TECHNISCHE UNIVERSITÄT DRESDEN

DRESDEN concept

Valentin Link and Walter Strunz
TU Dresden

Dynamical Phase Transition
in dissipative quantum dynamics

52 Symposium on Mathematical Physics, Torun
June 14-17, 2021

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in GKSL dynamics

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Motivation

Equilibrium Phase Transitions

Free energy nonanalytic w.r.t. parameter

$$Z = \mathrm{Tr}\, \mathrm{e}^{-\beta H} \qquad f = -\frac{1}{\beta N} \ln Z$$

Nonequilibrium Phase Transitions

Driven-dissipative system (GKSL):
Steady state changes nonanalytically w.r.t. parameter

$$\partial_t \rho = \mathcal{L} \rho \qquad \lim_{t\to\infty} \rho(t)$$

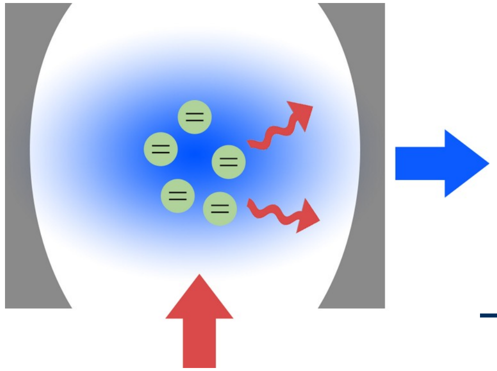
"Dynamical Quantum Phase Transitions"

Unitary dynamics,
Observable nonanalytic *in time*

$$\partial_t \left|\psi\right\rangle = -\mathrm{i} H \left|\psi\right\rangle \qquad \langle\psi(0)|\mathrm{e}^{-\mathrm{i}Ht}|\psi(0)\rangle$$

M. Heyl: Dynamical quantum phase transitions: a review,
Rep. Prog. Phys. 81, 054001 (2018)

Driven Dicke model



„cooperative resonance fluorescence model“
D. F. Walls, P. D. Drummond, S. S. Hassan, and H. J. Carmichael, Prog. Theor. Phys. Suppl. 64, 307 (1978).

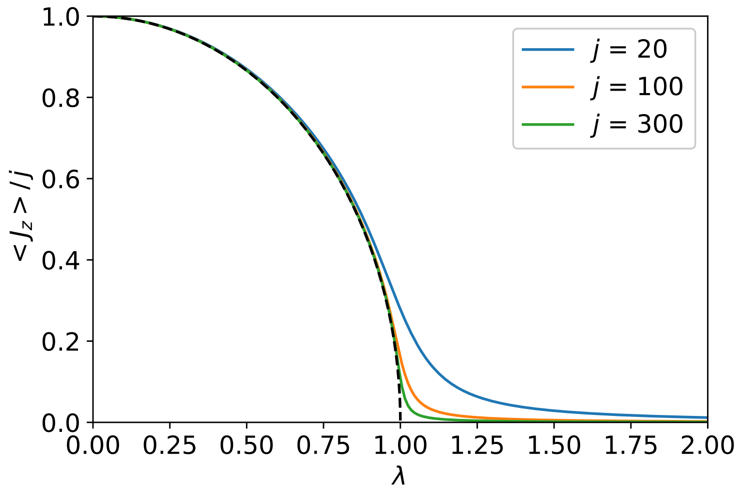
— GKSL Master equation, determine steady state:

$$\partial_t \rho_A(t) = -\mathrm{i}[\Delta_1 J_z + \omega J_x, \rho_A(t)] + \frac{\omega}{\lambda N}\Big(2J_-\rho_A(t)J_+ - \{J_+J_-, \rho_A(t)\}\Big)$$
$$+\frac{\omega}{\lambda N}\Big(2J_z\rho_A(t)J_z - \{J_z^2, \rho_A(t)\}\Big)$$

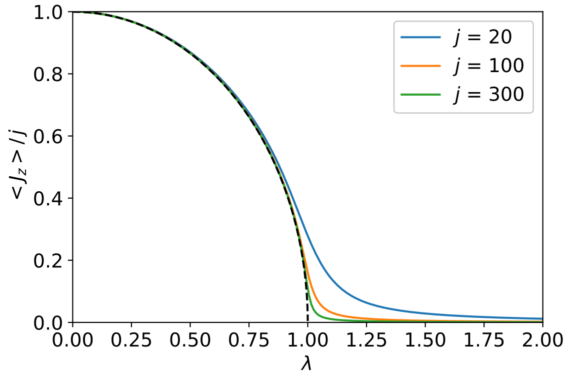
Interesting nonequilibrium steady state phases for $\Delta_1=0$

symmetry breaking phase transition

Valentin Link, Kimmo Luoma, and WS, Phys. Rev. A 99, 062120 (2019)



Driven Dicke model



GKSL Master equation:

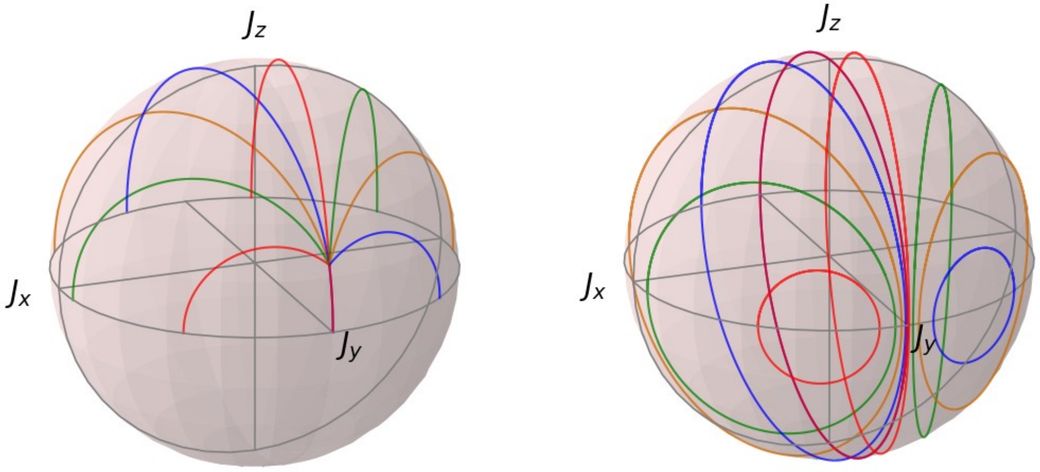
$$\rho_A(t) = -\mathrm{i}[\omega J_x, \rho_A(t)] + \frac{\omega}{\lambda N}\Big(2J_-\rho_A(t)J_+ - \{J_+J_-, \rho_A(t)\}\Big)$$
$$+\frac{\omega}{\lambda N}\Big(2J_z\rho_A(t)J_z - \{J_z^2, \rho_A(t)\}\Big)$$

Exact solution through spin coherent states

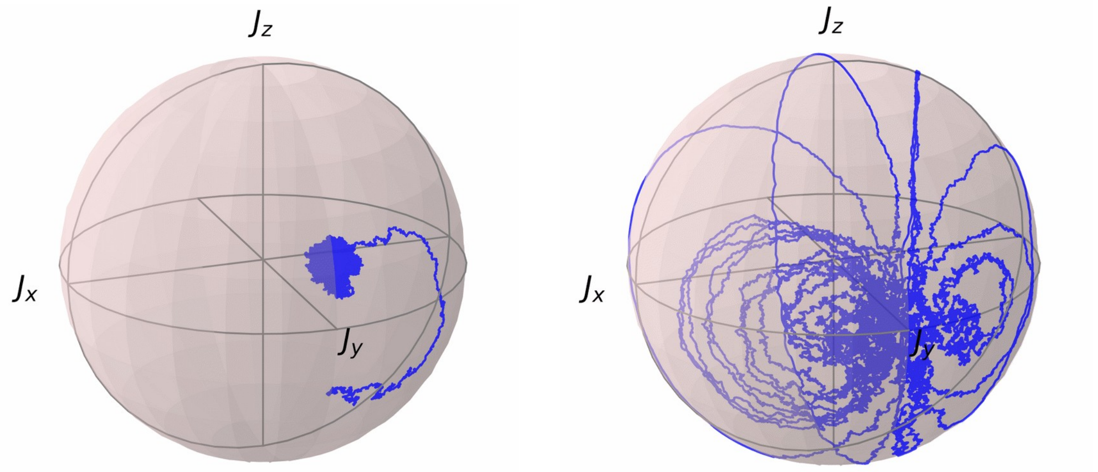
(Fokker Planck equation), also as N → ∞

Valentin Link, Kimmo Luoma and WS, Phys. Rev. A 99, 062120 (2019)

N → ∞ ; t→ ∞



N<∞, t→ ∞; N → ∞



Motivation

Equilibrium Phase Transitions

properties nonanalytic w.r.t. parameter

$$Z = \mathrm{Tr}\, \mathrm{e}^{-\beta H} \qquad f = -\frac{1}{\beta N} \ln Z$$

Nonequilibrium Phase Transitions

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$$\partial_t \rho = \mathcal{L}\rho \qquad \lim_{t\to\infty} \rho(t)$$

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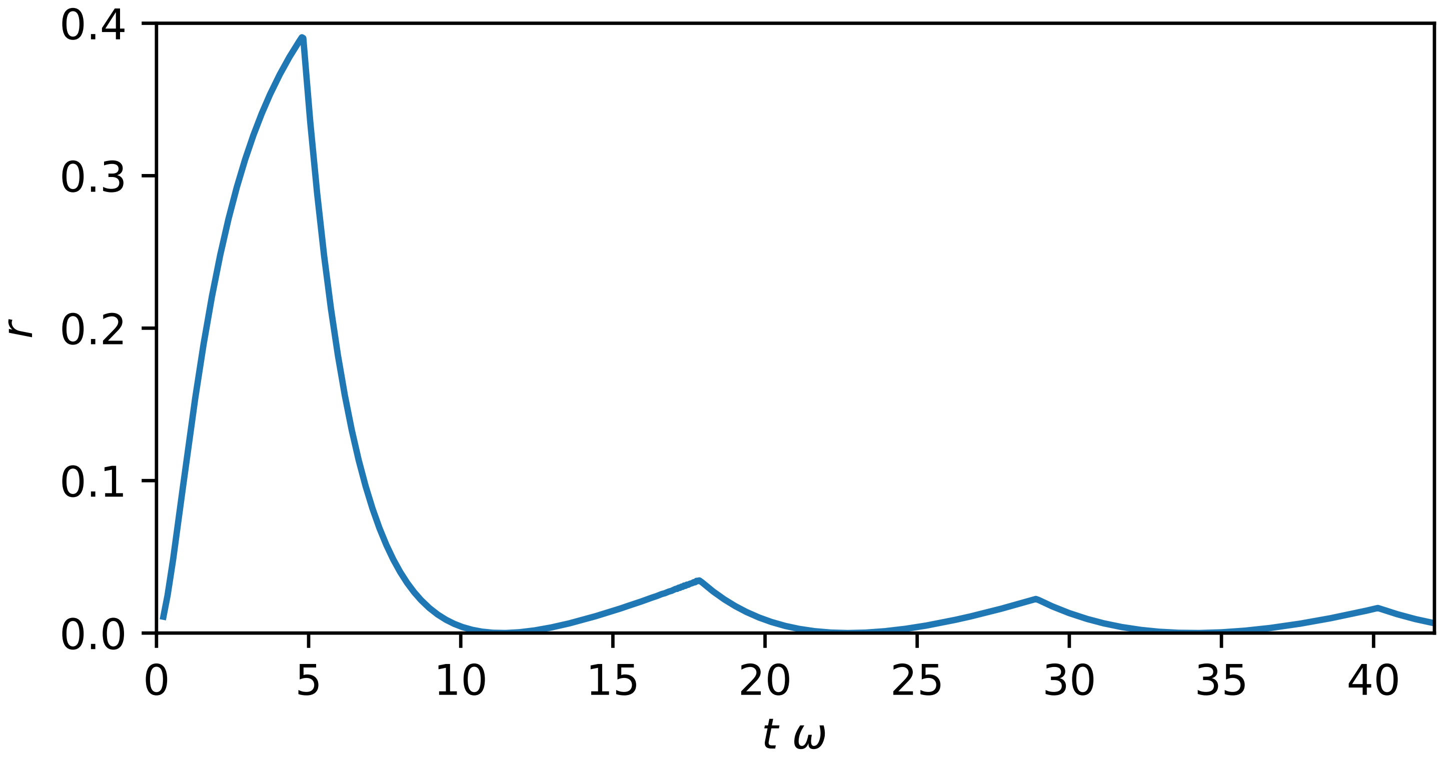
Observable nonanalytic *in time*

$$\partial_t |\psi\rangle = -\mathrm{i}H|\psi\rangle \qquad \langle\psi(0)|\mathrm{e}^{-\mathrm{i}Ht}|\psi(0)\rangle$$

M. Heyl: Dynamical quantum phase transitions: a review, Rep. Prog. Phys. 81, 054001 (2018)

Dynamical Phase Transition in GKSL master equation!
(Driven Dicke model)

$$\partial_t \rho = \mathcal{L}\rho$$



Exact (asymptotic N→ ∞)

Loschmidt echo:

$$L(t) = \langle 0_N | \rho(t) | 0_N \rangle$$

$$r(t) = -\frac{1}{N} \ln L(t)$$

Valentin Link and WS, Phys. Rev. Lett. 125,143602 (2020)

Outline

— **Dynamical Phase Transition (DPT)**

— Dicke model

— DPT in the Driven Dicke model

— Measurement scheme

Dynamical Phase Transition

Unitary time evolution

$$|\psi(t)\rangle = \mathrm{e}^{-\mathrm{i}Ht}\,|\psi(0)\rangle$$

Loschmidt amplitude

$$\langle\psi(0)|\psi(t)\rangle = \langle\psi(0)|\,\mathrm{e}^{-\mathrm{i}Ht}\,|\psi(0)\rangle$$

Loschmidt echo = return probability

$$L(t) = |\,\langle\psi(0)|\,\mathrm{e}^{-\mathrm{i}Ht}\,|\psi(0)\rangle\,|^2$$

Exponential scaling:
(N-body system)

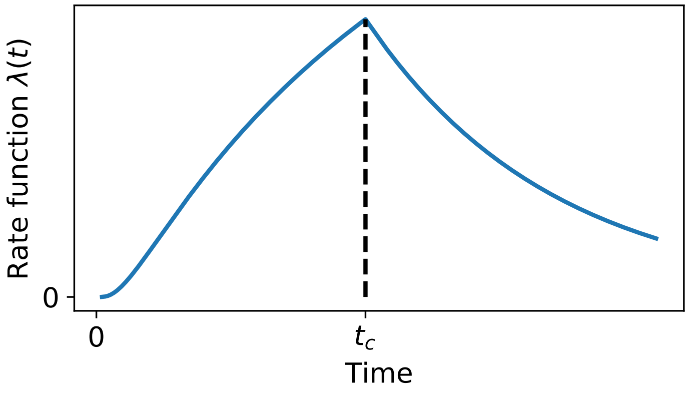
$$L(t) = \mathrm{e}^{-Nr(t)} \qquad r(t) = -\frac{1}{N}\ln L(t)$$

Dynamical Phase Transition

$$L(t) = |\langle\psi(0)|\,\mathrm{e}^{-\mathrm{i}Ht}\,|\psi(0)\rangle|^2$$

$$L(t) = \mathrm{e}^{-Nr(t)} \qquad r(t) = -\frac{1}{N}\ln L(t)$$

Illustration of a DPT[1]

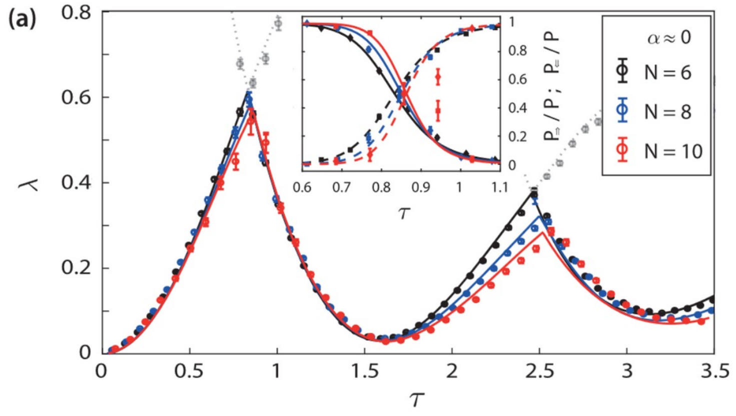


Kink at „critical time" t_c,

only possible for $N \to \infty$

Realization with trapped ions[2]:

(transverse Ising model)



[1] M. Heyl, Dynamical quantum phase transitions: a review, Rep. Prog. Phys. 81, 054001 (2018)

[2] Jurcevic, Shen, Hauke, Maier, Brydges, Hempel, Lanyon, Heyl, Blatt, and Roos, Phys. Rev. Lett. 119, 080501 (2017)

Dynamical Phase Transition: open quantum dynamics?

Density operator ρ (t)

Different generalisations for Loschmidt echo

$$\begin{aligned} L_{\mathrm{U}}(t) &= F(\rho(0),\rho(t)) = \mathrm{Tr}\left[\sqrt{\sqrt{\rho(0)}\rho(t)\sqrt{\rho(0)}}\right]^2 \qquad \text{(Uhlmann)} \\ L_{\mathrm{HS}}(t) &= \mathrm{Tr}\left[\rho(0)\rho(t)\right] \\ L(t) &= \ldots \end{aligned}$$

DPT: *overlap* of initial and current state changes nonanalytically in time

Scenarios:

- mixed initial state, finite T, (q)fluctuations: DPTs are smoothed out[2]
- here: DPT in dissipative quantum dynamics (zero temperature)[3]

[2]Sedlmayr, Fleischhauer, and Sirker, Phys. Rev. B 97, 045147 (2018); Mera, Vlachou, Paunkovic, Vieira, and Viyuela, Phys. Rev. B 97, 094110 (2018)
[3]Valentin Link and WS, Phys. Rev. Lett. 125,143602 (2020)

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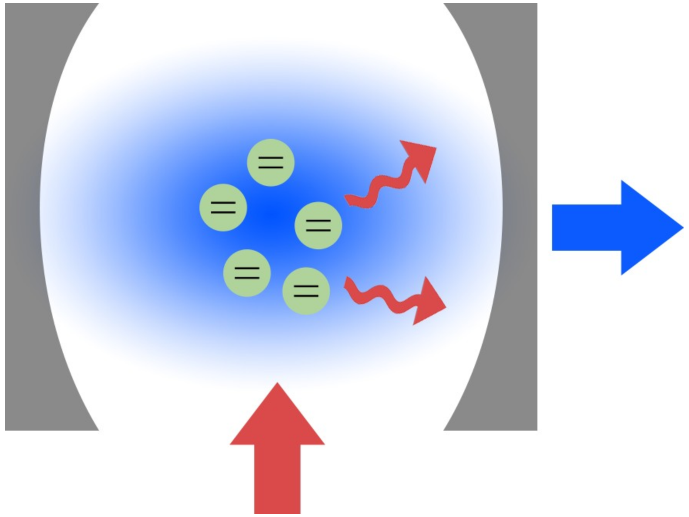
— DPTs in the Driven Dicke model

— Measurement scheme

Dicke model



effective Hamiltonian:

$$H = \omega_c a^\dagger a - \frac{\omega_1}{2} \sum_{\lambda=1}^{N} \sigma_{z\lambda} + \frac{g}{\sqrt{2N}} (a + a^\dagger) \sum_{\lambda=1}^{N} \sigma_{x\lambda}$$

Cavity loss, GKSL master equation:

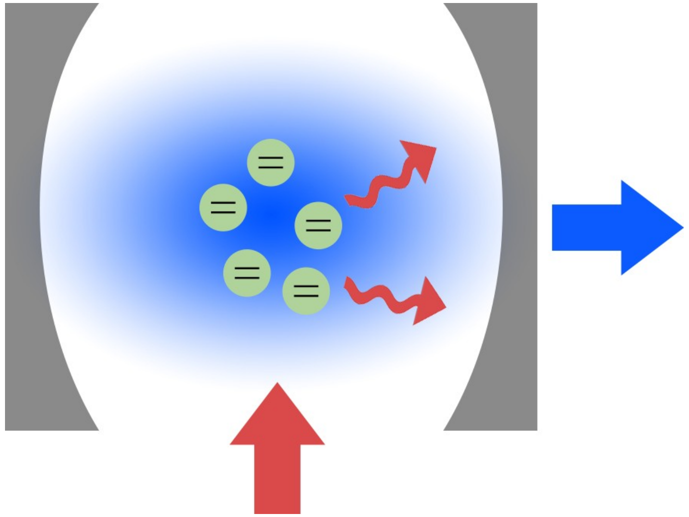
$$\partial_t \rho = -\mathrm{i}[H, \rho] + \gamma \big(2a\rho a^\dagger - a^\dagger a \rho - \rho a^\dagger a\big)$$

Collective spin: $\vec{J} = \sum_{\lambda=1}^{N} \frac{\vec{\sigma_\lambda}}{2}$

Experimental realization of non-equilibrium phase transition with superfluid gas in an optical cavity[4]:

[4]Baumann, Guerlin, Brennecke, and Esslinger, Nature 464,1301 (2010)

Dicke model: driven version



effective Hamiltonian:

$$H_0 = w_c a^\dagger a + \omega_a J_z + g\sqrt{\frac{2}{N}}(a + a^\dagger) J_x$$

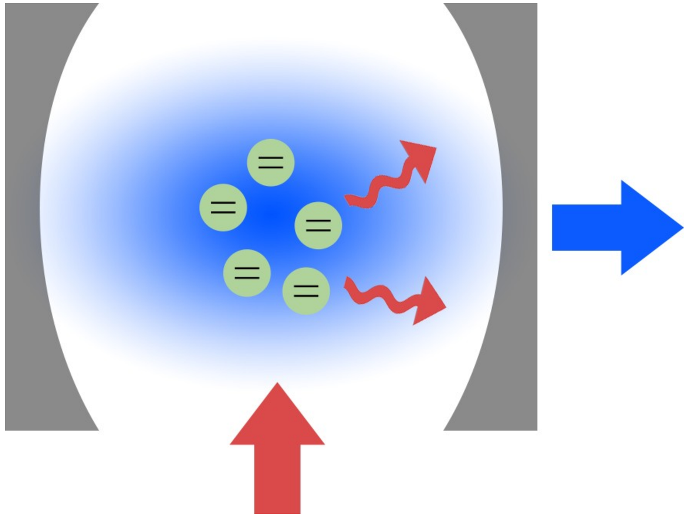
Driving frequency ν, detunings $\Delta_0=\omega_c-\nu$, $\Delta_1=\omega_a-\nu$

$$H = \Delta_0 a^\dagger a + \Delta_1 J_z + \omega J_x + g\sqrt{\frac{2}{N}}(J_- a^\dagger + J_+ a)$$

Cavity loss:

$$\partial_t \rho = -\mathrm{i}[H, \rho] + \gamma\left(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a\right)$$

Dicke model: driven version



effective Hamiltonian:

$$H = w_c a^\dagger a + \omega_a J_z + g\sqrt{\frac{2}{N}}(a + a^\dagger)J_x$$

Driving frequency ν, detunings $\Delta_0=\omega_c-\nu$, $\Delta_1=\omega_a-\nu$

$$H = \Delta_0 a^\dagger a + \Delta_1 J_z + \omega J_x + g\sqrt{\frac{2}{N}}(J_- a^\dagger + J_+ a)$$

Resonant driving, „bad cavity limit“:

$$\partial_t \rho_A(t) = -\mathrm{i}[\Delta_1 J_z + \omega J_x, \rho_A(t)] + \frac{g^2}{\gamma N}\Big(2J_-\rho_A(t)J_+ - J_+J_-\rho_A(t) - \rho_A(t)J_+J_-\Big)$$

Interesting nonequilibrium steady state phases for $\Delta_1=0$[5]

[5]Valentin Link, Kimmo Luoma and WS, Phys. Rev. A 99, 062120 (2019)

Outline

- Dynamical Phase Transition (DPT)
- Dicke model
- **DPT in the Driven Dicke model**
- Measurement scheme

Dynamical Phase Transition in driven Dicke model

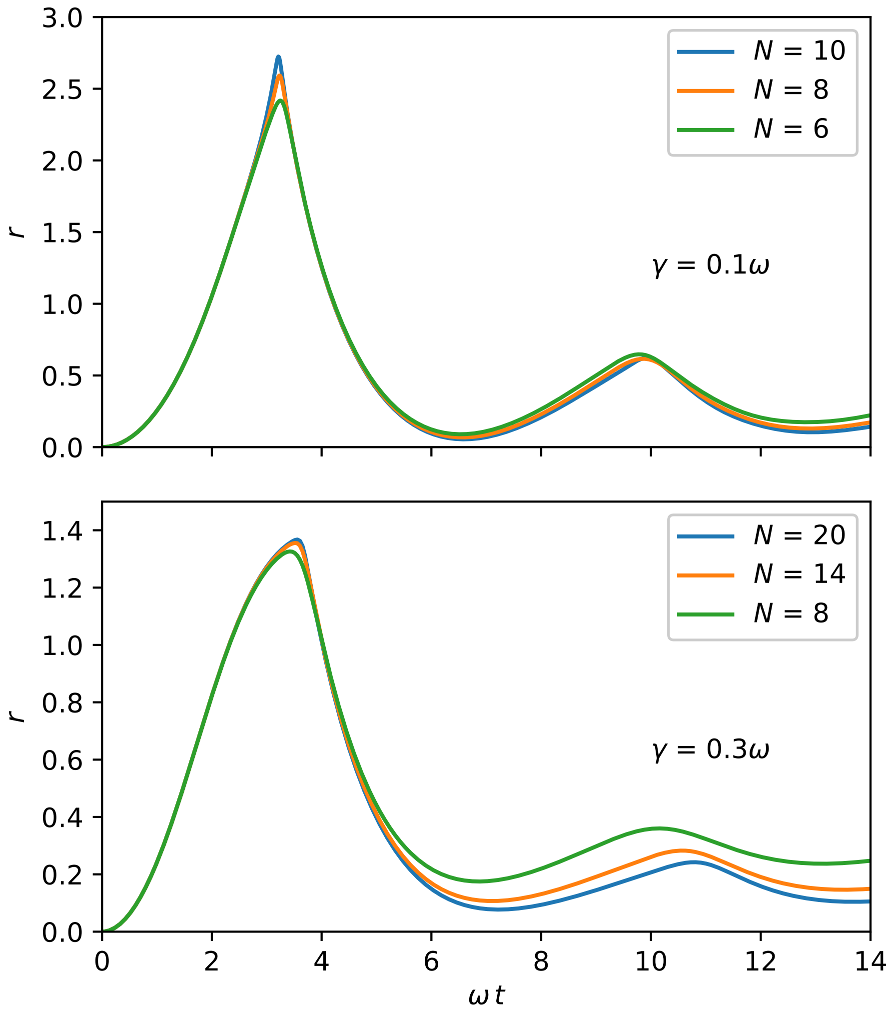
Initial ground state:

$$|\psi(0)\rangle = |\mathrm{vac}\rangle\,|0_N\rangle \qquad |0_N\rangle = \bigotimes_{\lambda=1}^{N} |0\rangle = |j, m=-j\rangle$$

$$L(t) = \langle 0_N|\rho_A(t)|0_N\rangle \qquad r(t) = -\frac{1}{N}\ln L(t)$$

Figure: $g^2 = 25/72\omega\gamma,\ \Delta_0 = 0.1\omega,\ \Delta_1 = 0$

DPT robust w.r.t. all parameters



Dynamical Phase Transition in driven Dicke model

Bad Cavity limit can be treated fairly analytically for *all* system sizes:

Express state in terms of spin-coherent states:

$$|\phi,\theta\rangle = \bigotimes_{\lambda=1}^{N}\left(\cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}e^{i\phi}|1\rangle\right) \qquad \rho_A(t) = \int d\Omega P(\phi,\theta,t)\,|\phi,\theta\rangle\langle\phi,\theta|$$

$$\partial_t\rho_A(t) = -i[\omega J_x, \rho_A(t)] + \frac{\omega}{\lambda N}\Big(2J_-\rho_A(t)J_+ - J_+J_-\rho_A(t) - \rho_A(t)J_+J_-\Big)$$

$$\partial_t P(t) = -\partial_\phi A_\phi P(t) - \partial_\theta A_\theta P(t) + \frac{1}{2N}\partial_\theta^2 D_\theta P(t) + \frac{1}{2N}\partial_\phi^2 D_\phi P(t)(0)$$

Fokker-Planck equation with diffusion scaling as 1/N

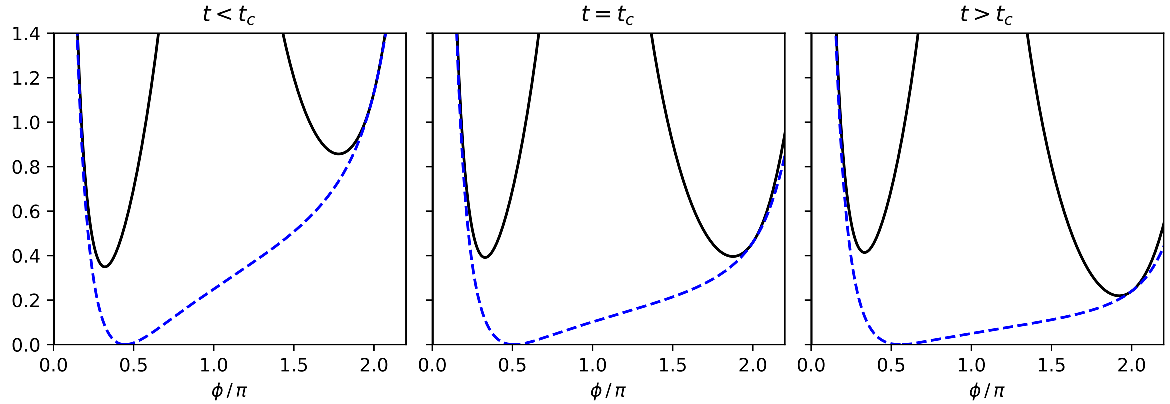
Large deviation theory of classical stochastic processes

$$P(\phi,\theta,t) = \int_{\phi_i,\theta_i}^{\phi,\theta}\mathcal{D}\phi\mathcal{D}\theta\int\mathcal{D}p_\phi\mathcal{D}p_\theta\, e^{-NS([\phi,\eta,p_\phi,p_\eta],t)} \approx F(\phi,\theta,t)\,e^{-NS(\phi,\theta,t)}$$

Dynamical Phase Transition in driven Dicke model

$$L(t) = \mathrm{tr}\{\rho_A(t)\rho_A(0)\} = \int \mathrm{d}\Omega\, P(\phi,\theta,t)\langle\phi,\theta|\rho_A(0)|\phi,\theta\rangle$$

$$\mathrm{P}(\phi,\theta,t) \propto \mathrm{e}^{-NS(\phi,\theta,t)} \qquad \langle\phi,\theta|\rho_A(0)|\phi,\theta\rangle \propto \mathrm{e}^{-NW(\phi,\theta)} \qquad K(\phi,\theta,t) = S(\phi,\theta,t) + W(\phi,\theta)$$

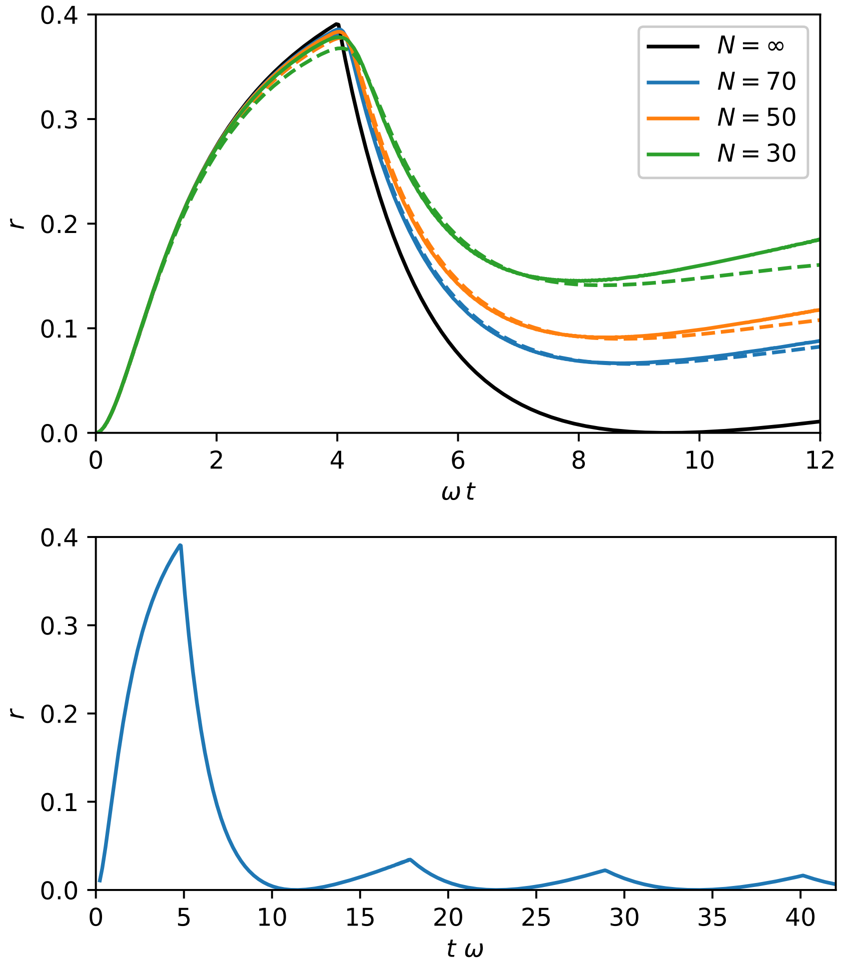


Minimum of S: “mean field dynamics”

Minimum of K: Loschmidt echo

$$L(t) = \int \mathrm{d}\Omega\, F\mathrm{e}^{-NK(\phi,\theta,t)} \approx \sum_{\min} \frac{2\pi F}{N\sqrt{\det K''}}\mathrm{e}^{-NK(t)} \qquad r(t) = -\frac{1}{N}\ln L(t) = \min_{\theta,\phi} K(t) + \mathcal{O}(1/N)$$

Dynamical Phase Transition GKSL dynamics
(Driven Dicke model)



Exact asymptotic (N→ ∞) Loschmidt echo in bad cavity case:

Figure:

- DPT occurs
- robustness w.r.t.~all parameters obvious
- steepest descent also for finite size

Questions:

- what about full model?
- Can non-analytical behaviour be observed?

Outline

— Dynamical Phase Transition (DPT)

— Dicke model

— DPTs in the Driven Dicke model

— **Measurement scheme**

Measurement scheme

Issues for direct measurement:

- Observable of interest is exponentially small:

  Can only be measured for small systems

- Nonanalyticity requires large system

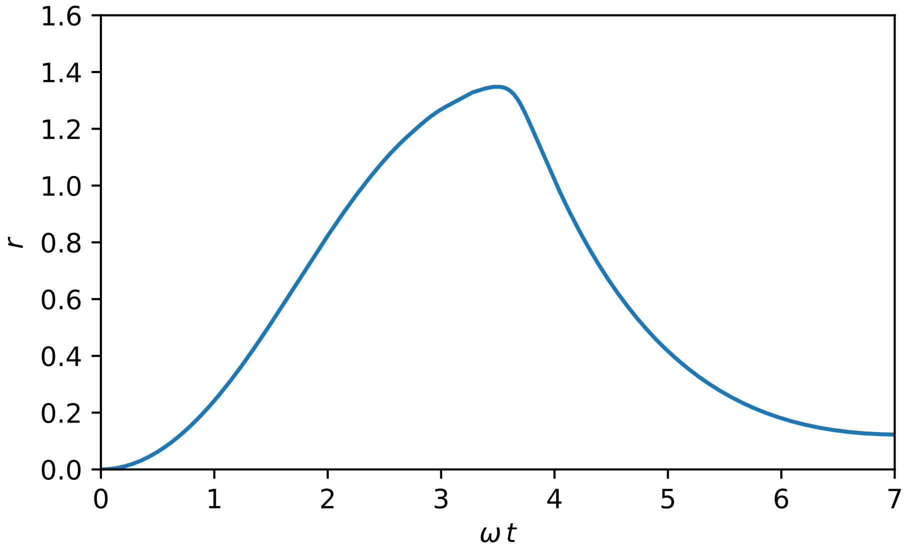


Figure: Driven Dicke model

$$N = 12,\ g^2 = 25/72\omega\gamma, \Delta_0 = 0.1\omega, \Delta_1 = 0, \gamma = 0.3\omega.$$

Further theoretical input is required to measure DPT!

Realized by exploiting correlations between the cavity field and the atoms

Measurement scheme: condition on homodyne outcome

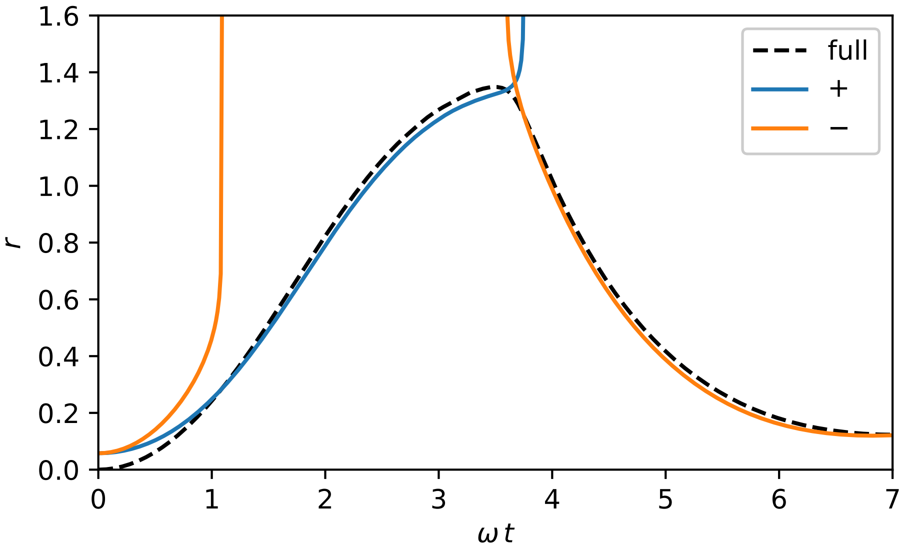
Proposed scheme:

Measure atomic Loschmidt echo (overlap) $L_\pm$, *conditioned* on

homodyne measurement result of cavity field (quantum trajectories):

If homodyne outcome at t is >0: Add quantum trajectory to group '+'

If homodyne outcome at t is <0: Add quantum trajectory to group '-'



Combining all homodyne outcomes recovers unconditioned echo:

$$L(t) = L_+(t) + L_-(t)$$

Theoretical Input:

L_+ and L_- are exponentially small, *can be measured separately, for any* N

Only maximum of the two contributes for N → ∞

Proof of DPT if L_+ and L_- cross! (can be measured for any N).

Outline

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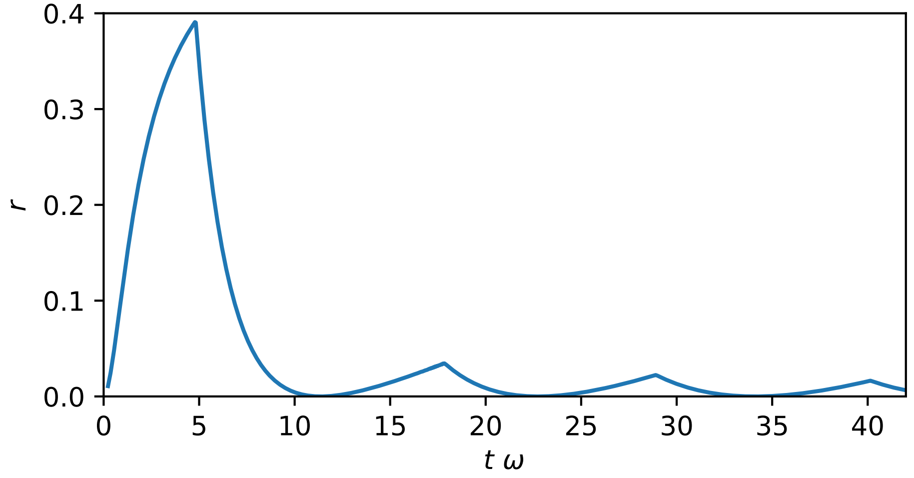
Conclusions:

- Dynamical Phase Transitions occur in GKSL open quantum system dynamics
- Here: driven Dicke model
- Asymptotic ($N \to \infty$) exact treatment possible
- Proof of kink (Landau-type „free energy“)
- Can be experimentally shown by exploiting measurements on environment
- Non-analyticities in “tails” of overlaps



- More general observation: kinks in “Fock state” overlaps

$$r_m(t) = -\frac{1}{N} \ln \langle m, j | \rho_A(t) | m, j \rangle$$