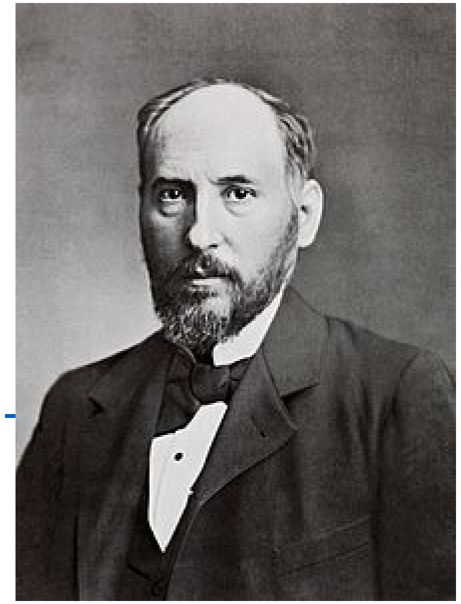
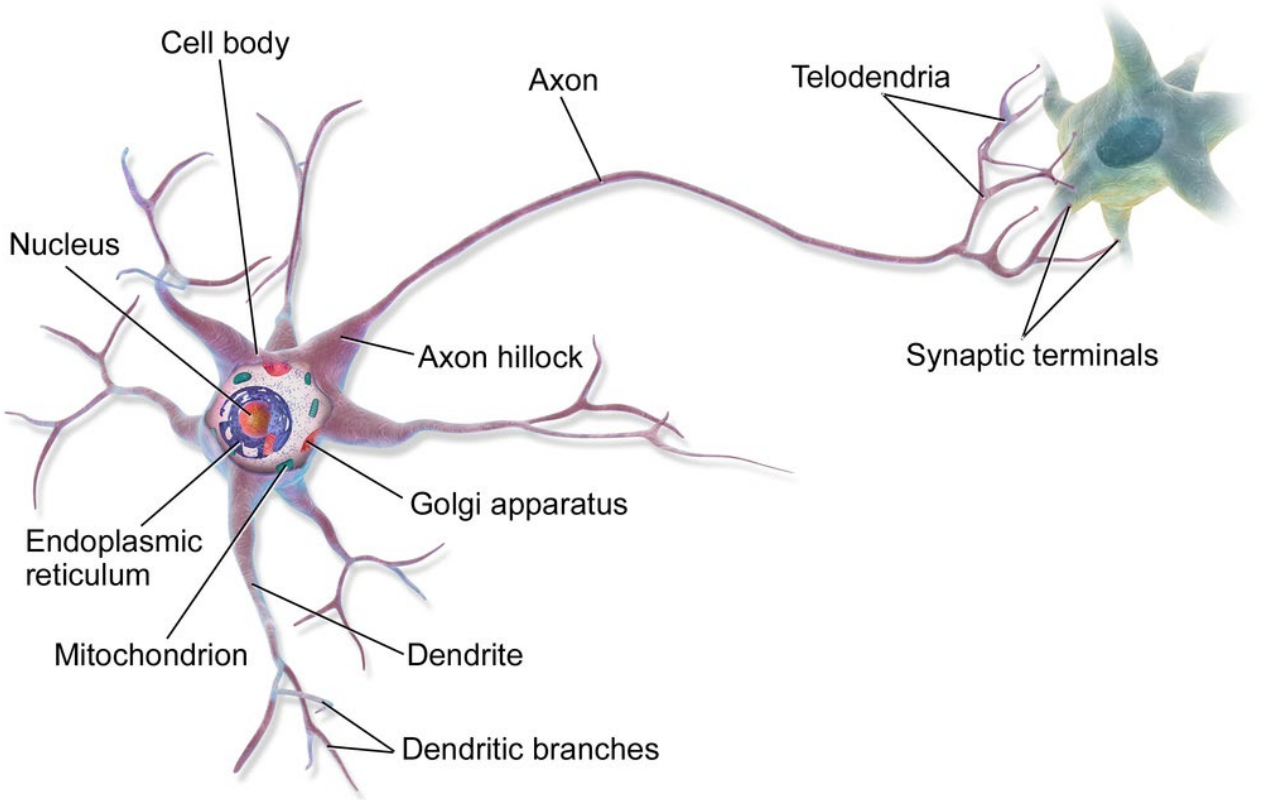
A neuron is an electrically excitable cell that receives, processes, and transmits information through electrical and chemical signals. These signals between neurons occur via specialized connections called synapses. Neurons can connect to each other to form neural networks. Neurons are major components of the brain and spinal cord of the central nervous system, and of the autonomic ganglia of the peripheral nervous system.



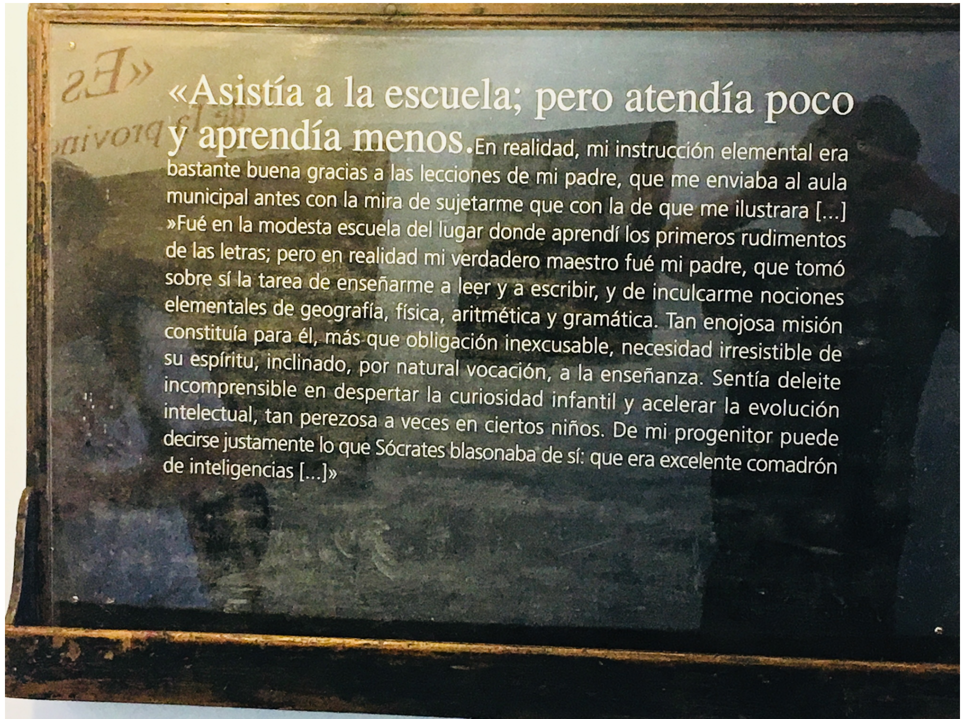
Santiago Felipe Ramón y Cajal (1852-1934)



Santiago Ramón y Cajal (1 May 1852 – 17 October 1934) was a Spanish neuroscientist and pathologist, specializing in neuroanatomy, particularly the histology of the central nervous system. He and Camillo Golgi received the Nobel Prize in Physiology or Medicine in 1906, with Ramón y Cajal thereby becoming the first person of Spanish origin who won a scientific Nobel Prize. His original investigations of the microscopic structure of the brain made him a pioneer of modern neuroscience. Hundreds of his drawings illustrating the delicate arborizations of brain cells are still in use for educational and training purposes.

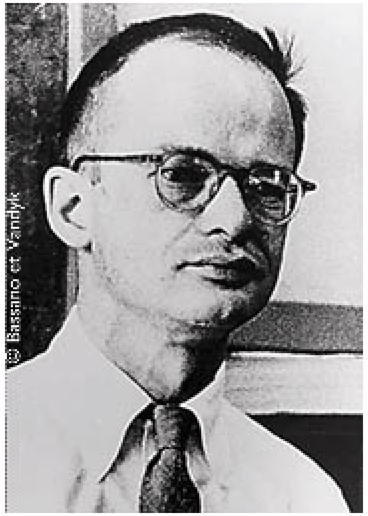
As a child he was transferred many times from one school to another because of behaviour that was declared poor, rebellious, and showing an anti-authoritarian attitude. An extreme example of his precociousness and rebelliousness at the age of eleven is his 1863 imprisonment for destroying his neighbour's yard gate with a homemade cannon.

Museum of RyC, Ayerbe, Navarra



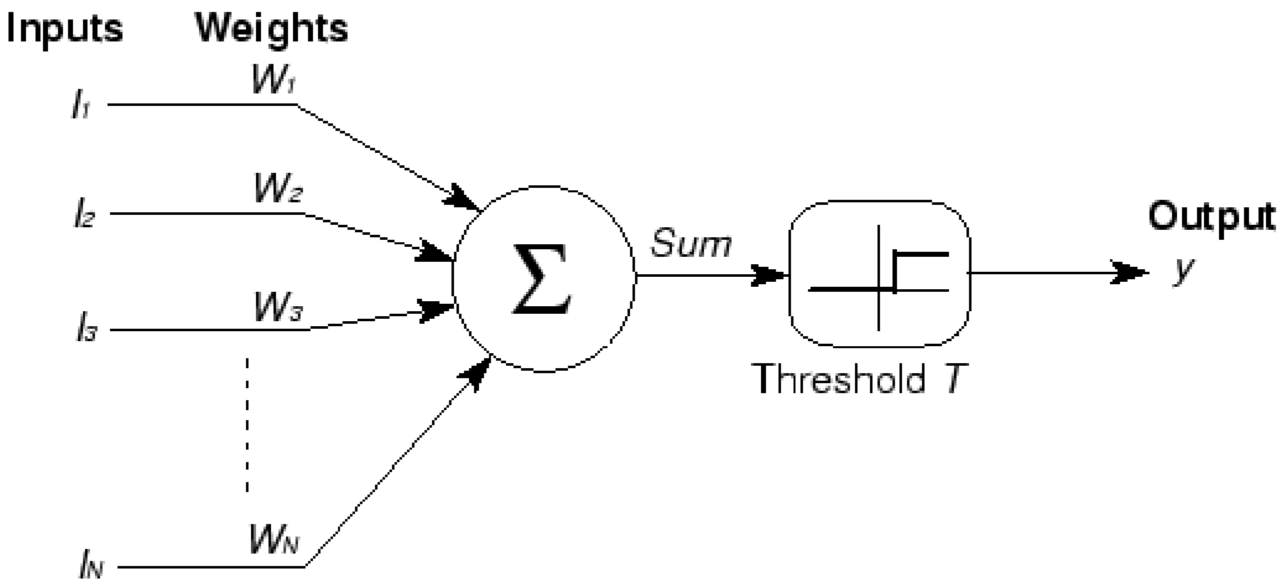
• 1.1 Real neurons and McCullogh-Pitts neuron

An artificial neuron is a mathematical function conceived as a model of biological neurons, a neural network. Artificial neurons are elementary units in an artificial neural network. The artificial neuron receives one or more inputs (representing dendrites) and sums them to produce an output (or activation) (representing a neuron's axon).



Walter Pitts

Warren Sturgis McCullogh



• 1.2 Perceptrons, Minsky-Pappert no-go theorem

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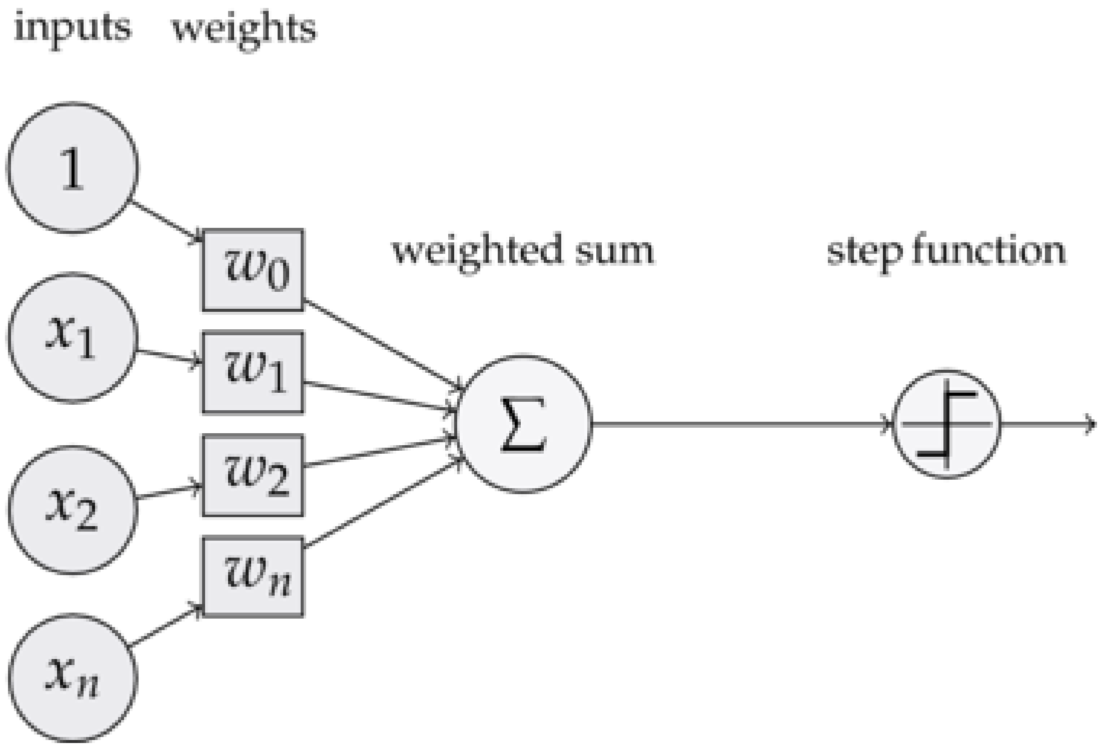
In machine learning, the perceptron is an algorithm for supervised learning of binary classifiers (functions that can decide whether an input, represented by a vector of numbers, belongs to some specific class or not). It is a type of linear classifier, i.e. a classification algorithm that makes its predictions based on a linear predictor function combining a set of weights with the feature vector.

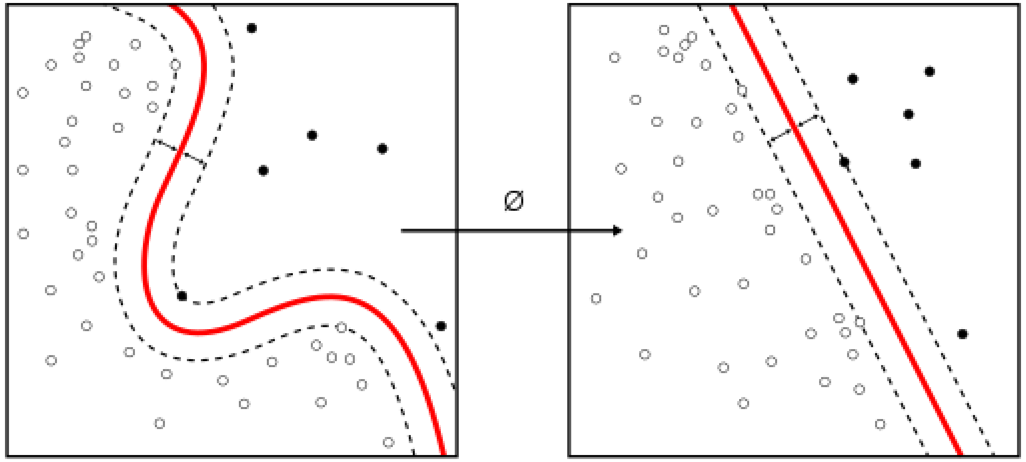
Frank Rosenblatt

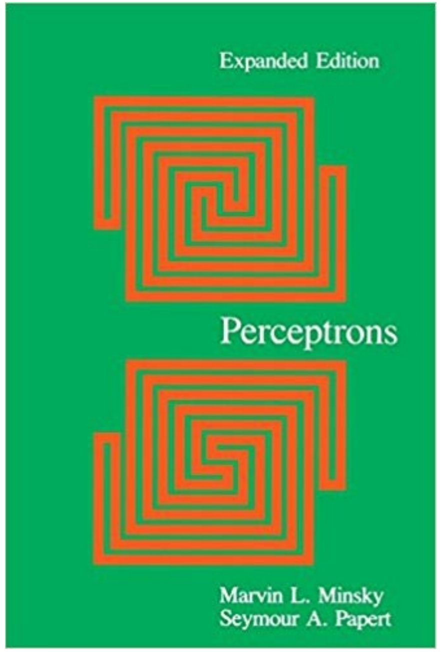


Minsky-Papert no-go theorem

Marvin Lee Minsky

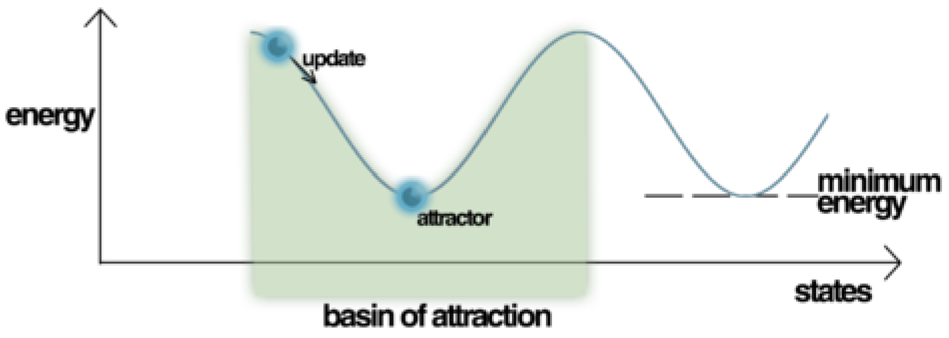
Seymour Aubrey Papert





• 1.3 Attractor Neural Networks

An attractor network is a type of recurrent dynamical network, that evolves toward a stable pattern over time. Nodes in the attractor network converge toward a pattern that may either be fixed-point (a single state), cyclic (with regularly recurring states), chaotic (locally but not globally unstable) or random (stochastic). Attractor networks have largely been used in computational neuroscience to model neuronal processes such as associative memory[2] and motor behavior, as well as in biologically inspired methods of machine learning. An attractor network contains a set of *n* nodes, which can be represented as vectors in a *d*-dimensional space where *n*>*d*. Over time, the network state tends toward one of a set of predefined states on a *d*-manifold; these are the attractors.



$$s_i(t+1)=\mathrm{sign}(\Sigma_j J_{ij} s_j(t) - t_i)$$

Stationary states = memorized patterns = memories

ANN =pattern recognizer = associative memory

Recognition = attraction toward a certain stationary state

• 1.3 Attractor Neural Networks

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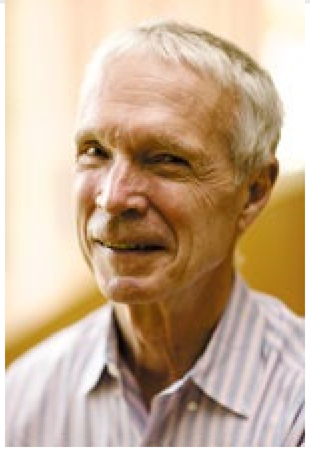
Example: Hopfield model

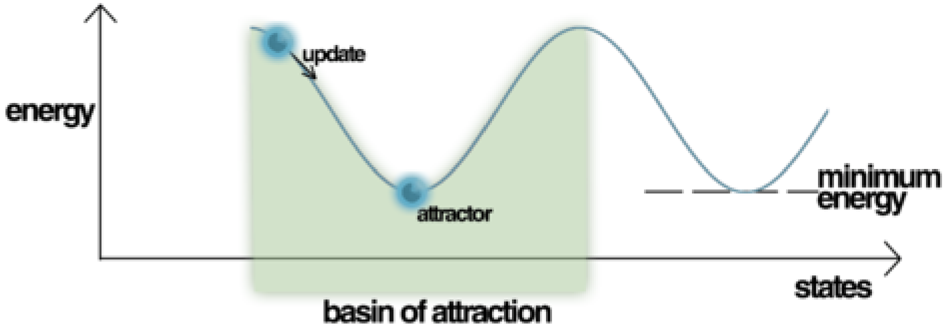
$$s_i(t+1)=\mathrm{sign}(\Sigma_j J_{ij} s_j(t) - t_i)$$

with $J_{ij} = J_{ji}$

and $H = \frac{1}{2} \Sigma_j J_{ij} s_i s_j(t) - \Sigma_i t_i s_i$



John J. Hopfield



Stationary states = minima of (free) energy)

ANN =pattern recognizer = content-addressable memory

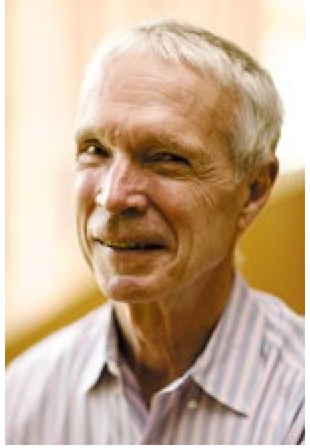
Recognition = attraction toward a certain stationary state

• 1.4 Storage capacity

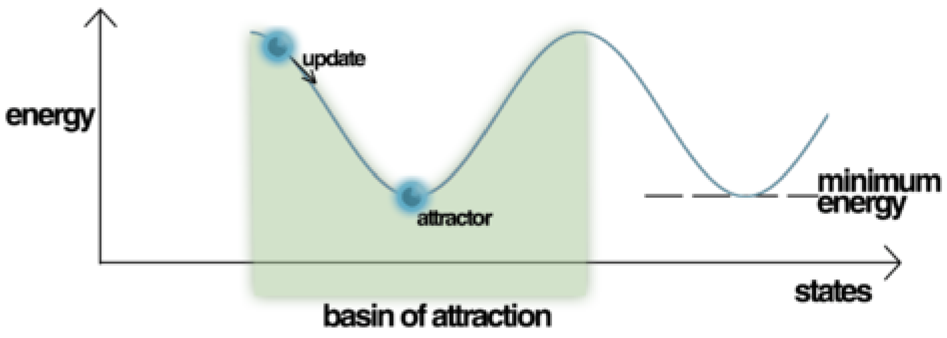
We want to store p patterns/memories/stationary states

$$\{\xi_i^\mu\},\ i = 1, \ldots, N,\ \mu = 1, \ldots, p$$

$$\xi_i^\mu = \mathrm{sign}(\Sigma_j J_{ij}\ \xi_j^\mu - t_i)$$



John J. Hopfield



If p<<N storage is possible with small errors

If $p=\alpha N$, storage possible for $\alpha < \alpha_c$

ANN =pattern recognizer = content-addressable memory

• 1.4.1 Learning = adopting connections, interactions, thresholds

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We want to store p patterns/memories/stationary states

$$\{\xi_i^\mu\},\ i = 1, \ldots, N,\ \mu = 1, \ldots, p$$

$$\xi_i^\mu = \text{sign}(\Sigma_j J_{ij}\ \xi_j^\mu - t_i)$$

Learning = adjusting/modifying J_{ij}'s, h_j's etc.

We want to store p random patterns/memories/stationary states

$$\{\xi_i^\mu\},\ i = 1, \ldots, N,\ \mu = 1, \ldots, p$$

$$\xi_i^\mu = \mathrm{sign}(\Sigma_j J_{ij}\ \xi_j^\mu - t_i)$$

Hebbian rule

$$J_{ij} = 1/pN\ (\Sigma_\mu\ \xi_i^\mu \xi_j^\mu),$$

with ξ_i^μ riidv with $P(\xi_j^\mu = \pm 1) = \frac{1}{2}$.

1.4.2 Learning rules / Storage capacity

$$s_i = \text{sign}\Big(\underbrace{\sum_j J_{ij} s_j}_{h_i} - t\Big) \qquad \text{We take } t = 0$$

Stationary state

$$s_i h_i > 0 \quad \text{for all} \quad i = 1, \dots, N$$

For the, so called, Hebbian rule

$$h_i(s) = \frac{1}{N} \sum_{j \neq i} \sum_{\mu} \xi_i^\mu \xi_j^\mu s_j \longleftarrow \text{random patterns } \xi^\mu$$

$$h_i(\xi^1) = \frac{N-1}{N} \xi_i^1 + \frac{1}{N} \sum_{j \neq i} \sum_{\mu \geq 2} \xi_i^\mu \xi_j^\mu \xi_j^1$$

The pattern is „stable" iff

$$\xi_i^1 h_i(\xi^1) = \frac{N-1}{N} + \frac{1}{N}\sum_{j\neq i}\sum_{\mu\geq 2} \xi_i^\mu \xi_j^\mu \xi_i^1 \xi_j^1 > 0$$

$$\xi_i^1 h_i(\xi^1) = 1 + R \quad \text{as } N\to\infty$$

1 ← signal, R ← noise

$$\langle\langle R \rangle\rangle = 0$$

$$\langle\langle R^2 \rangle\rangle \approx \frac{P}{N}$$

so typically $R \sim \sqrt{\frac{P}{N}}$!

2. Learning

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• 2.1 Learning = adopting connections, interactions, thresholds

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We want to store p patterns/memories/stationary states

$$\{\xi_i^\mu\},\ i = 1, \ldots, N,\ \mu = 1, \ldots, p$$

$$\xi_i^\mu = \mathrm{sign}(\Sigma_j J_{ij}\, \xi_i^\mu - h_i)$$

Learning = adjusting/modifying J_{ij}'s, h_j's etc.

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We want to store p random patterns/memories/stationary states

$$\{\xi_i^\mu\},\ i = 1, \ldots, N,\ \mu = 1, \ldots, p$$

$$\xi_i^\mu = \mathrm{sign}(\Sigma_j J_{ij} \xi_j^\mu - h_i)$$

Hebbian rule

$$w_{ij} = 1/pN\ (\Sigma_\mu\ \xi_i^\mu\ \xi_j^\mu)$$

2.2 Learning from examples

Let us consider a perceptron. Suppose we know p examples patterns/memories/stationary states

$$\{\xi_i^\mu\},\ i = 1, \ldots, N,\ \mu = 1, \ldots, p$$

$$y^\mu = \mathrm{sign}(\Sigma_j w_j \xi_j^\mu - h_j)$$

For perceptrons there exist the famous perceptron learning algorithm

For feed-forward MNN there exist the famous error back propagation algorithm

For feed-forward ANN there exist similar error corrections algorithm

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The space of interactions in neural network models

The network is defined as follows. Ising spins, $S_i = \pm 1$, are defined on each site i, $i = 1, \ldots, N$. They are updated according to the rule

$$S_i(t+1) = \mathrm{sgn}(h_i(t) - T_i) \tag{1}$$

where $S_i(t)$ is the Ising spin at time t and the internal magnetic field $h_i(t)$ at time t and site i is given by

$$h_i(t) = \frac{1}{\sqrt{N}} \sum_{j \neq i} J_{ij} S_j(t) \tag{2a}$$

where J_{ij} is the interaction strength for the bond from site j to site i. The interactions J_{ij} and J_{ji} need not in general be equal. The field T_i is a local threshold at the site i

a solution of given κ provided such solutions exist.

Gardner's program (for a perceptron)

normalize

$$\sum_{j=1}^{N} J_j^2 = N$$

$$S_{out} = \text{sign}\left(\sum_{j=1}^{N} J_j S_{in}\right)$$

We want $\xi^{\mu}_{out} = \text{sign}\left(\sum_{j=1}^{N} J_j \xi^{\mu}_{in,j}\right)$

$$V_{total} = \int \mathcal{D}J\, \delta\left(\sum_{j=1}^{N} J_j^2 - N\right)$$

$$V_{task} = \int \mathcal{D}J\, \delta\left(\sum_{j=1}^{N} J_j^2 - N\right) \prod_{\mu=1}^{P} \Theta\left(\xi^{\mu}_{out} \sum_{j=1}^{N} J_j \xi^{\mu}_{in,j} - \kappa\right)$$

Relative volume $V_{rel} = V_{task} / V_{total}$

If V_{rel} - big ~ good , if $V_{rel} \to 0$, no chance 

3. Storage Capacity of Quantum Neural Networks

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• 3.1 Some history: Gardner's program for quantum perceptrons

In a recent letter Lloyd [9] has formulated three necessary conditions that must be met by any computational system:

(1) One has to be able to prepare the input states of the computer, without knowing beforehand the results of the computation.
(2) The system must transform the input states into the output states according to its internal 'computational' dynamics and interactions with its environment.
(3) One must be able to perform measurements on the system that allow extraction of the results of computation.

Quantum perceptrons that I discuss in the present paper fulfil the above formulated criteria, and are in fact closely related to Lloyd's unitary computers.

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• 3.1 Some history: Gardner's

JOURNAL OF MODERN OPTICS, 1994, VOL. 41, NO. 12, 2491–2501

Quantum perceptrons

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(*Received 3 November 1993; revision received 29 April 1994*)

Abstract. I formulate a statistical theory of quantum perceptrons, i.e. ideal quantum computing elements that process input states into output states through unitary transforms.

1. Introduction

In recent years problems of quantum computing and information processing have received increasing attention. Although quantum computation suffers from various obstacles [1], several models of it have been formulated: ballistic quantum computers of Benioff [2], Deutsch [3], and Feynman [4], dissipative enzymatic and Brownian motion computers of Bennett [5], quantum cellular automata computers of Margolus [6], error-correcting computers of Żurek [7] and Peres [8], or quantum unitary computers of Lloyd [9]. Quite recently Deutsch and Jozsa [10] have shown that quantum mechanical interference between the computational paths might be very useful and might allow quantum computers to be superior to classical ones for some computational problems, such as recognition of 'unanimous' and 'balanced' Boolean functions [11].

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• 3.1 Some history: Exponential storage capacity of QNN

Preprint submitted to IEEE Transactions on Neural Networks, June 16 1998

Quantum Associative Memory

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Neural Networks and Machine Learning Laboratory (*http://axon.cs.byu.edu*)

Department of Computer Science

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Abstract This paper combines quantum computation with classical neural network theory to produce a quantum computational learning algorithm. Quantum computation uses microscopic quantum level effects to perform computational tasks and has produced results that in some cases are exponentially faster than their classical counterparts. The unique characteristics of quantum theory may also be used to create a quantum associative memory with a capacity exponential in the number of neurons. This paper combines two quantum computational algorithms to produce such a quantum associative memory. The result is an exponential increase in the capacity of the memory when compared to traditional associative memories such as the Hopfield network. The paper covers necessary high-level quantum mechanical and quantum computational ideas and introduces a quantum associative memory. Theoretical analysis proves the utility of the memory, and it is noted that a small version should be physically realizable in the near future.

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• 3.1 Some history: Exponential storage capacity of QNN

PHYSICAL REVIEW A **98**, 042308 (2018)

Editors' Suggestion

Quantum Hopfield neural network

Patrick Rebentrost,[1,*] Thomas R. Bromley,[1,†] Christian Weedbrook,[1] and Seth Lloyd[2]

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(Received 19 June 2018; published 5 October 2018)

Quantum computing allows for the potential of significant advancements in both the speed and the capacity of widely used machine learning techniques. Here we employ quantum algorithms for the Hopfield network, which can be used for pattern recognition, reconstruction, and optimization as a realization of a content-addressable memory system. We show that an exponentially large network can be stored in a polynomial number of quantum bits by encoding the network into the amplitudes of quantum states. By introducing a classical technique for operating the Hopfield network, we can leverage quantum algorithms to obtain a quantum computational complexity that is logarithmic in the dimension of the data. We also present an application of our method as a genetic sequence recognizer.

DOI: 10.1103/PhysRevA.98.042308

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• 3.2 Quantum Attractor Neural Networks (QANN) – mathematical preliminaries

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PAPER

Storage capacity and learning capability of quantum neural networks

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Keywords: quantum neural networks, attractor neural networks, storage capacity

Abstract

We study the storage capacity of quantum neural networks (QNNs), described by completely positive trace preserving (CPTP) maps acting on an N-dimensional Hilbert space. We demonstrate that attractor QNNs can store in a non-trivial manner up to N linearly independent pure states. For n qubits, QNNs can reach an exponential storage capacity, $\mathcal{O}(2^n)$, clearly outperforming standard classical neural networks whose storage capacity scales linearly with the number of neurons n. We estimate, employing the Gardner program, the relative volume of CPTP maps with $M \leqslant N$ stationary states and show that this volume decreases exponentially with M and shrinks to zero for $M \geqslant N+1$. We generalize our results to QNNs storing mixed states as well as input–output relations for feed-forward QNNs. Our approach opens the path to relate storage properties of QNNs to the quantum features of the input–output states. This paper is dedicated to the memory of Peter Wittek.

arXiv:2011.06113, in print Quantum Science and Technology (IOP).

• 3.2 Quantum Attractor Neural Networks (QANN) – mathematical preliminaries

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Attractor QNN

$$\rho^{out} = \Lambda(\rho^{in})$$

Λ - CPTP map

Λ ~ linear

$$\Lambda(\rho^{+}) = \Lambda(\rho)^{+}$$

$$\Lambda(\rho \geq 0) \geq 0$$

$$(I \otimes \Lambda)(w \geq 0) \geq 0$$

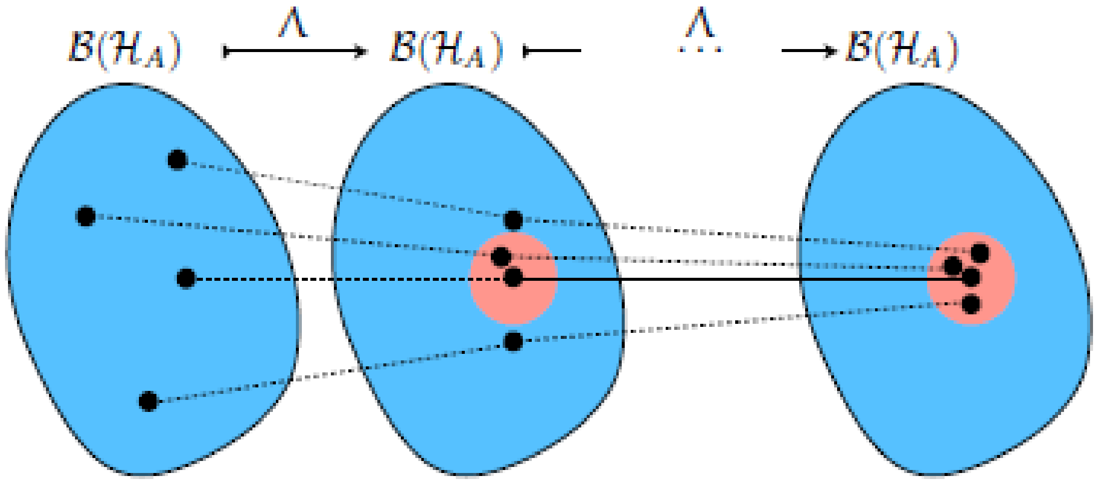

FIG. 1: Schematical representation of the action caused by repeated applications of the CPTP map $\Lambda : \mathcal{B}(\mathcal{H}_A) \mapsto \mathcal{B}(\mathcal{H}_A)$ bringing arbitrary states $\rho \in \mathcal{B}(\mathcal{H}_A)$ to the set $\{\rho_\mu\}$ of stationary states of the map fulfilling $\Lambda(\rho_\mu) = \rho_\mu$ (red area) .

$$\mathrm{Tr}\,(\Lambda(\rho)) = \mathrm{Tr}\,(\rho)$$

3.2 Quantum Attractor Neural Networks (QANN) – mathematical preliminaries

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Attractor QNN

Λ can be interpeted as

$$\Lambda(\rho(0)) = \rho(t)$$

$$\Lambda(\rho(0)) = \rho(t=\infty)$$

$$\Lambda(\rho(t)) = \rho(t+\delta t)$$

Stationary states/ Stored patterns

$$\Lambda(\rho) = \rho$$

• 3.3 Our theorem 1: For n qubit-neurons, a quantum ANN can store up to $p=2^n$ "patterns"

Theorem 1: For n qubits-neurons, a quantum ANN can store up to $p=2^n$ "patterns".

Proof: We consider $N=2^n$ dimensional Hilbert space and projectors on N linearly independent vectors $|e_\mu\rangle$. We transform them to the canonical orthonormal basis $|e_\mu\rangle \to |\mu\rangle$. The desired map has as stationary states projectors on $|\mu\rangle$.

• 3.3 Our theorem 1: For n qubit-neurons, a quantum ANN c… store up to $p=2^n$ "patterns"

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The family of Maps

$$\rho = \sum_{\mu,\nu} \rho_{\mu\nu} |\mu\rangle\langle\nu| \xrightarrow{\Lambda(\rho)} \sum_{\mu,\nu} \alpha_{\mu\nu}\rho_{\mu\nu}|\mu\rangle\langle\nu|$$

with $\alpha_{\mu\mu} = 1$

and $\alpha_{\mu\nu} = \alpha^*_{\nu\mu}$

$$|\alpha_{\mu\nu}| \leq 1$$

• 3.4 Our theorem 2: Relative volume of QANN, storing p patterns, decreases with p as $\exp[-p^2/d]$

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Theorem 2: Relative volume of QANN, storing p patterns, decreases with p as $\exp(-p^2/(N^4-N^2))$ and shrinks strictly to zero for $p > N=2^n$

Proof: The last point can be proven explicitly considering one more stationary state for The Map. It cannot have coherences… We consider $N=2^n$ dimensional Hilbert space, and estimate the total volume CPTP as a ball of radius $\exp(-1/4)$ in dimension $d= N^4-N^2$.

The result for relative volume is

$$V = \exp[-p^2/d]$$

$$V_{\mathrm{CPTP}}(d) = \frac{\pi^{d/2}}{\Gamma(d/2+1)} \exp(-d/4). \qquad (2)$$

$$V_{\mathrm{aQNN}}(\epsilon, M, d) \simeq \frac{\epsilon^M \pi^{(d-M)/2}}{\Gamma((d-M)/2+1)} e^{-(d-M)/4}. \qquad (4)$$

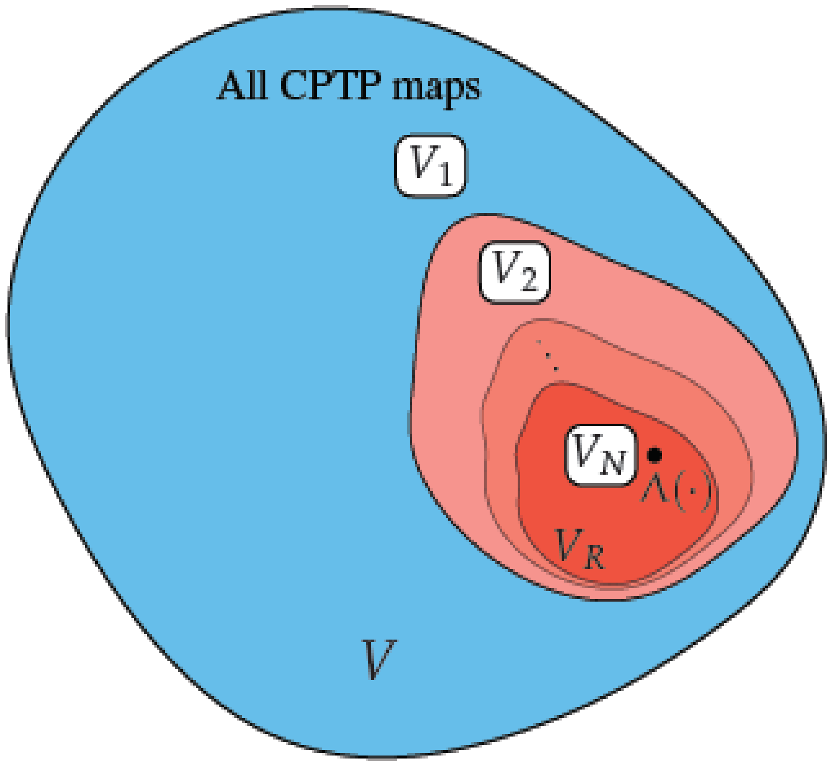

FIG. 2: Schematic representation of the volume V_M of CPPT maps with M stationary states. The volume shrinks as we increase the number of fixed states from V_{CPTP} for $M=1$ to V_N for $M=N$.

V. Cappellini, H.-J. Sommers, and K. Życzkowski, J. Math. Phys. 48, 52110 (2007).
S. J. Szarek, E. Werner, and K. Życzkowski, J. Math. Phys. 49, 32113 (2008).

• 3.4 Our theorem 3: Relative volume of QANN, storing p patterns, decreases with p as exp[-p²/d]

Definition 1 Let $\mathcal{E} = \{\rho_\mu\}$ with $\mu = 1, \cdots, M$ be an ensemble of N-dimensional density matrices in $\mathcal{B}(\mathbb{C}^N)$. The ensemble $\mathcal{E}$ is called *classical* if there exists a single invertible operation T that diagonalizes all elements of the ensemble; i.e., $T\rho_\mu T^\dagger = D_\mu$, where all D_μ are simultaneously diagonal. We call this basis the *computational basis.*

D. A. Kronberg, Lobachevskii J. Math. 40, 1507 (2019).

Theorem 1' There exist non-trivial CPTP maps Λ, s.t. $\Lambda(\rho_\mu) = \rho_\mu$, where $\rho_\mu \in \mathcal{E}$ with $\mu = 1, \ldots, M$, and arbitrary M.

4. Outlook: Experiments?

Conclusions?

Enjoy physics and beyond!!!

Quantum Narcissism

The Amateur's Guide to Avantgarde: Catalunya, Poland, Portugal, and More...

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