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Self-testing within the stabiliser formalism

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PRL **125**, 260507 (2020)

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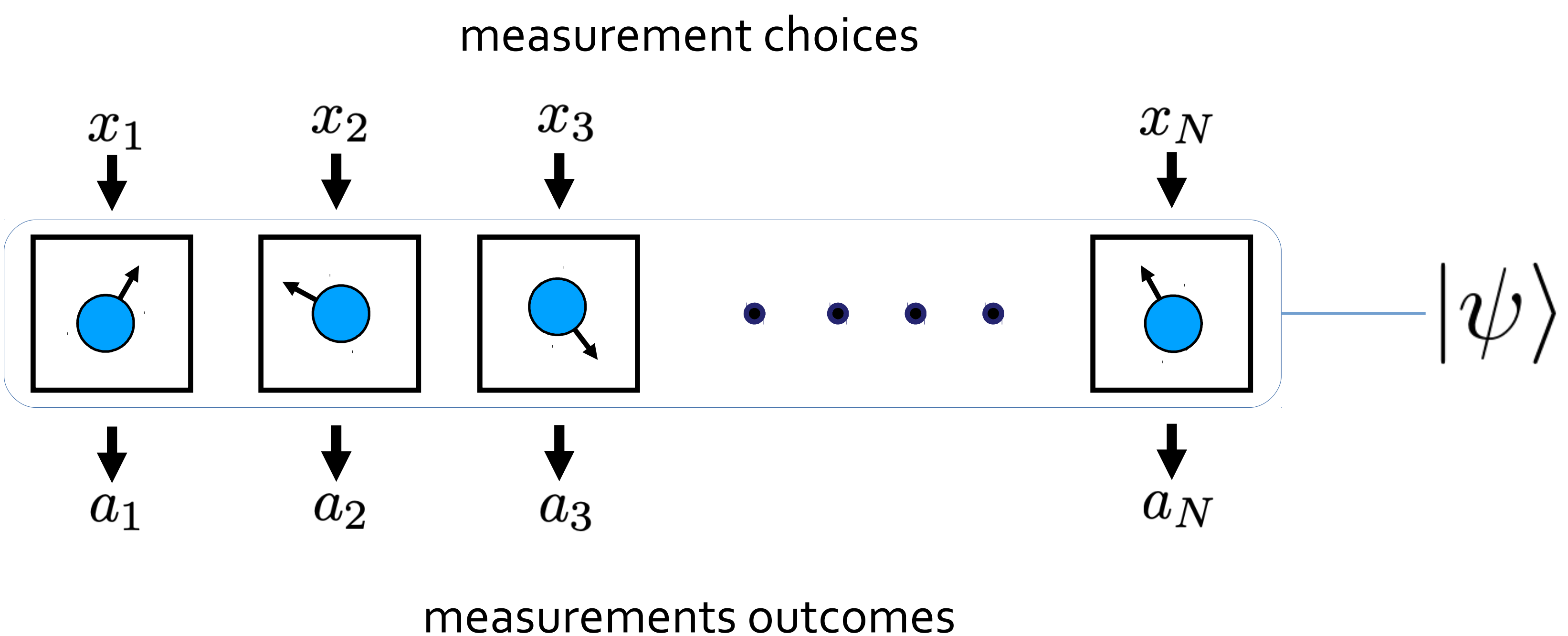
Preliminaries

- **Bell scenario:** *N observers performing measurements on their shares of the state*



$$p(\vec{a}|\vec{x}) := p(a_1, \ldots, a_N | x_1, \ldots, x_N) = \langle\psi| M_{x_1}^{a_1} \otimes \ldots \otimes M_{x_N}^{a_N} |\psi\rangle$$

correlations

measurement operators

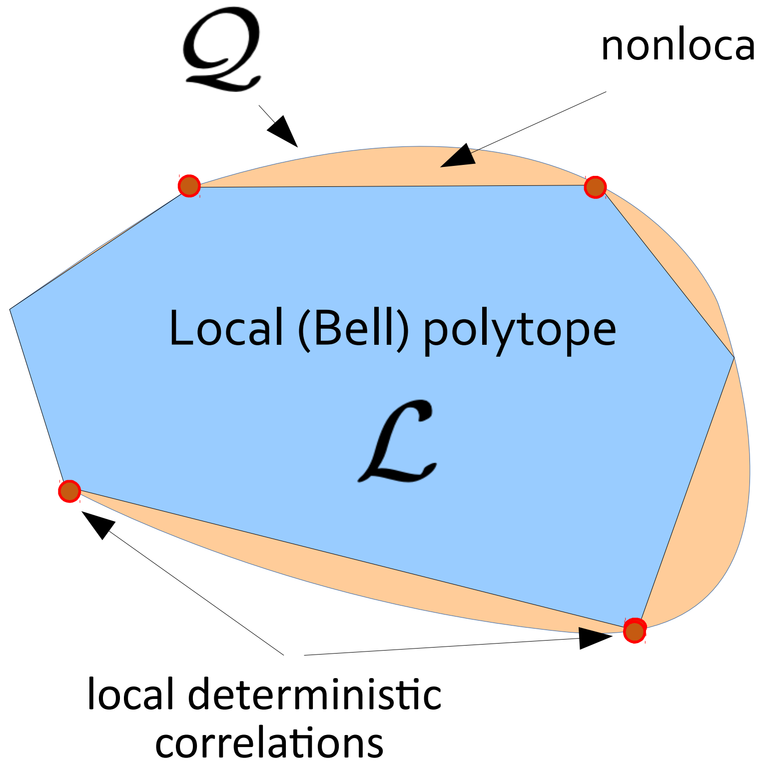
[J. S. Bell, Physics **1**, 195 (1964)]

- **Local (classical) correlations**

$$p(\vec{a}|\vec{x}) = \sum_{\lambda} p(\lambda) p(a_1|x_1,\lambda) \cdot \ldots \cdot p(a_N|x_N,\lambda)$$

$$\forall_{a_i,x_i,\lambda} \qquad p(a_i|x_i,\lambda) \in \{0,1\}$$

λ – hidden variable



- Otherwise they are called **nonlocal** → **nonlocality**

$$\mathcal{P}_{m,d} \subsetneq \mathcal{Q}_{m,d}$$

- **Bell inequalities**: *Hyperplanes constraining the local set*

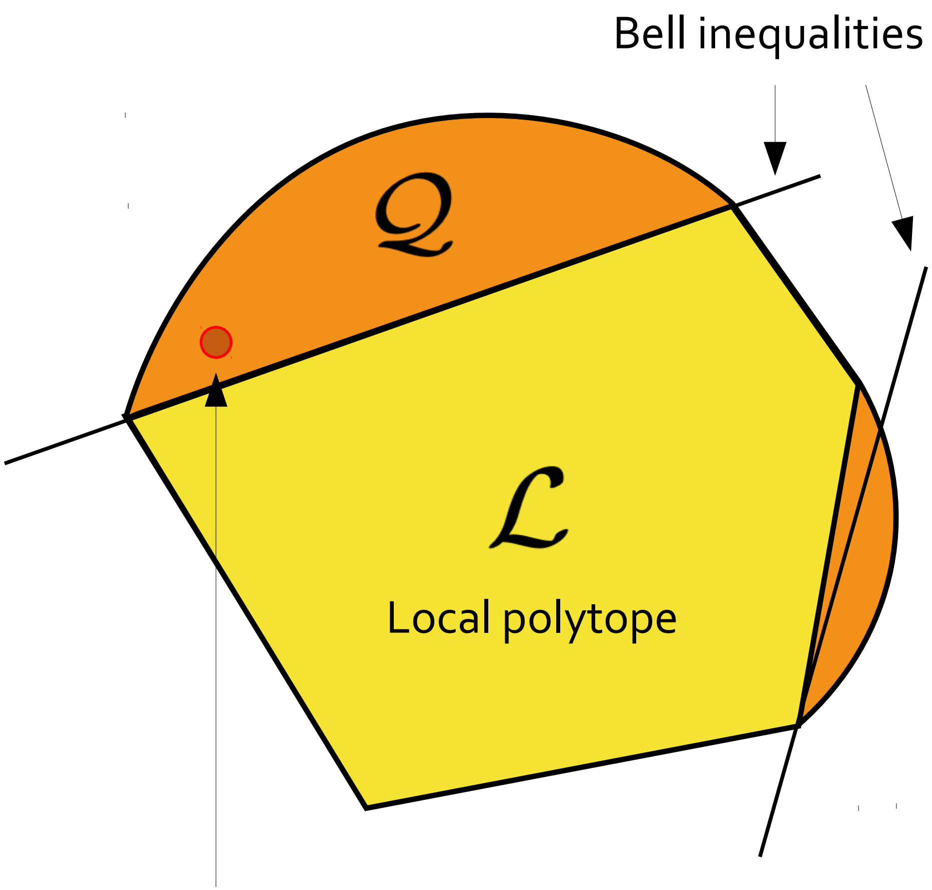
$$I := \sum_{\vec{a},\vec{x}} \alpha_{\vec{a},\vec{x}}\, p(\vec{a}|\vec{x}) \leq \beta_C$$

$$\beta_C = \max_{\mathcal{P}_{m,d}} I \qquad \text{(classical bound)}$$

$$\beta_Q = \sup_{\mathcal{Q}_{m,d}} I \qquad \text{(quantum bound)}$$

Examples

Clauser, Horne, Shimony, Holt (1969);
Collins *et al.* (CGLMP) (2002);
Barrett, Kent, Pironio (BKP) (2006);

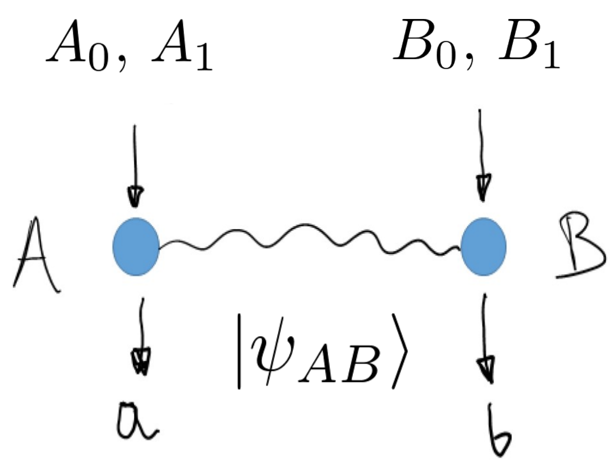


$$I := \sum_{\vec{a},\vec{x}} \alpha_{\vec{a},\vec{x}}\, p(\vec{a}|\vec{x}) > \beta_C$$

$$\Downarrow$$

nonlocality

- **Example**: *Clauser-Horne-Shimony-Holt (CHSH) Bell inequality*



$$a, b = \pm 1$$

$$\langle A_0 B_0 \rangle + \langle A_0 B_1 \rangle + \langle A_1 B_0 \rangle - \langle A_1 B_1 \rangle \leq 2$$

- **Example**: *Clauser-Horne-Shimony-Holt (CHSH) Bell inequality*

$$\langle A_0 B_0 \rangle + \langle A_0 B_1 \rangle + \langle A_1 B_0 \rangle - \langle A_1 B_1 \rangle \leq 2$$

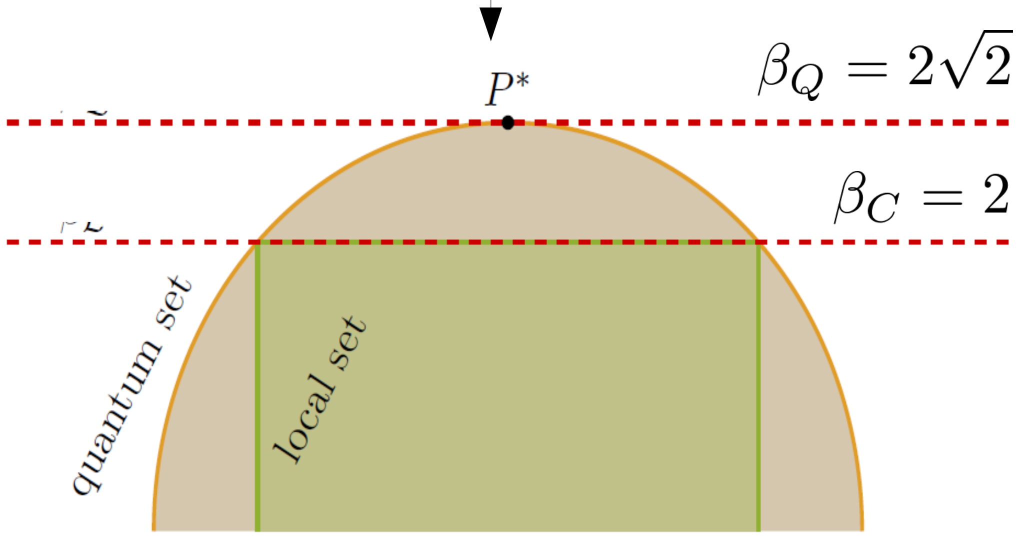
Maximal quantum violation

$$|\psi_2^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$A_0 = X \quad B_0 = \frac{1}{\sqrt{2}}(X + Z)$$

$$A_1 = Z \quad B_1 = \frac{1}{\sqrt{2}}(X - Z)$$

mutually unbiased bases (MUB)



Non-locality

- Non-locality is a resource for device-independent applications

- Quantum key distribution
  [Ekert, PRL (1991); A. Acín *et al.*, PRL (2007)]

- Randomness certification/amplification
  [Pironio *et al.*, Nature (2010); Colbeck, Renner, Nat. Phys. (2012)]

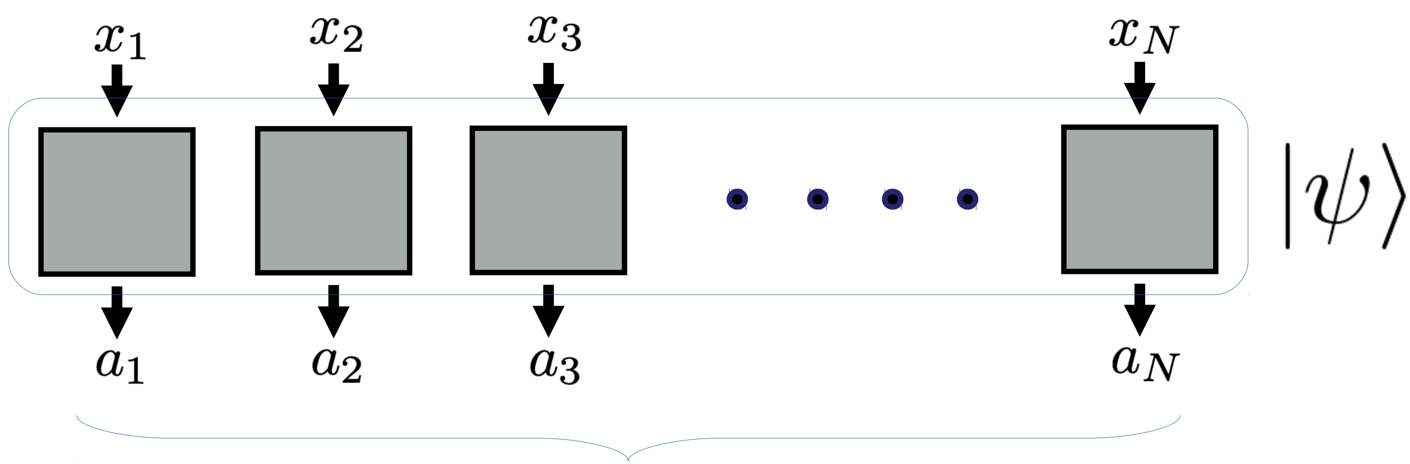
- Device-independent entanglement certification
  [J.-D. Bancal *et al.*, PRL (2011)]

- Self-testing
  [Mayers, Yao, QIC (2004)]

Self-testing

[Mayers, Yao, 2004]

- The idea of device-independent certification

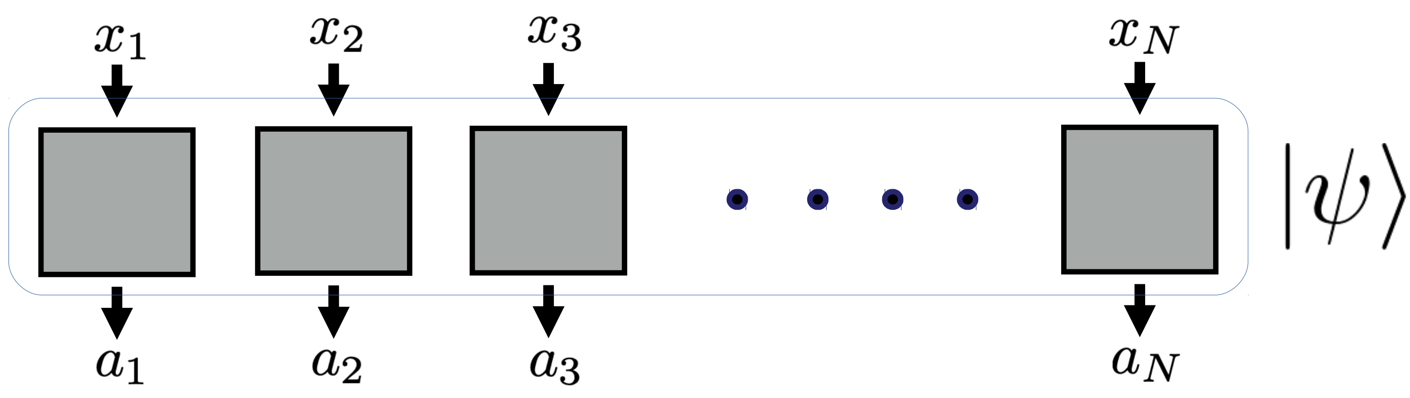


$$p(a_1, \ldots, a_N | x_1, \ldots, x_N)$$

Self-testing

[Mayers, Yao, 2004]

- The idea of device-independent certification



$$p(a_1, \ldots, a_N | x_1, \ldots, x_N)$$

- Given $\{p(\vec{a}|\vec{x})\}$
- or violation of some Bell inequality

$$\sum_{\vec{a},\vec{x}} \alpha_{\vec{a},\vec{x}}\, p(\vec{a}|\vec{x}) = \beta > \beta_C$$

- deduce properties of the state $|\psi\rangle$ and the underlying measurements

- **Self-testing:**

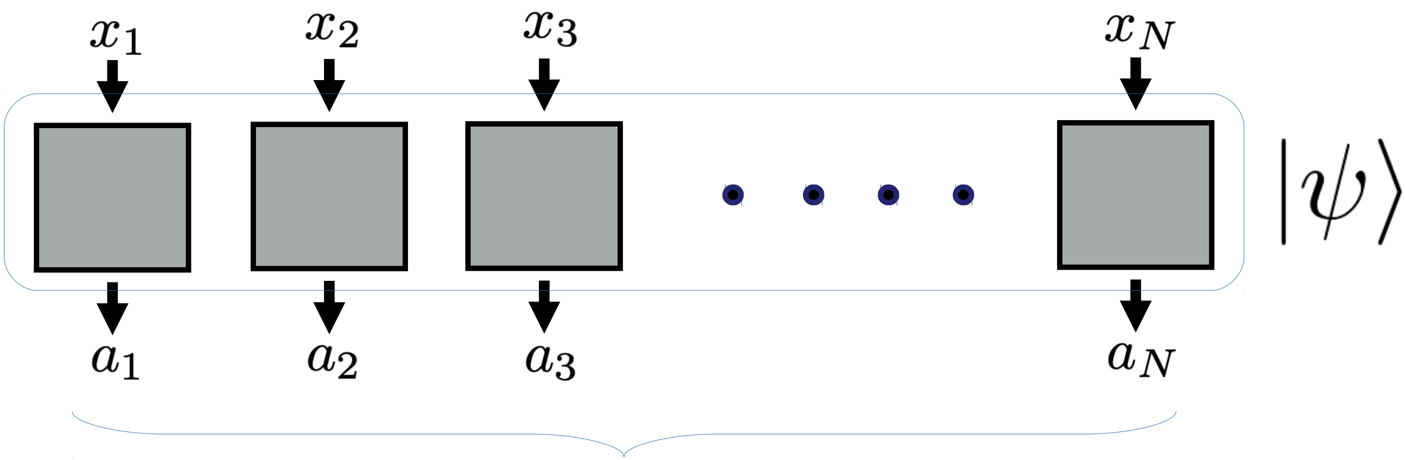
$$\exists_{U_1,\ldots,U_N}$$

$$(U_1 \otimes \ldots \otimes U_N)|\psi\rangle = |\phi\rangle \otimes |\text{aux}\rangle$$

local isometries

the state we want to certify

The idea of device-independent certification



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$$(U_1 \otimes \ldots \otimes U_N)|\psi\rangle = |\phi\rangle \otimes |\text{aux}\rangle$$

Seems like a hopeless task!

but
often one can deduce **everything**!

Self-testing

[Tsirelson 93; S. Popescu, D. Rohrlich, 92]

▸ **Example**: *Self-testing from violation of the CHSH Bell inequality*

$$\langle A_0 B_0\rangle + \langle A_0 B_1\rangle + \langle A_1 B_0\rangle - \langle A_1 B_1\rangle \leq 2\sqrt{2}$$

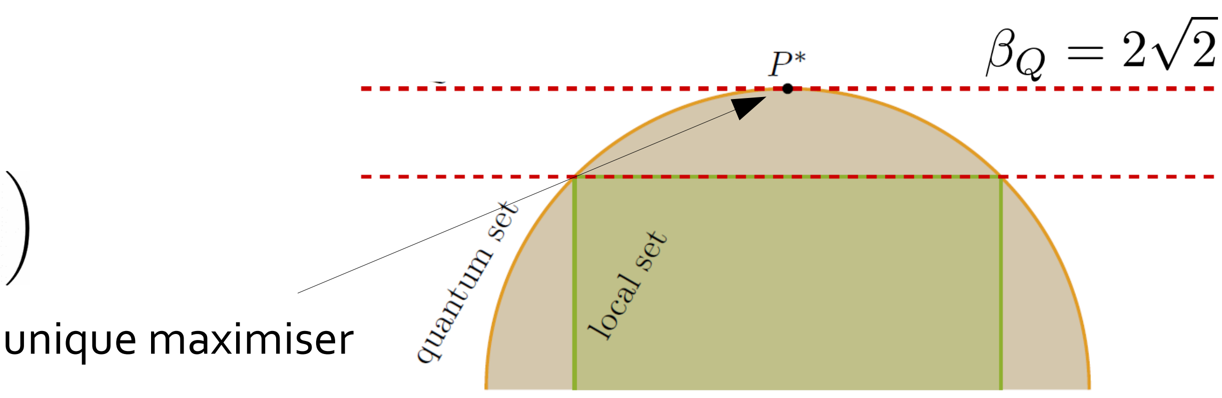
⇓

$$\exists U_A, U_B \qquad U_A \otimes U_B|\psi_{AB}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \otimes |\mathrm{aux}\rangle$$

$$U_A\, A_0\, U_A^\dagger = X \otimes \mathbb{1} \qquad U_B\, B_0\, U_B^\dagger = \frac{1}{\sqrt{2}}(X+Z) \otimes \mathbb{1}$$

$$U_A\, A_1\, U_A^\dagger = Z \otimes \mathbb{1} \qquad U_B\, B_1\, U_B^\dagger = \frac{1}{\sqrt{2}}(X-Z) \otimes \mathbb{1}$$

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$



Scalable Bell inequalities for graph states

F. Baccari, R.A., I. Šupić, J. Tura, A. Acín
PRL **124**, 020402 (2020)

Stabilizer formalism

- *N*-qubit Pauli group

$$\mathbb{G}_N = \{\lambda\, g_1 \otimes \ldots \otimes g_N \,|\, g_i \in \{\mathbb{1}, X, Y, Z\};\ \lambda \in \{\pm 1, \pm \mathrm{i}\}\}$$

N-fold tensor products of the Pauli matrices

- Consider a subgroup

$$\mathbb{S}_N = \langle G_1, \ldots, G_k \rangle$$

$G_i \in \mathbb{G}_N$ – generators (independent elements of the group)

Stabilizer if it stabilizes a nontrivial subspace V in $\mathcal{H}_N = (\mathbb{C}^2)^{\otimes N}$

$$\forall_{|\psi\rangle \in V} \quad \mathbb{S}_N |\psi\rangle = |\psi\rangle \qquad \dim\, V = 2^{N-k}$$

Stabilizer formalism

- Necessary and sufficient condition

$$\left.\begin{array}{l} \forall_{G_i, G_j \in \mathbb{S}_N} \quad [G_i, G_j] = 0 \\ -\mathbb{1}_2^{\otimes N} \notin \mathbb{S}_N \end{array}\right\} \iff \mathbb{S}_N$$

– nontrivial stabilizer

- Applications:
  - Quantum computing
  - Quantum error correction

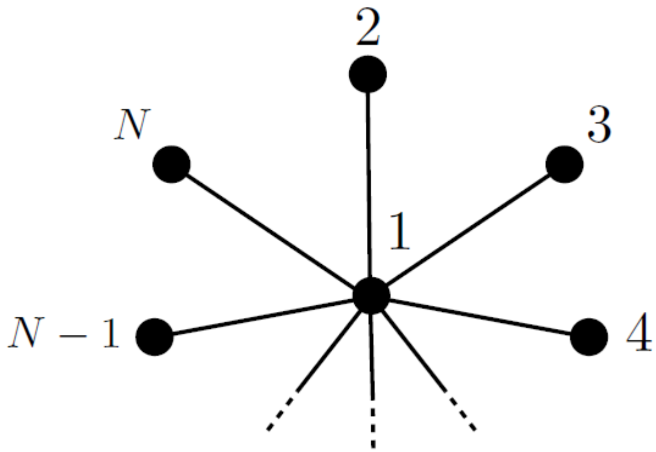
- Useful description of a class of multipartite systems

→ constructing Bell inequalities

Graph states

[Hein *et al.* (2004)]

- Multiqubit graph states $G = (V, E)$



$$G_i = X_i \otimes \bigotimes_{j \in n(i)} Z_j$$

- Graph state associated to a graph *G*:

$$G_i|\psi_G\rangle = |\psi_G\rangle \qquad i = 1, \ldots, N$$

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Graph states

[Hein *et al.* (2004)]

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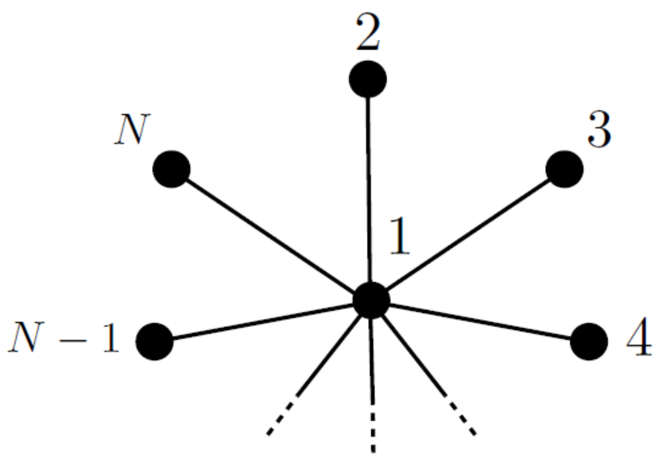

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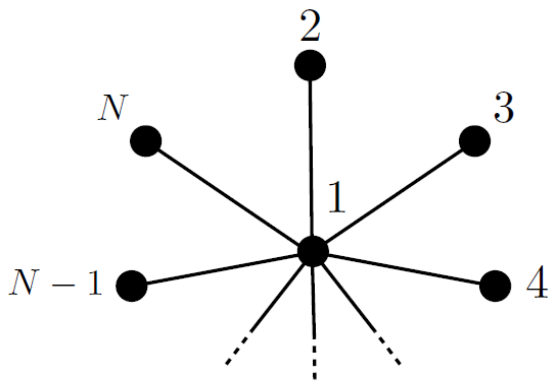
$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- A representative class of multiqubit entangled states
  - GHZ states – multipartite cryptography $\frac{1}{\sqrt{2}}\left(|0\rangle^{\otimes N} + |1\rangle^{\otimes N}\right)$
  - Cluster states – quantum computing
  - Absolutely maximally entangled states
  - All stabilizer states are LU equivalent to graph states

CHSH-like Bell inequalities for graph states

- **Step 1:** *Take a graph and the generators* $G = (V, E)$



$$G_i = X_i \otimes \bigotimes_{j \in n(i)} Z_j \qquad n(1) = \max_{i=1,\ldots,N} n(i)$$

- **Step 2:** *Make a substitution*

$$\widetilde{G}_i = \widetilde{X}_i \otimes \bigotimes_{j \in n(i)} \widetilde{Z}_j$$

$$\widetilde{X}_1 = A_0^{(1)} + A_1^{(1)} \qquad \widetilde{X}_i = A_0^{(i)}$$

$$\widetilde{Z}_1 = A_0^{(1)} - A_1^{(1)} \qquad \widetilde{Z}_i \to A_1^{(i)} \qquad i = 2, \ldots, N$$

$A_j^{(i)}$
arbitrary
± 1 observables

- **Step 3:** *Construct Bell expression*

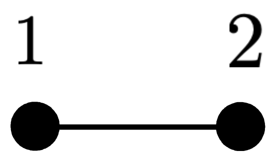
$$I_G := \sqrt{2}\,|n(1)|\,\langle \widetilde{G}_1 \rangle + \sqrt{2} \sum_{j \in n(1)} \langle \widetilde{G}_j \rangle + \sum_{j \notin n(1)} \langle \widetilde{G}_j \rangle$$

Bell inequalities for graph states – example

- **Example:** *the simplest graph*

$$G_1 = X_1 \otimes Z_2$$

$$G_2 = Z_1 \otimes X_2$$



2-qubit maximally entangled state

$$|\psi_+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Step 2

$$\left.\begin{array}{ll} X_1 \to \widetilde{X}_1 = A_0 + A_1 & X_2 \to \widetilde{X}_2 = B_0 \\ Z_1 \to \widetilde{Z}_1 = A_0 - A_1 & Z_2 \to \widetilde{Z}_2 = B_1 \end{array}\right\} \Rightarrow \begin{array}{l} \widetilde{G}_1 = \widetilde{X}_1 \otimes \widetilde{Z}_2 \\ \widetilde{G}_2 = \widetilde{Z}_1 \otimes \widetilde{X}_2 \end{array}$$

Step 3

$$I_G = \langle \widetilde{G}_1 \rangle + \langle \widetilde{G}_1 \rangle = \langle (A_0 + A_1) B_0 \rangle + \langle (A_0 - A_1) B_1 \rangle \leq 2$$

The CHSH Bell inequality

Bell inequalities for graph states

Properties

- Number of expectation values linear in *N*

previous constructions
– exponential scaling
O. Gühne *et al.*, PRL (2005)

- Analytical expressions for maximal classical and quantum values

$$\beta_G^C = N + n_{\max} - 1$$

$$\beta_G^Q = N + (2\sqrt{2} - 1)n_{\max} - 1$$

$\beta_G^Q > \beta_G^C$

for any connected graph

- sum-of-squares decomposition

$$\beta_G^Q \mathbb{1} - \mathcal{B}_G = \frac{|n(1)|}{\sqrt{2}}(\mathbb{1} - \widetilde{G}_1)^2 + \frac{1}{\sqrt{2}} \sum_{j \in n(1)} (\mathbb{1} - \widetilde{G}_j)^2 + \sum_{j \neq n(1)} (\mathbb{1} - \widetilde{G}_j)^2 \quad \Big\} \quad \mathrm{Tr}(\mathcal{B}_G \rho) \leq \beta_Q$$

- optimal quantum realisation

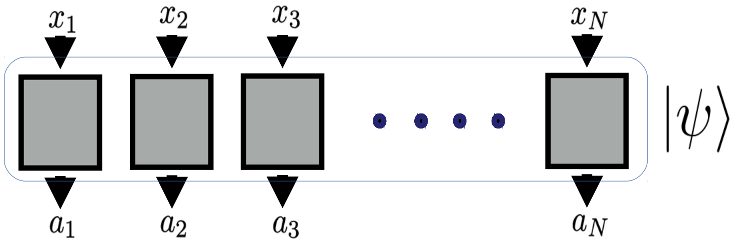
$$A_{0/1}^{(1)} = (X \pm Z)/\sqrt{2}$$

$$A_{0/1}^{(i)} = X/Z \qquad (i = 2, \ldots, N)$$

$$\Big\} \quad \langle \psi_G | \mathcal{B}_G | \psi_G \rangle = \beta_G^Q$$

Bell inequalities for graph states

- Self-testing of all graph states



$$I_G = \beta_G^Q \quad \begin{cases} |\psi\rangle \in (\mathbb{C}^D)^{\otimes N} \\ A_j^{(i)} \end{cases}$$

?

$$|\psi\rangle = |\psi_G\rangle$$

sum-of-squares decomposition

$$\widetilde{G}_i|\psi\rangle = |\psi\rangle \qquad i = 1, \ldots, N$$

$$\Downarrow$$

$$\left\{A_0^{(i)}, A_1^{(i)}\right\} = 0 \qquad \left[A_j^{(i)}\right]^2 = \mathbb{1}$$

$$\Downarrow$$

$$U_1 \otimes \ldots \otimes U_N|\psi\rangle = |\psi_G\rangle \otimes |\mathrm{aux}\rangle$$

Bell inequalities for graph states

- We are not the first to provide Bell inequalities and self-testing methods for graph states

O. Gühne *et al.*, Phys. Rev. Lett. (2005)
G. Tóth *et al.*, Phys. Rev. A (2006)

M.McKague,
Lecture Notes in Computer Science (2014)

- **But:**

  - Scalable Bell inequalities → minimal information?
  - Maximal classical and quantum values direct to determine $\beta_G^Q > \beta_G^C$
  - Self-testing
  - Potentially robust
  - **Possible generalization to entangled subspaces**

Recent experiment:
D. Wu *et al.*, arXiv:2105.10298

Self-testing of genuinely entangled subspaces

F. Baccari, R.A., I. Šupić, A. Acín
PRL **125**, 260507 (2020)

Self-testing of subspaces

- What about stabilizer subspaces of higher dimension?

$$\mathbb{S}_N = \langle G_1, \ldots, G_k \rangle \qquad \mathbb{S}_N V = V$$

$$\dim V = 2^{N-k}$$

- Can we construct Bell inequalities maximally violated by whole subspace?

$$I_{\mathbb{S}}(|\psi\rangle) = \beta^Q \qquad |\psi\rangle \in V$$

$$\Downarrow$$

genuinely entangled subspaces

$\forall_{|\psi\rangle \in V}$ $\quad |\psi\rangle$ – genuinely entangled

$$|\psi\rangle \neq |\psi_1\rangle \otimes |\psi_2\rangle$$

M. Demianowicz, RA, Phys. Rev. A (2020)

Self-testing of subspaces

- **Example:** *five-qubit code* $\mathcal{H} = (\mathbb{C}^2)^{\otimes 5}$ (allows for encoding 1 logical qubit)

$$\mathbb{S}_5 = \langle G_1, G_2, G_3, G_4 \rangle$$

$$\left.\begin{aligned} G_1 &= X_1 Z_2 Z_3 X_4 \\ G_2 &= X_2 Z_3 Z_4 X_5 \\ G_3 &= X_1 X_3 Z_4 Z_5 \\ G_4 &= Z_1 X_2 X_4 Z_5 \end{aligned}\right\} \qquad V = \mathrm{span}\{|\psi_0\rangle, |\psi_1\rangle\}$$

genuinely entangled subspace

Self-testing of subspaces

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genuinely entangled subspace

- Constructing a Bell inequality

$$X_1 \to A_0^{(1)} + A_1^{(1)} \qquad X_i \to A_0^{(i)}$$

$$Z_1 \to A_0^{(1)} - A_1^{(1)} \qquad Z_i \to A_1^{(i)} \qquad i = 2, \ldots, 5$$

$$I_5 = \langle (A_0^{(1)} + A_1^{(1)}) A_1^{(2)} A_1^{(3)} A_0^{(4)} \rangle + \langle A_0^{(2)} A_1^{(3)} A_1^{(4)} A_0^{(5)} \rangle$$
$$+ \langle (A_0^{(1)} + A_1^{(1)}) A_0^{(3)} A_1^{(4)} A_1^{(5)} \rangle + 2 \langle (A_0^{(1)} - A_1^{(1)}) A_0^{(2)} A_0^{(4)} A_1^{(5)} \rangle \leq 5$$

Self-testing of subspaces

$$I_5 = \langle (A_0^{(1)} + A_1^{(1)}) A_1^{(2)} A_1^{(3)} A_0^{(4)} \rangle + \langle A_0^{(2)} A_1^{(3)} A_1^{(4)} A_0^{(5)} \rangle + \langle (A_0^{(1)} + A_1^{(1)}) A_0^{(3)} A_1^{(4)} A_1^{(5)} \rangle + 2\langle (A_0^{(1)} - A_1^{(1)}) A_0^{(2)} A_0^{(4)} A_1^{(5)} \rangle \leq 5$$

- Quantum violations

$$A_{0/1}^{(1)} = (X \pm Z)/\sqrt{2}$$

$$A_{0/1}^{(i)} = X/Z \qquad (i = 2, \ldots, 5)$$

$$\mathcal{B}_5 = \sqrt{2}(G_1 + G_2 + 2G_3) + G_4$$

$$\langle \psi | \mathcal{B}_5 | \psi \rangle = 4\sqrt{2} + 1 > 5 \qquad \forall_{|\psi\rangle \in V}$$

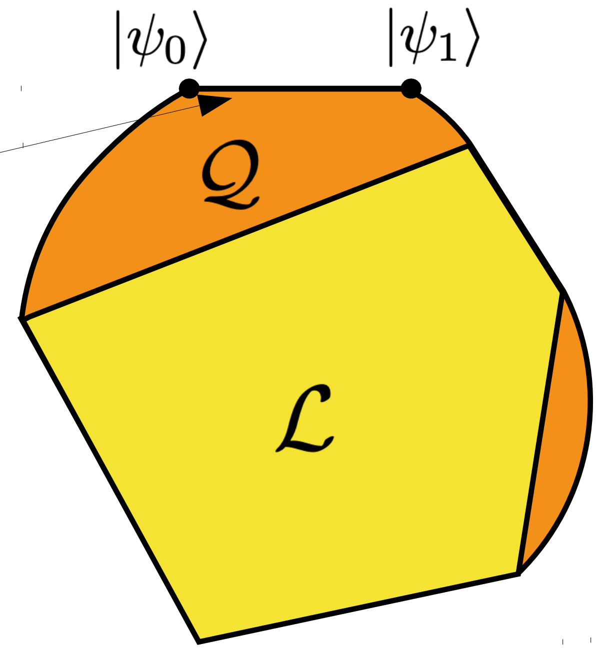
Sum-of-squares decomposition

$$(4\sqrt{2}+1)\mathbb{1} - \mathcal{B}_5 = \frac{1}{\sqrt{2}}(\mathbb{1} - \widetilde{G}_1)^2 + \frac{1}{2}(\mathbb{1} - \widetilde{G}_2)^2 + \frac{1}{\sqrt{2}}(\mathbb{1} - \widetilde{G}_3)^2 + \sqrt{2}(\mathbb{1} - \widetilde{G}_4)^2$$

any vector from the subspace violates this inequality maximally

Self-testing of subspaces

- Geometric picture



Our Bell inequality identifies a nontrivial face structure in the set of quantum correlations

Maximal violation by mixed states

$$\rho = p|\psi_0\rangle\langle\psi_0| + (1-p)p|\psi_1\rangle\langle\psi_1|$$

Can we self-test this entangled subspace?

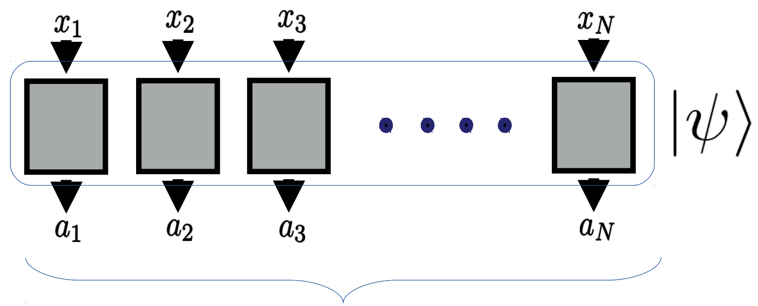
Self-testing of subspaces

But how one defines self-testing of a subspace?

- State self-testing

$\vec{p}$ → $\exists_{U_1,\ldots,U_N}$ such that

$$U_1 \otimes \ldots \otimes U_N |\psi\rangle = |\phi\rangle \otimes |\text{aux}\rangle$$



$$\vec{p} = \{p(a_1, \ldots, a_n | x_1, \ldots, x_N)\}$$

Self-testing of subspaces

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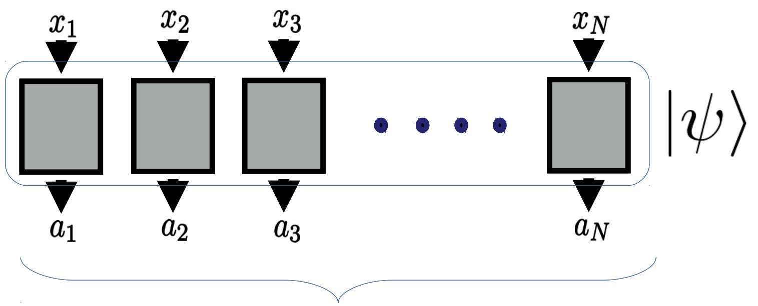


$$\vec{p} = \{p(a_1,\ldots,a_n|x_1,\ldots,x_N)\}$$

- Subspaces

consider a subspace $V = \mathrm{span}\{|\phi_1\rangle,\ldots,|\phi_k\rangle\}$

$\vec{p}$ → $\exists_{U_1,\ldots,U_N}$ such that

$$U_1 \otimes \ldots \otimes U_N|\psi\rangle = \sum_i p_i|\phi_i\rangle \otimes |\mathrm{aux}_i\rangle$$

additional symmetry that does not change the observed correlations

partially correlated additional degrees of freedom

Self-testing of subspaces

- **Example:** *five-qubit code*

$$\mathbb{S}_5 = \langle G_1, G_2, G_3, G_4 \rangle$$

$$\left.\begin{aligned} G_1 &= X_1 Z_2 Z_3 X_4 \\ G_2 &= X_2 Z_3 Z_4 X_5 \\ G_3 &= X_1 X_3 Z_4 Z_5 \\ G_4 &= Z_1 X_2 X_4 Z_5 \end{aligned}\right\} \quad V = \mathrm{span}\{|\psi_0\rangle, |\psi_1\rangle\}$$

genuinely entangled subspace

$$I_5 = \langle (A_0^{(1)} + A_1^{(1)}) A_1^{(2)} A_1^{(3)} A_0^{(4)} \rangle + \langle A_0^{(2)} A_1^{(3)} A_1^{(4)} A_0^{(5)} \rangle$$
$$+ \langle (A_0^{(1)} + A_1^{(1)}) A_0^{(3)} A_1^{(4)} A_1^{(5)} \rangle + 2\langle (A_0^{(1)} - A_1^{(1)}) A_0^{(2)} A_0^{(4)} A_1^{(5)} \rangle \leq 2$$

$$I_5(|\psi\rangle) = 4\sqrt{2} + 1$$

maximal violation

$$U_1 \otimes \ldots \otimes U_5 |\psi\rangle = p|\psi_1\rangle \otimes |\mathrm{aux}_1\rangle + \sqrt{1-p}|\psi_2\rangle \otimes |\mathrm{aux}_2\rangle$$

Generalizations

- Graph states of local dimension *d* prime $G = (V, E)$

$$G_i = X_i \otimes \bigotimes_{j \in n(i)} Z^{r_{ij}}$$

$$G_i |\psi_G\rangle = |\psi_G\rangle$$

$$X|i\rangle = |i+1\rangle \qquad \forall\, |i\rangle$$
$$Z|i\rangle = \omega^i |i\rangle$$

generalizations of Pauli matrices

- Naive approach to constructing Bell inequalities

$$X_1 \to A_0^{(1)} + A_1^{(1)} \qquad X_i \to A_0^{(i)} \qquad A_j^{(i)}$$

$$Z_1 \to A_0^{(1)} - A_1^{(1)} \qquad Z_i \to A_1^{(i)}$$

d-outcome unitary observables

- **But**

$\nexists$ $\alpha X + \beta Z$ – unitary → how to determine maximal quantum violation?

$\alpha, \beta \in \mathbb{C}$

 [J. Kaniewski *et al.*, Quantum (2020)]

- A possible solution

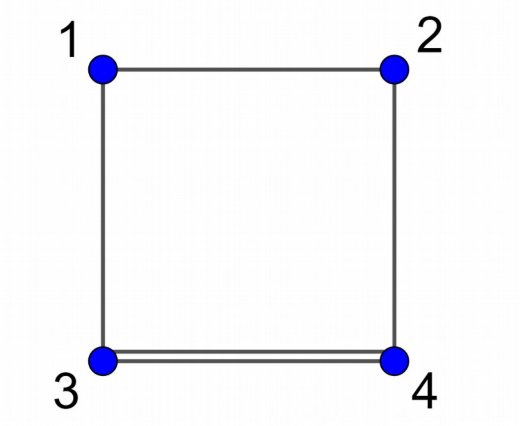
$$\omega^{k(k+1)} X\, Z^k \qquad k = 0, \ldots, d-1$$

✔ mutually unbiased bases in prime *d*

✔ certain combinations of these give proper observables

- **Example**: *AME(4,3) state* $\mathcal{H} = (\mathbb{C}^3)^{\otimes 4}$

$$\left.\begin{array}{ll} G_1 = X_1 Z_2 Z_3 & G_3 = Z_1 X_3 Z_4^2 \\ G_2 = Z_1 X_2 Z_4 & G_4 = Z_2 Z_3^2 X_4 \end{array}\right\} \quad |\psi_{AME(3,4)}\rangle$$



- Constructing a Bell inequality

$$G_1 = X_1 Z_2 Z_3$$

$$G_1 G_2 = (XZ)_1 (ZX)_2 Z_3 Z_4$$

$$G_1 G_2^2 = (XZ^2)_1 (ZX^2)_2 Z_3 Z_4^2$$

$$G_1 G_3 = (XZ)_1 Z_2 (ZX)_3 Z_4^2$$

$$G_4 = Z_2 Z_3^2 X_4$$

Generalizations

- Substitution

$$X_1 \to \frac{1}{\sqrt{3}\lambda}(A_0^{(1)} + A_1^{(1)} + A_2^{(1)}) \qquad Z_2 \to A_0^{(2)}$$

$$(\omega XZ)_1 \to \frac{1}{\sqrt{3}\lambda}(A_0^{(1)} + \omega A_1^{(1)} + \omega^2 A_2^{(1)}) \qquad (ZX)_2 \to A_1^{(2)}$$

etc.

$$(XZ^2)_1 \to \frac{1}{\sqrt{3}\lambda}(A_0^{(1)} + \omega^2 A_1^{(1)} + \omega A_2^{(1)}) \qquad (ZX^2)_2 \to A_2^{(2)}$$

$$\lambda \in \mathbb{C}$$

$$\omega = \exp(2\pi \mathbb{i}/3)$$

- Bell inequality

$$\begin{aligned} I_{\mathrm{AME}} = &\frac{1}{\sqrt{3}\lambda}\langle (A_0^{(1)} + A_1^{(1)} + A_2^{(1)})B_0C_0\rangle + \frac{1}{\sqrt{3}\lambda\omega}\langle (A_0^{(1)} + \omega A_1^{(1)} + \omega^2 A_2^{(1)})B_1C_0D_0\rangle \\ &+ \frac{1}{\sqrt{3}\lambda}\langle (A_0^{(1)} + \omega^2 A_1^{(1)} + \omega A_2^{(1)})B_2C_0D_0^2\rangle \\ &+ \frac{1}{\sqrt{3}\lambda\omega}\langle (A_0^{(1)} + \omega A_1^{(1)} + \omega^2 A_2^{(1)})B_0C_1D_0^2\rangle + \langle B_0C_0^2D_1\rangle + \mathrm{c.c.} \leq \beta_C \end{aligned}$$

maximally violated by the AME(4,3) state

Conclusion/Outlook

- Scalable Bell inequalities for multiqubit graph states + self-testing

- Self-testing of genuinely entangled subspaces (5-qubit and toric codes)

- Possible generalizations → graph states of arbitrary local dimension

  J. Kaniewski *et al.*, Quantum (2021)

  → maximally-dimensional stabilizer subspaces

  O. Makuta, R. A., NJP (2021)

- Further questions

  - Self-testing of multipartite states from minimal information

  - Are all genuinely entangled subspaces self-testable?

    → are all multipartite genuinely entangled states self-testable?