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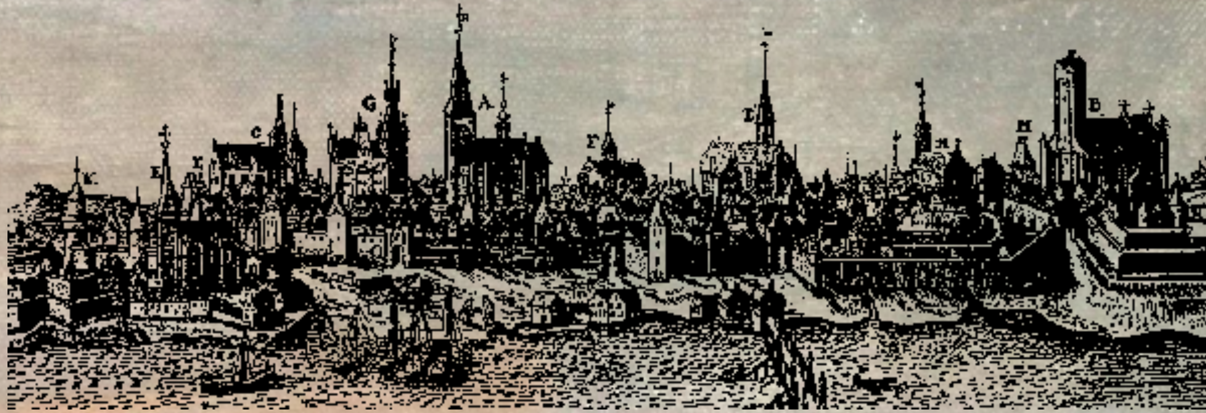


On the relation between time local and non-local master equations

Nina Megier

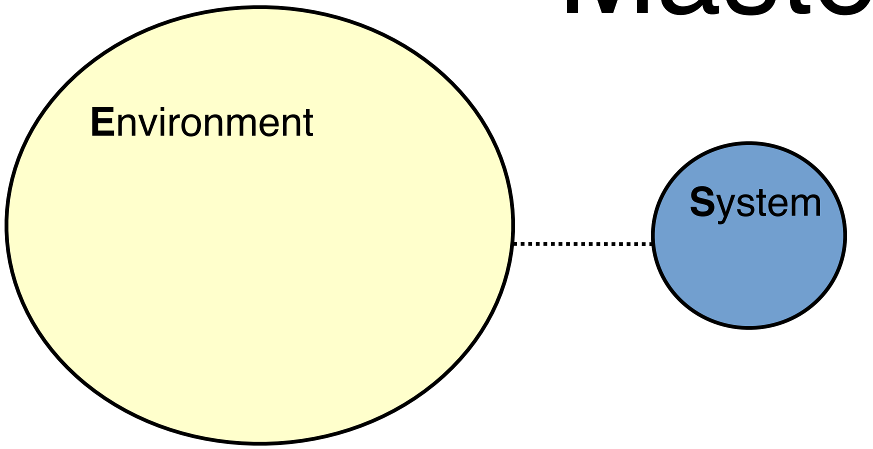
Andrea Smirne, Bassano Vacchini

UNIVERSITAS STUDIORUM MEDIOLANENSIS

INFN

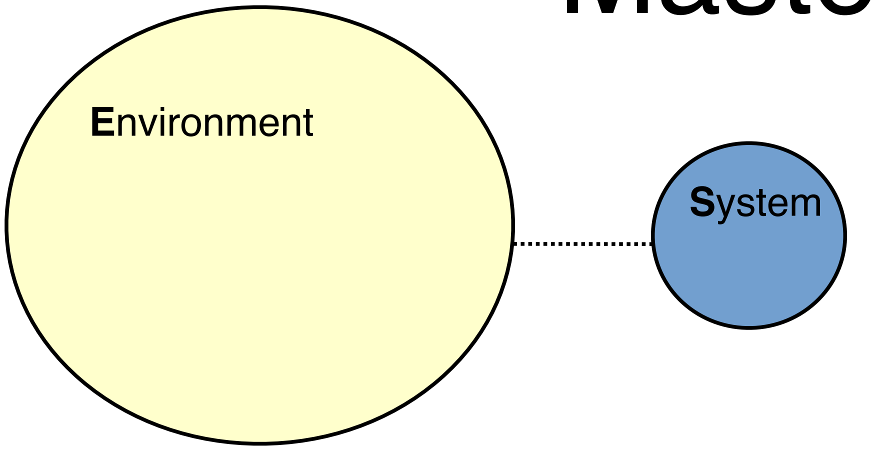
Alexander von Humboldt Stiftung/Foundation

Master equations



$$\rho_S(t) = \mathrm{Tr}_E(\rho_{tot}(t))$$

Master equations



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Time non-local description

$$\dot{\rho}_S(t) = \mathscr{D}_t[\rho_S(s)] = \int_0^t ds \mathscr{K}^{NL}_{t-s}\rho_S(s)$$

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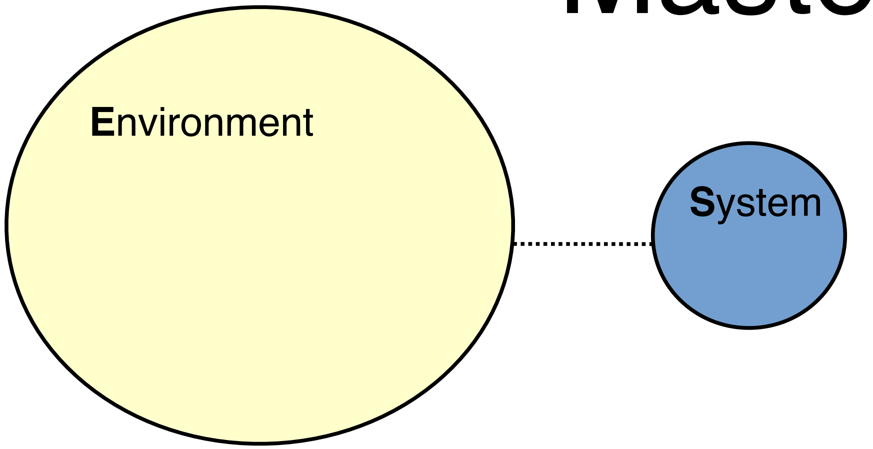

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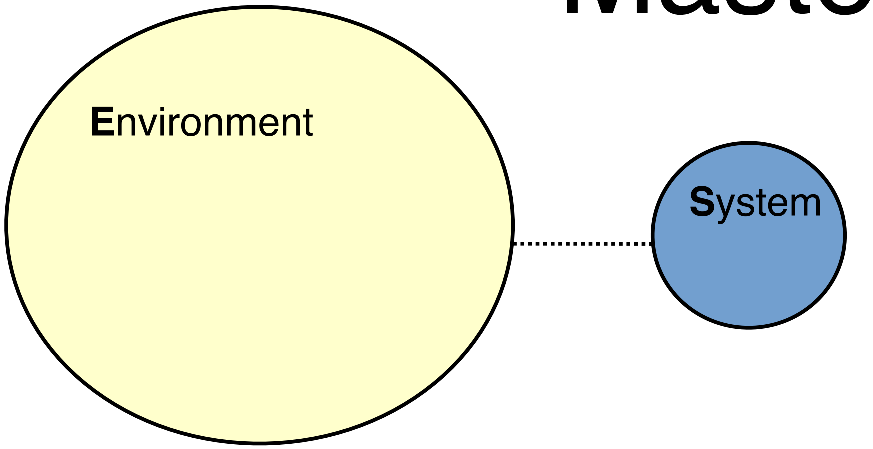

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Time non-local description

$$\dot{\rho}_S(t) = \mathcal{D}_t[\rho_S(s)] = \int_0^t ds \mathcal{K}^{NL}_{t-s} \rho_S(s)$$

Time local description

$$\dot{\rho}_S(t) = \mathcal{D}_t[\rho_S(t)] = \mathcal{K}^L_t[\rho_S(t)]$$

Master equations

Both descriptions are equivalent.

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Advantages to know both:

- Easier access to some properties of the dynamics

- Better understanding of the physical origin of the dynamics

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Quantum non-Markovianity

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Quantum non-Markovianity

$$\dot{\rho}_S(t) = -\frac{i}{\hbar}[H_S, \rho_S(t)] + \sum_i \gamma_i(t)\left(L_i(t)\rho_S(t)L_i^\dagger(t) - \frac{1}{2}\{L_i^\dagger(t)L_i(t), \rho_S(t)\}\right)$$

V. Gorini, A. Kossakowski, E.C.G. Sudarshan, J.Math. Phys. 17, 821 (1976)
G. Lindblad, Commun. Math. Phys. 48, 119 (1976)

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CP-divisibility iff $\gamma_i(t) \geq 0$

$$\Lambda_t[\rho_S(0)] = \rho_S(t), \quad \Lambda_t = \Lambda_{t,s}\Lambda_s$$

M.J.W. Hall, J.D. Cresser, E. Andersson, Phys. Rev. A 89, 042120 (2014)

- Better understanding of the physical origin of the dynamics

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Quantum semi-Markov dynamics

$$\rho_S(t) = p_0(t)\mathscr{F}_t\rho_S(t) + \sum_{n=0}^{\infty}\int_0^t dt_n \ldots \int_0^{t_2} dt_1 p_n(t; t_n, \ldots, t_1) \ldots \mathscr{E}\mathscr{F}_{t_2-t_1}\mathscr{E}\mathscr{F}_{t_1}\rho_S(0)$$

H.P. Breuer, B. Vacchini, Phys. Rev. Lett. 101, 140402 (2008).

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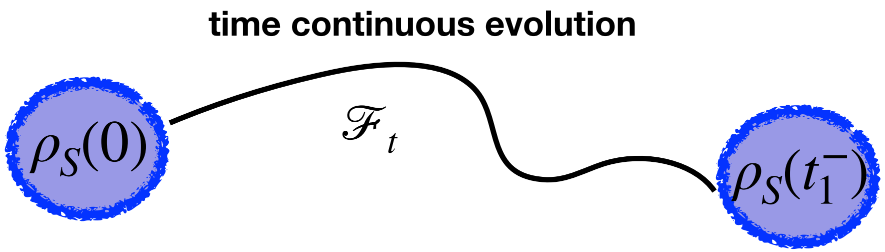

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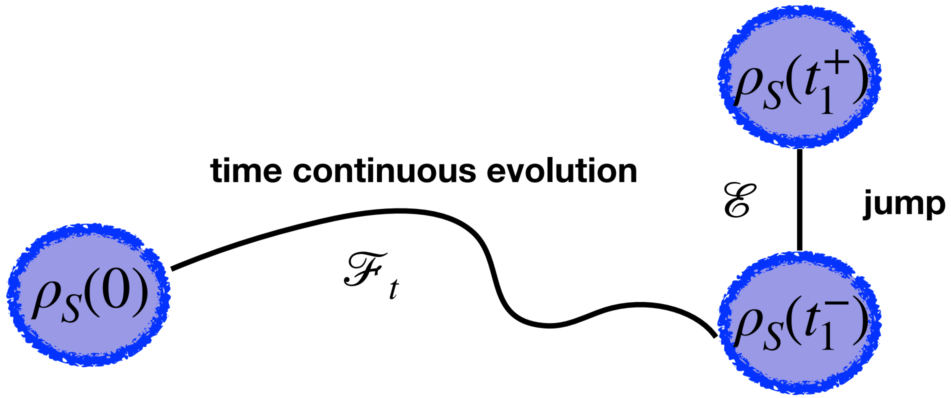

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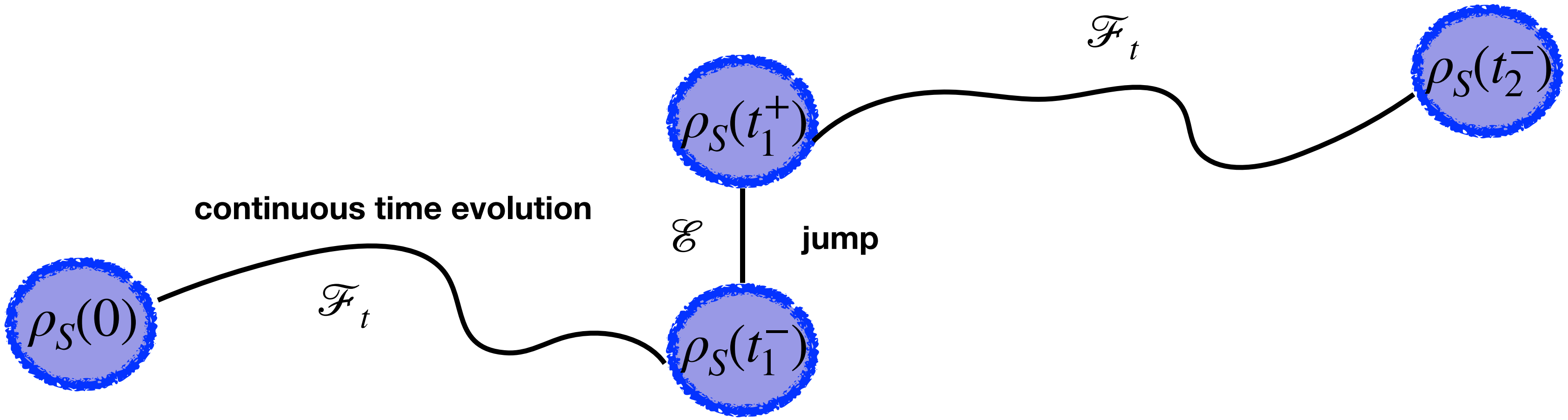

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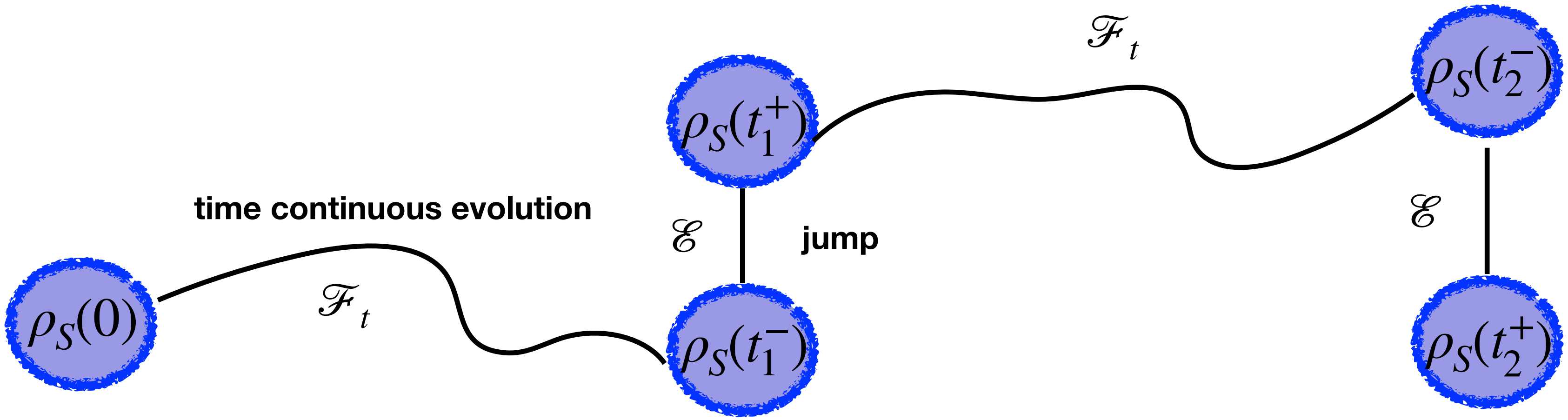

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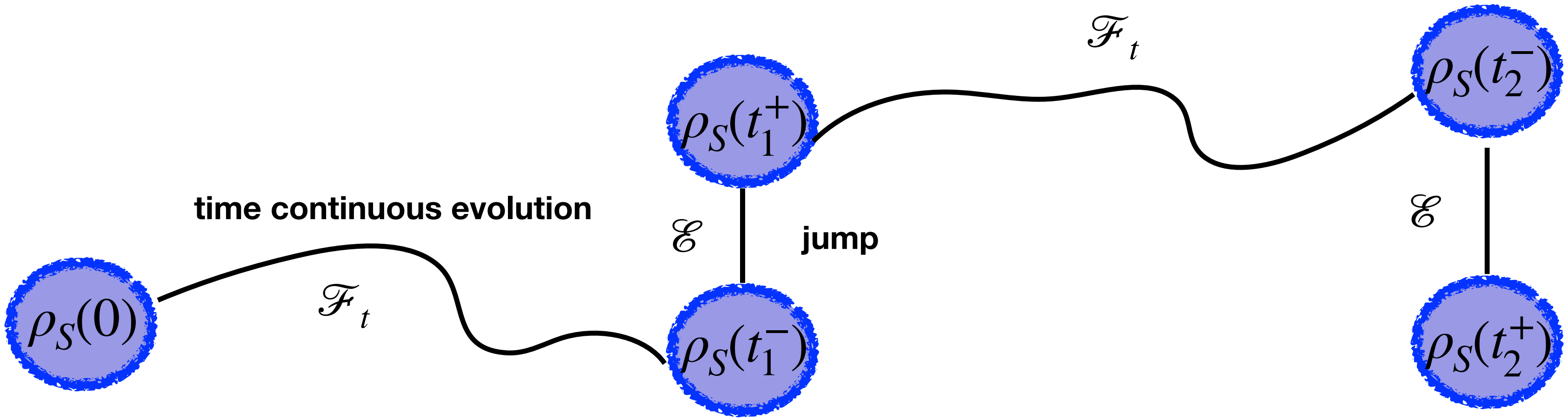

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N. Megier, A.Smirne, B. Vacchini, Entropy 22, 796 (2020)

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$$\Xi(\rho) = \sum_{\alpha\beta=1}^{N^2} M^{\Xi}_{\alpha\beta} \mathrm{Tr}\left[\sigma^{\dagger}_{\beta}\rho\right]\sigma_{\alpha}, \qquad M^{\Xi}_{\alpha\beta} = \mathrm{Tr}\left[\sigma^{\dagger}_{\alpha}\Xi(\sigma_{\beta})\right]$$

Hilbert-Schmidt scalar product

$$\langle\sigma_{\alpha}, \sigma_{\beta}\rangle = \mathrm{Tr}[\,\sigma^{\dagger}_{\alpha}\,\sigma_{\beta}] = \delta_{\alpha\beta},$$

D. Chruscinski, A. Kossakowski, P. Aniello, G. Marmo, and F. Ventriglia.: Op. Sys. Inf. Dyn., 17:255, 2010

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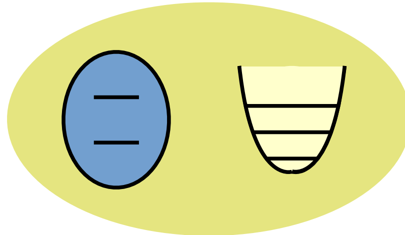
Example: Jaynes-Cummings model



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A.Smirne and B.Vacchini, Phys. Rev. A 82, 022110 (2010)

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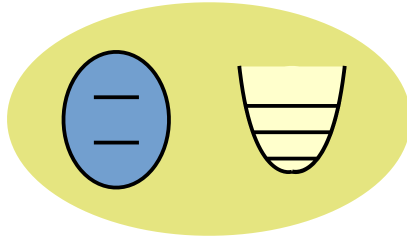
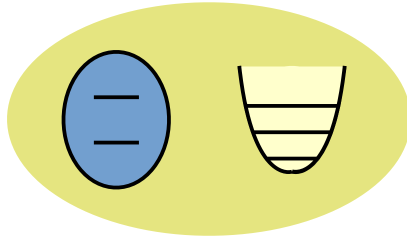


$$H = \frac{\omega_S}{2}\sigma_z \otimes I_E + g(\sigma_+ \otimes b + \sigma_- \otimes b^\dagger) + \omega_E I_S \otimes b^\dagger b$$

$$[\rho_E(0), H_E] = 0$$

A.Smirne and B.Vacchini, Phys. Rev. A 82, 022110 (2010)

Example: Jaynes-Cummings model



$$\frac{d}{dt}\rho_S(t) = -ih(t)[H, \rho_S(t)]$$

$$+\gamma_-(t)\left(\sigma_-\rho_S(t)\sigma_+ - \frac{1}{2}\{\rho_S(t), \sigma_+\sigma_-\}\right)$$

$$+\gamma_+(t)\left(\sigma_+\rho_S(t)\sigma_- - \frac{1}{2}\{\rho_S(t), \sigma_-\sigma_+\}\right)$$

$$+\gamma_z(t)\left(\sigma_z\rho_S(t)\sigma_z - \rho_S(t)\right)$$

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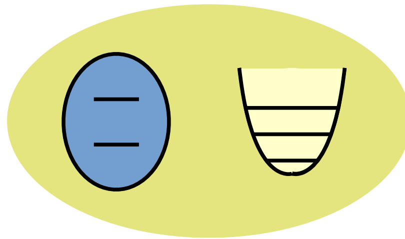

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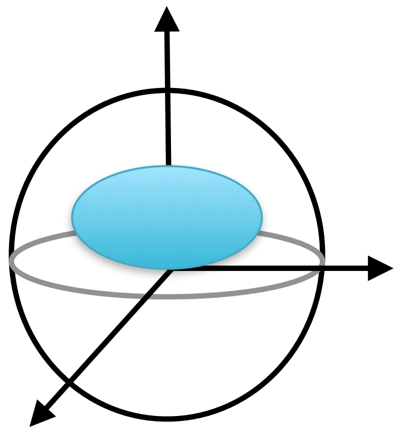
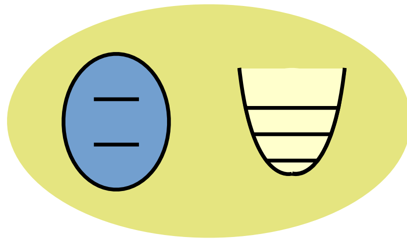
$$+\gamma_z(t)\big(\sigma_z\rho_S(t)\sigma_z - \rho_S(t)\big)$$



Phase covariant dynamics

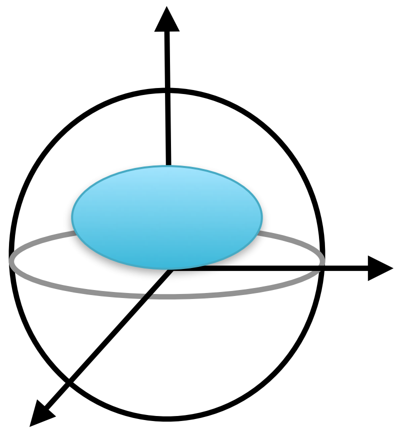
$$U_S(t)\Lambda_t[\rho_S(0)]U_S^\dagger(t) = \Lambda_t[U_S(t)\rho_S(0)U_S^\dagger(t)]$$

Example: Jaynes-Cummings model



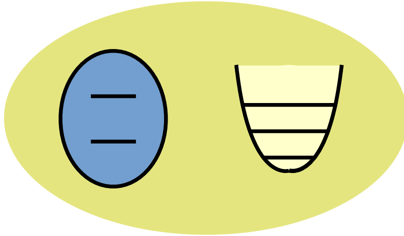
$$\{\varsigma\}_\alpha = \frac{1}{\sqrt{2}} \left\{ I - \frac{\gamma_-(t) - \gamma_+(t)}{\gamma_-(t) + \gamma_+(t)} \sigma_z, \sigma_x, \sigma_y, \sigma_z \right\}$$

$$\{\tau\}_\alpha = \frac{1}{\sqrt{2}} \left\{ I, \sigma_x, \sigma_y, \frac{\gamma_-(t) - \gamma_+(t)}{\gamma_-(t) + \gamma_+(t)} I + \sigma_z \right\}$$



D. Roie, N. Megier, R. Kosloff, arXiv:2106.05295 (2021)

Example: Jaynes-Cummings model



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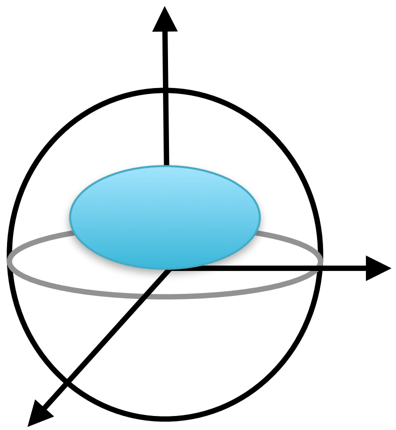

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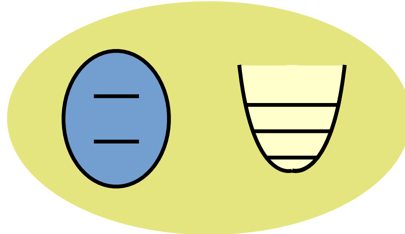

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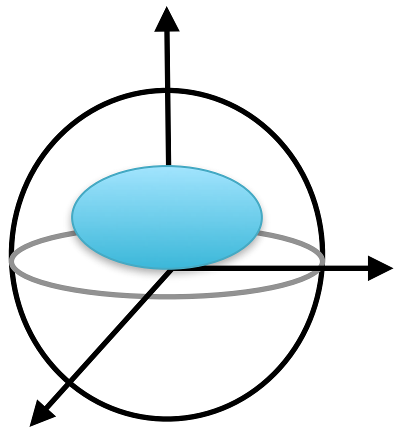
$$[\Lambda_t, \Lambda_s] = 0 \Leftrightarrow \gamma_+(t) = \kappa \gamma_-(t)$$



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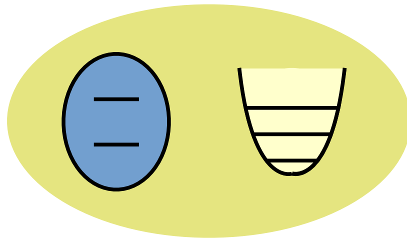
J. Teittinen, H. Lyyra, B. Sokolov and S. Maniscalco, New J. Phys. 20 073012 (2018)

Example: Jaynes-Cummings model



$$\{\varsigma\}_\alpha = \frac{1}{\sqrt{2}}\left\{ I - \frac{1-\kappa}{1+\kappa}\sigma_z, \sigma_x, \sigma_y, \sigma_z \right\}$$

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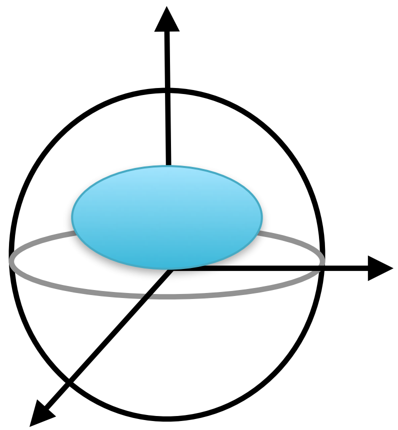

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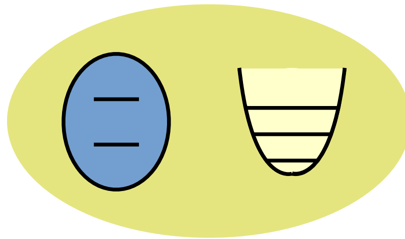

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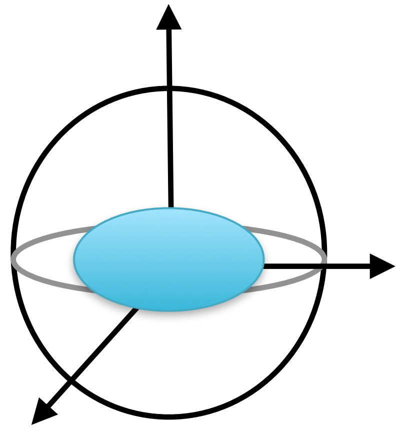
$\kappa = 1$ for unital dynamics



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Goal: connection between local and non-local description

Commutative, diagonalisable dynamics

$$\Lambda_t = \sum_{\alpha=1}^{N^2} m_\alpha(t)\mathrm{Tr}\left[\varsigma_\alpha^\dagger \omega\right] \tau_\alpha = \sum_{\alpha=1}^{N^2} m_\alpha(t)\mathscr{M}_\alpha,$$

D. Chruscinski, A. Kossakowski, P. Aniello, G. Marmo, and F. Ventriglia.: Op. Sys. Inf. Dyn., 17:255, 2010

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$$\mathscr{K}_t^L = \sum_{\alpha=1}^{N^2} m_\alpha^L(t) \mathscr{M}_\alpha,$$

$$\mathscr{K}_t^{NL} = \sum_{\alpha=1}^{N^2} m_\alpha^{NL}(t) \mathscr{M}_\alpha$$

N. Megier, A. Smirne, B. Vacchini, New J. Phys. 22, 083011 (2020)

Goal: connection between local and non-local description

Commutative, diagonalisable dynamics

$$\Lambda_t = \sum m_\alpha(t)\mathscr{M}_\alpha \qquad \mathscr{K}_t^L = \sum m_\alpha^L(t)\mathscr{M}_\alpha \qquad \mathscr{K}_t^{NL} = \sum m_\alpha^{NL}(t)\mathscr{M}_\alpha$$

$$m_\alpha(t) = e^{\int_0^t d\tau m_\alpha^L(\tau)},$$

$$m_\alpha^{NL}(t) = \mathfrak{I}\left(\frac{u\widetilde{G_\alpha}(u)}{1+\widetilde{G_\alpha}(u)}\right)(t), \qquad G_\alpha(t) = \frac{d}{dt}e^{\int_0^t d\tau m_\alpha^L(\tau)}$$

N. Megier, A. Smirne, B. Vacchini, New J. Phys. 22, 083011 (2020)

Master equations

Both local and non-local descriptions are equivalent.

Advantages to know both:

- Easier access to some properties of the dynamics

→ CP-divisibility

- Better understanding of the physical origin of the dynamics

→ Quantum semi-Markov dynamics

Different terms in local and non-local master equations

$$\dot{\rho}_S(t) = \sum_i \gamma_i(t) \left(L_i \rho_S(t) L_i^\dagger - \frac{1}{2} \{ L_i^\dagger L_i, \rho_S(t) \} \right)$$

N. Megier, A. Smirne, B. Vacchini, New J. Phys. 22, 083011 (2020)

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$$\dot{\rho}_S(t) = \sum_i \int_0^t \gamma_i^{NL}(t-s) \left(L_i \rho_S(s) L_i^\dagger - \frac{1}{2} \{ L_i^\dagger L_i, \rho_S(s) \} \right)$$

N. Megier, A. Smirne, B. Vacchini, New J. Phys. 22, 083011 (2020)

Different terms in local and non-local master equations

$$\frac{d}{dt}\rho_S(t) = h(t)(\sigma_-\rho_S(t)\sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-, \rho_S(s)\})$$

$$||$$

$$\frac{d}{dt}\rho_S(t) = \int_0^t ds k(t-s)(\sigma_-\rho_S(s)\sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-, \rho_S(s)\})$$

$$+\int_0^t ds(k_{\surd}(t-s) - \frac{k(t-s)}{2})(\sigma_z\rho_S(s)\sigma_z - \rho_S(s))$$

$$\frac{d}{dt}\rho_S(t) = \mu(t)(\sigma_-\rho_S(t)\sigma_+ + \sigma_+\rho_S(t)\sigma_- - \rho_S(t))$$

$$+(h(t) - \mu(t))(\sigma_z\rho_S(t)\sigma_z - \rho_S(t))$$

$$||$$

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N. Megier, A. Smirne, B. Vacchini, New J. Phys. 22, 083011 (2020)

N. Megier, A.Smirne, B. Vacchini, Entropy 22, 796 (2020)

Damping basis

$$\mathscr{K}_t^L = \sum_{\alpha=1}^{N^2} m_\alpha^L(t)\mathscr{M}_\alpha,$$

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N. Megier, A. Smirne, B. Vacchini, New J. Phys. 22, 083011 (2020)

Redfield-like approximation

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$$\dot{\rho}_S(t) = \int_0^t ds \mathscr{K}^{NL}_{t-s} \rho_S(s)$$

$$\dot{\rho}_S(t) \approx \mathscr{K}^{Red}_t \rho_S(t), \qquad \mathscr{K}^{Red}_t = \int_0^t d\tau \mathscr{K}^{NL}_\tau$$

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N. Megier, A. Smirne, B. Vacchini, New J. Phys. 22, 083011 (2020)

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$$\mathscr{K}_t^{L} = \sum m_\alpha^{L}(t)\mathscr{M}_\alpha$$

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$$\mathscr{K}_t^{Red} = \sum m_\alpha^{Red}(t)\mathscr{M}_\alpha$$

$$m_\alpha^{Red}(t) = \int_0^t d\tau m_\alpha^{NL}(\tau)$$

N. Megier, A. Smirne, B. Vacchini, New J. Phys. 22, 083011 (2020)

Redfield-like approximation

$$\mathscr{F}_t = 1 \Rightarrow \dot{\rho}_S(t) = \int_0^t ds\, k(t-s)(\mathscr{E} - 1)\rho_S(s)$$

N. Megier, A. Smirne, B. Vacchini, New J. Phys. 22, 083011 (2020)

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$$\dot{\rho}_S(t) \approx \int_0^t d\tau k(\tau)(\mathscr{E}-1)\rho_S(t) = S(t)(\mathscr{E}-1)\rho_S(t)$$

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S(t) - renewal density/sprinkling distribution

N. Megier, A. Smirne, B. Vacchini, New J. Phys. 22, 083011 (2020)

Both Markovian and non-Markovian dynamics can result approximated Markovian evolution

$$\mathcal{E}_x \bullet = \sigma_+\sigma_- \bullet \sigma_+\sigma_- + \sigma_-\sigma_+ \bullet \sigma_-\sigma_+$$

$$\frac{d}{dt}\rho_S(t) = h(t)(\mathcal{E}_x - 1)\rho_S(t)$$

$$\geq 0$$

$$\approx$$

$$S(t)(\mathcal{E}_x - 1)\rho_S(t)$$

$$\geq 0$$

N. Megier, A. Smirne, B. Vacchini, New J. Phys. 22, 083011 (2020)

Both Markovian and non-Markovian dynamics can result approximated Markovian evolution

$$\mathcal{E}_z \bullet = \sigma_z \bullet \sigma_z$$

$$\frac{d}{dt}\rho_S(t) = \underset{0}{\underset{\gtrless}{\mu(t)}}(\mathcal{E}_z - 1)\rho_S(t)$$

$$\approx$$

$$\underset{0}{\underset{\geq}{S(t)}}(\mathcal{E}_z - 1)\rho_S(t)$$

N. Megier, A. Smirne, B. Vacchini, New J. Phys. 22, 083011 (2020)

exact		approx.
CP-div	$\Rightarrow$	CP-div

N. Megier, A. Smirne, B. Vacchini, New J. Phys. 22, 083011 (2020)

	exact		approx.
Single eigenvalue	CP-div	$\Rightarrow$	CP-div

$$\mathscr{K}_t^L = m^L(t)\mathscr{M}$$

N. Megier, A. Smirne, B. Vacchini, New J. Phys. 22, 083011 (2020)

	exact		approx.
Single eigenvalue $\mathscr{K}_t^L = m^L(t)\mathscr{M}$	CP-div	$\Rightarrow$	CP-div
Pauli channel	P-div	$\Rightarrow$	P-div
	CP-div	$\not\Rightarrow$	CP-div

N. Megier, A. Smirne, B. Vacchini, New J. Phys. 22, 083011 (2020)

Redfield-like approximation

Mixture of GKSL dynamics

$$\rho_S(t) = \sum_{i=1}^{3} x_i e^{\mathscr{L}_i t} \rho_S(0)$$

$$\mathscr{L}_i[\omega] = \sigma_i \omega \sigma_i - \omega, \qquad x_i \geq 0, \qquad x_1 + x_2 + x_3 = 1$$

Qubit dephasing in random direction

N. Megier, D. Chruściński, J. Piilo, W. T. Strunz, Sci. Rep., 7:6379 (2017)

Redfield-like approximation

Mixture of GKSL dynamics

$$\dot{\rho}_S(t) = \frac{1}{2}\sum_{k=1}^{3} \gamma_k(t)(\sigma_k \rho_S(t)\sigma_k - \rho_S(t))$$

$$\gamma_k(t) \to \gamma_k(t, x_1, x_2, x_3)$$

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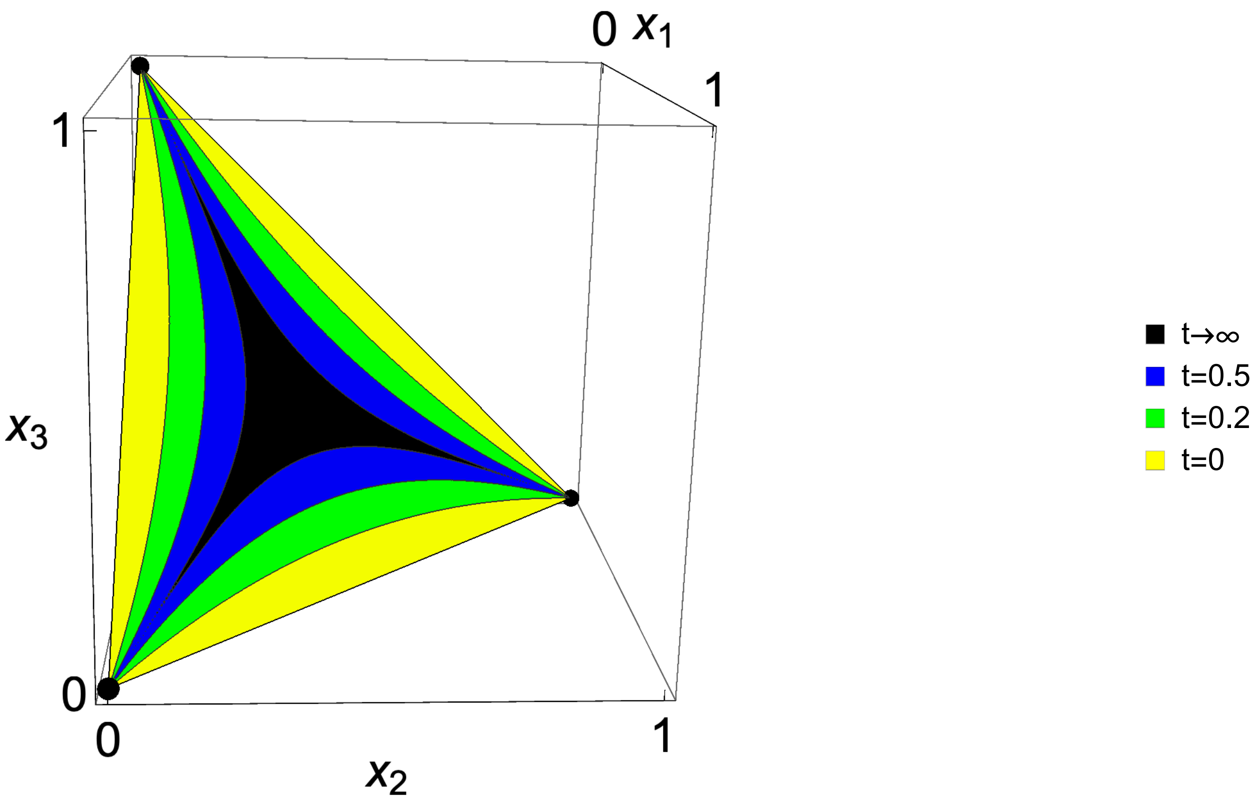
Exact dynamics

$$x_1 + x_2 + x_3 = 1$$



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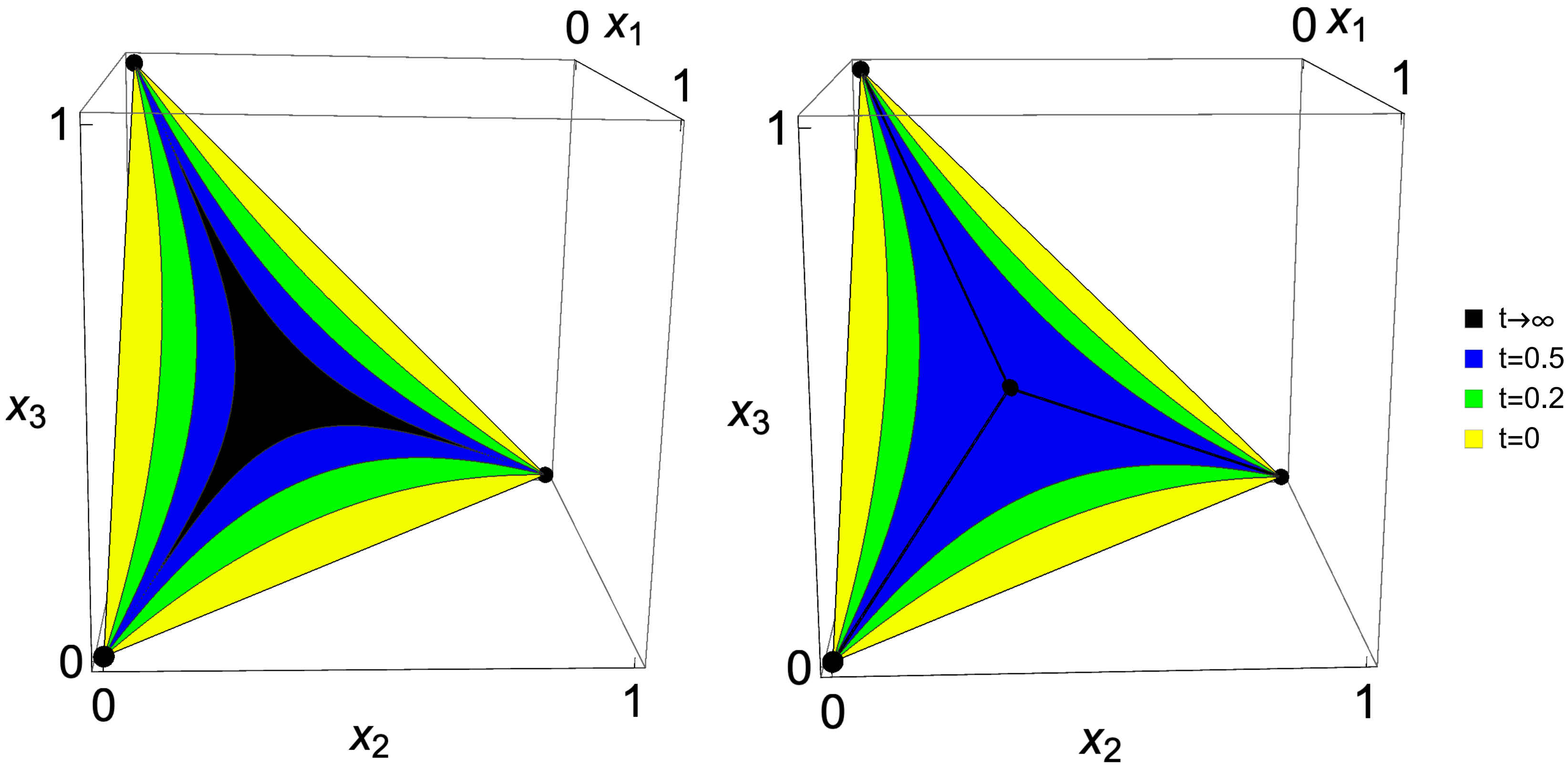
Exact vs. approximated dynamics



N. Megier, A. Smirne, B. Vacchini, New J. Phys. 22, 083011 (2020)

Markovian dynamics can result in approximated non-Markovian evolution

$$\frac{d}{dt}\rho_S(t) = \frac{1}{2}\sum_{k=1}^{3} \underset{\geq 0}{\gamma_k(t)} (\sigma_k \rho_S(t) \sigma_k - \rho_S(t))$$

$$\approx$$

$$\frac{1}{2}\sum_{k=1}^{3} \underset{\gtrless 0}{\bar{\gamma}_k(t)} (\sigma_k \rho_S(t) \sigma_k - \rho_S(t))$$

N. Megier, A. Smirne, B. Vacchini, New J. Phys. 22, 083011 (2020)

$$\mathscr{K}_t^L = \sum_\alpha m_\alpha^L(t)\mathscr{M}_\alpha$$

Single eigenvalue	CP-div	$\Rightarrow$	CP-div
Pauli channel	P-div	$\Rightarrow$	P-div
	CP-div	$\not\Rightarrow$	CP-div

N. Megier, A. Smirne, B. Vacchini, New J. Phys. 22, 083011 (2020)

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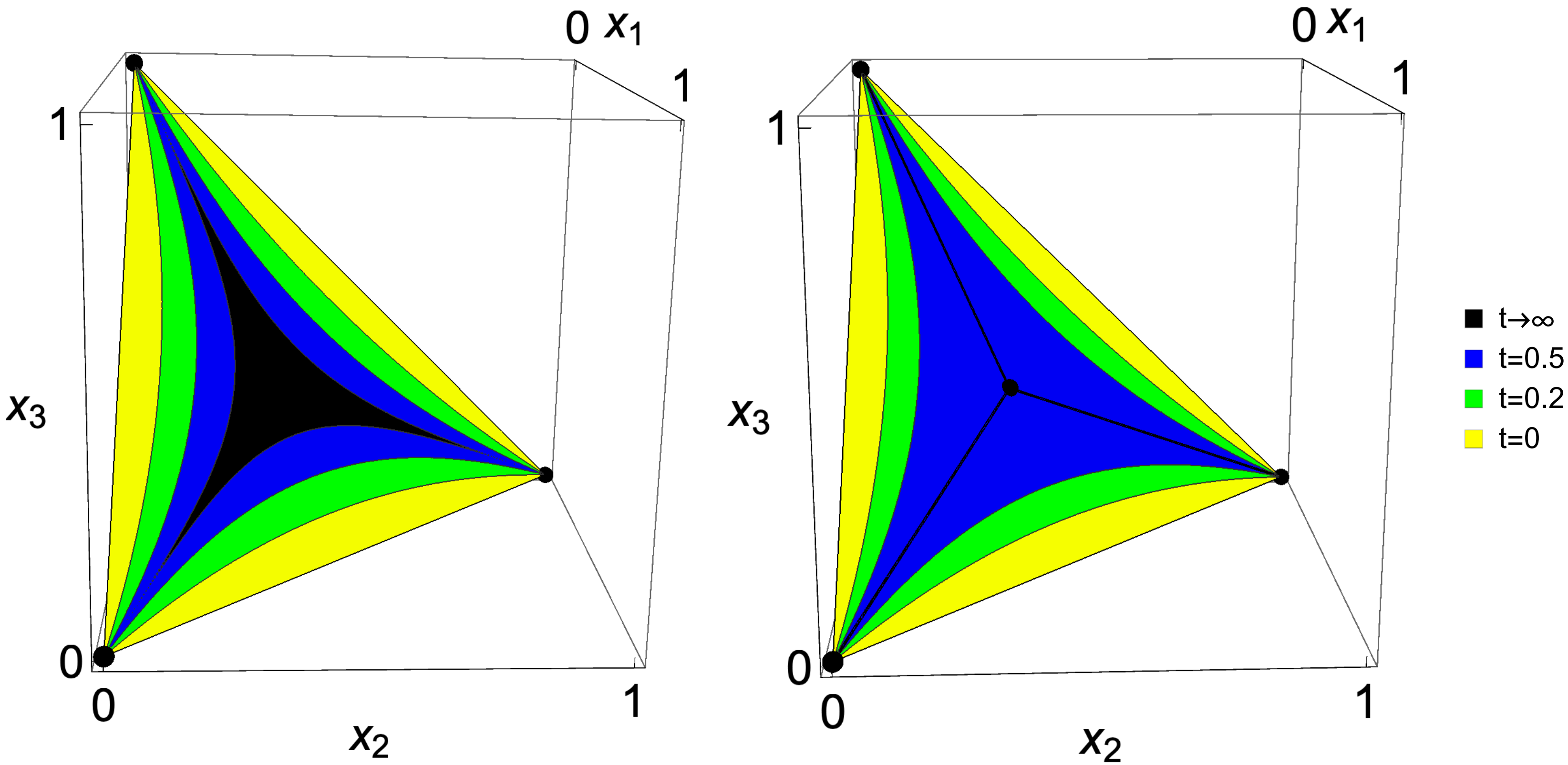
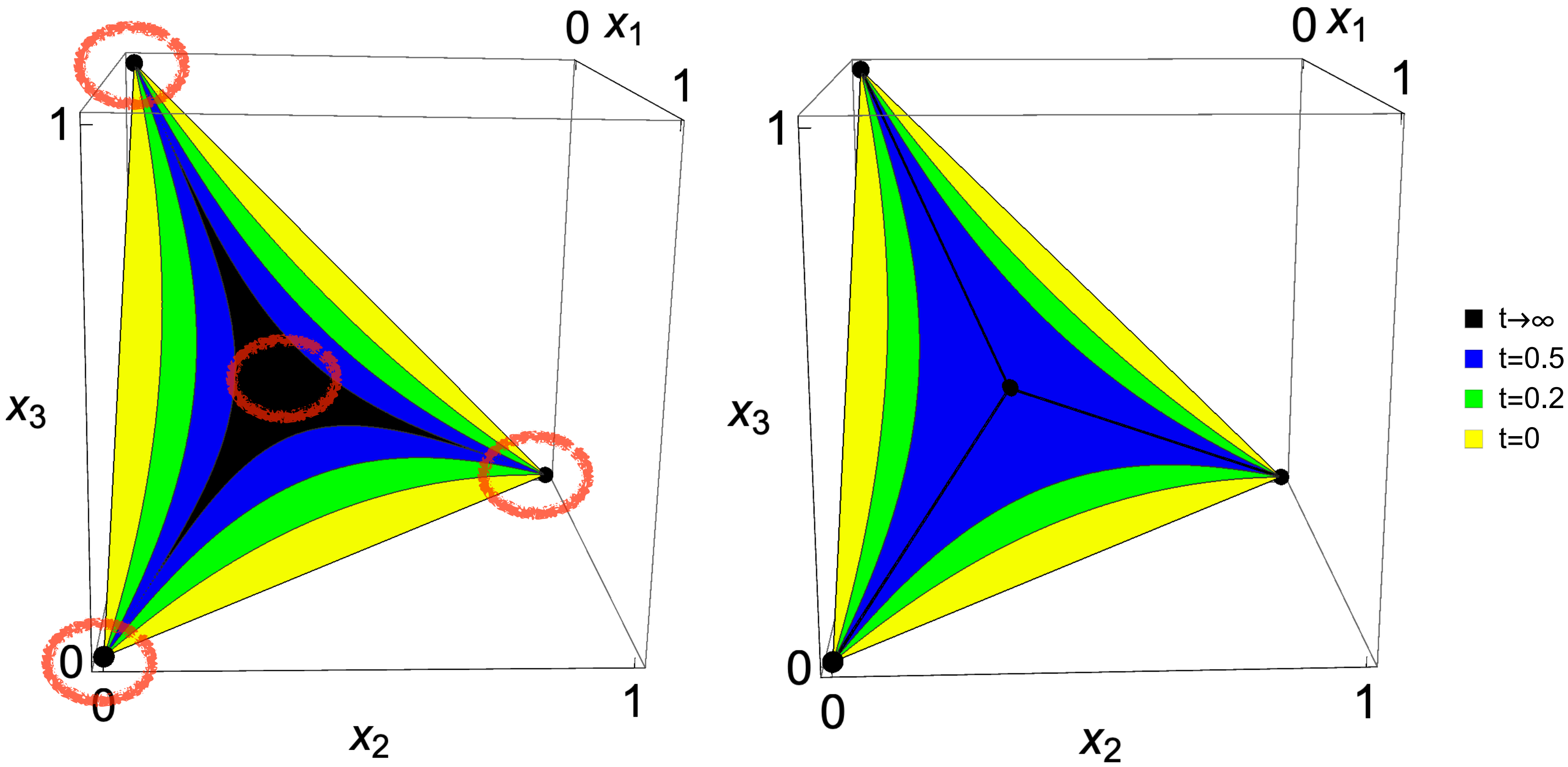
N. Megier, A. Smirne, B. Vacchini, New J. Phys. 22, 083011 (2020)

Exact vs. approximated dynamics



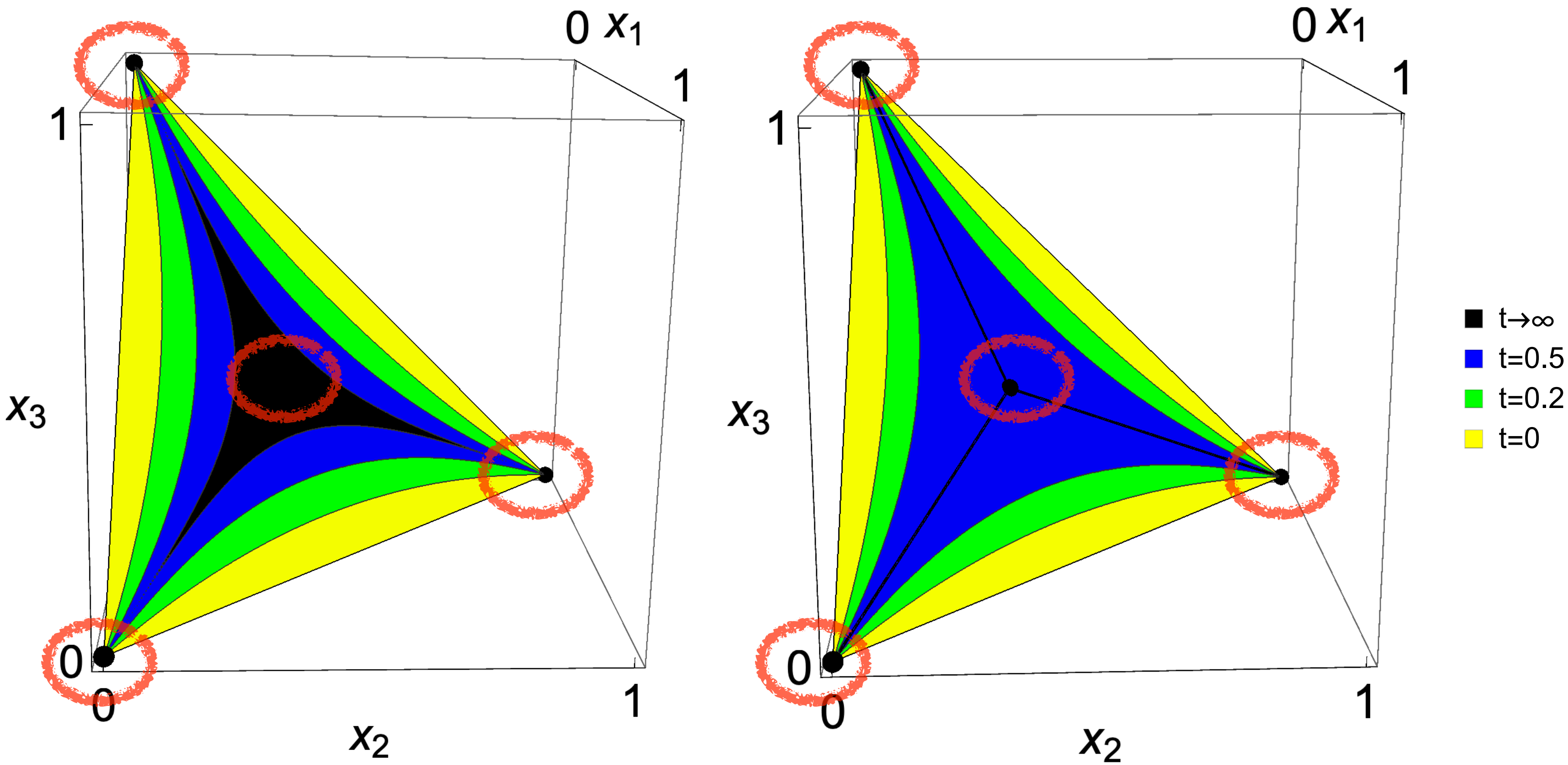
N. Megier, A. Smirne, B. Vacchini, New J. Phys. 22, 083011 (2020)

Exact vs. approximated dynamics



N. Megier, A. Smirne, B. Vacchini, New J. Phys. 22, 083011 (2020)

Exact vs. approximated dynamics



N. Megier, A. Smirne, B. Vacchini, New J. Phys. 22, 083011 (2020)

Summary

- master equations can be of time local and time-non local type

- knowledge of both forms can be beneficial
  - some properties: non-Markovianity
  - physical origin of the dynamics

- an easy connection for commutative, diagonalisable dynamics
- shed some light on occurrence of different dephasing channels in local/non-local master equations
- applications to Redfield-like approximated dynamics