INFORMATIONAL STEADY-STATES AND CONDITIONAL ENTROPY PRODUCTION IN CONTINUOUSLY MONITORED SYSTEMS

Mauro Paternostro
School of Mathematics & Physics, Queen's University Belfast

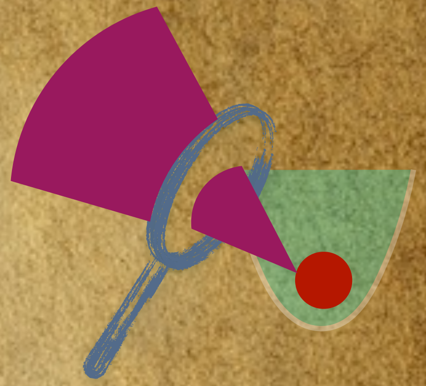
52th Symposium on Mathematical Physics

"Channels, Maps and All That"

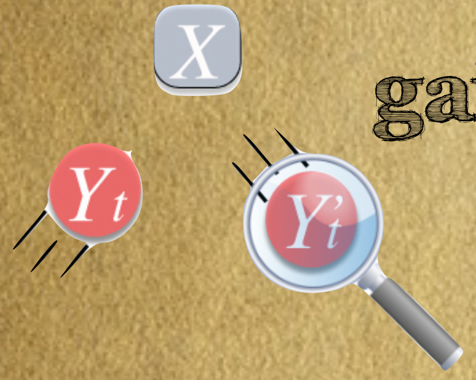
16 June 2021

STRUCTURE OF THE TALK

1



Formalism for entropy production in continuously measured quantum systems



Informational steady states: gaining & losing through measurement

2

3

Observing irreversible entropy in measured mesoscopic quantum settings

QTEQ QUANTUM TECHNOLOGY at QUEEN'S

DUE CREDIT

Alessio Belenchia (Tubingen)

Massimiliano Rossi (ETH Zürich)

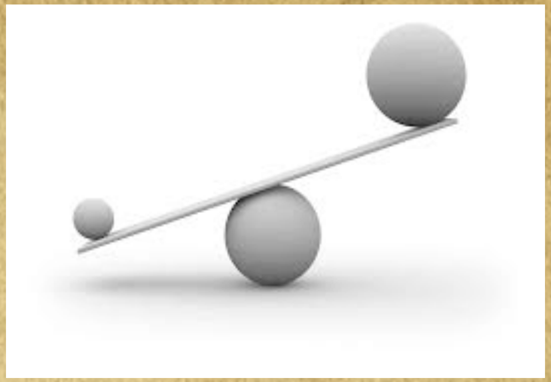
Luca Mancino (Belfast)

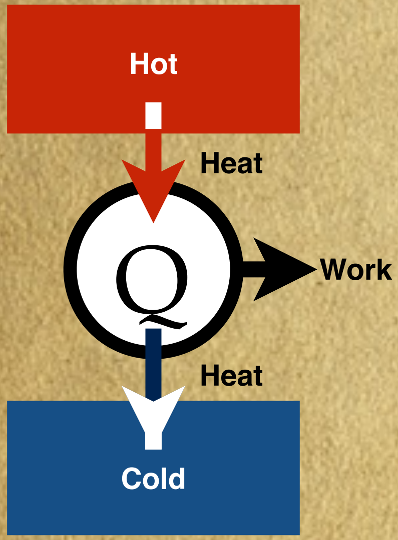
Gabriel T. Landi (Sao Paulo)

Albert Schliesser (Copenhagen)

Why entropy production?

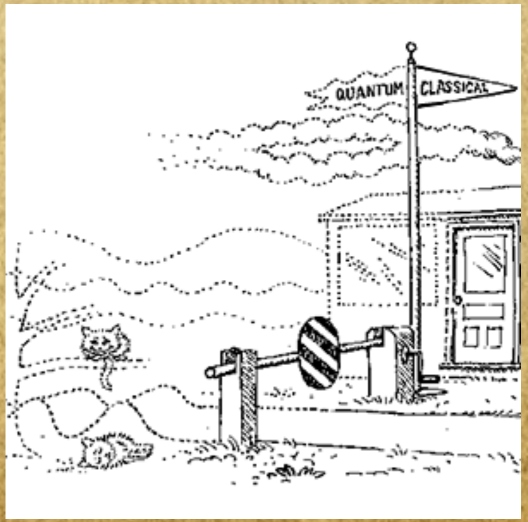
Non-equilibrium processes dissipate energy. This produces irreversible increase of entropy





Entropy production for estimating the performance of devices (exergy is reduced by irreversibility)

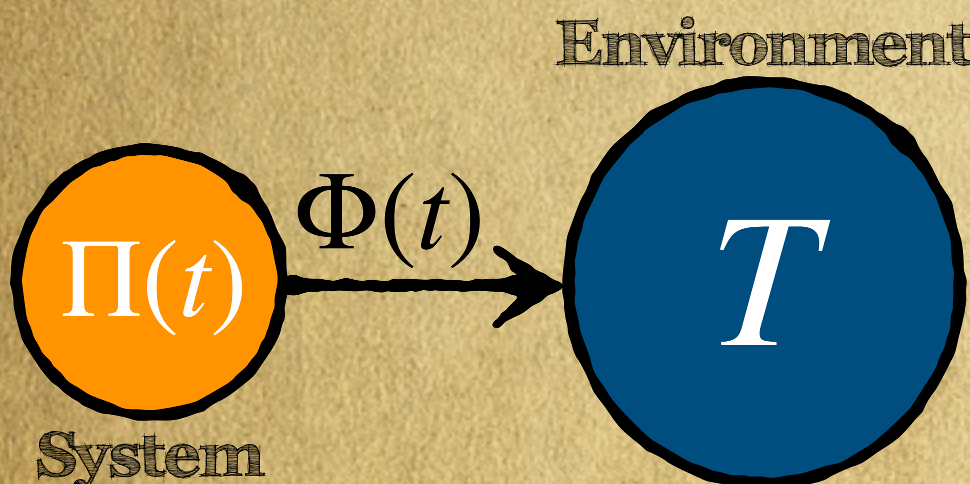
Fantastic framework for pinpointing the quantum-to-classical transition



Entropy production

Second Law: $\Delta S \geq \int \frac{\delta Q}{T} \quad \Rightarrow \quad \Sigma = \Delta S - \int \frac{\delta Q}{T}$

Clausius: "Uncompensated transformation" — Entropy production



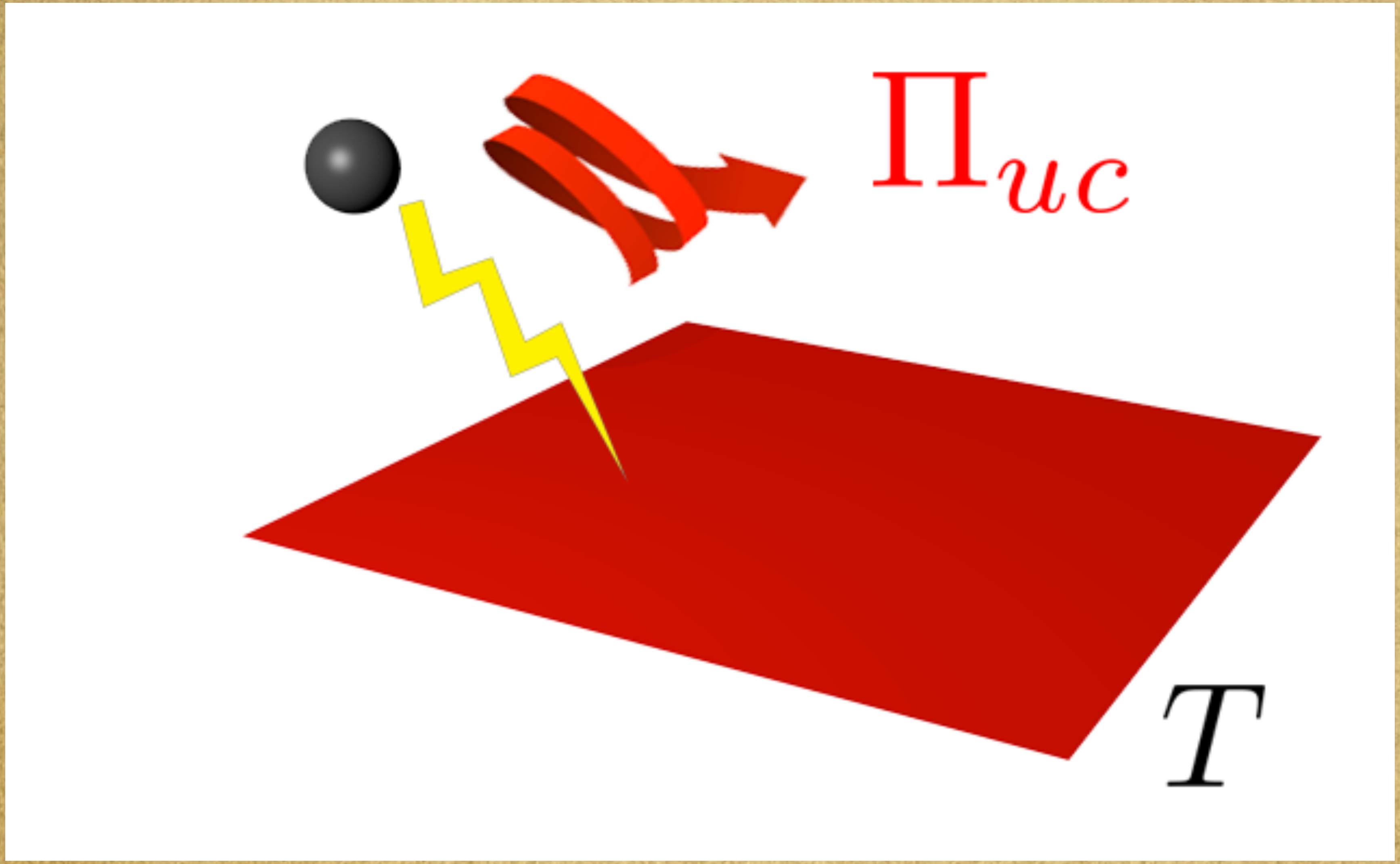
$$\frac{dS}{dt} = \Pi(t) + \Phi(t)$$

Entropy production rate — Entropy flux rate

Which is the role of quantum fluctuations on entropy production?

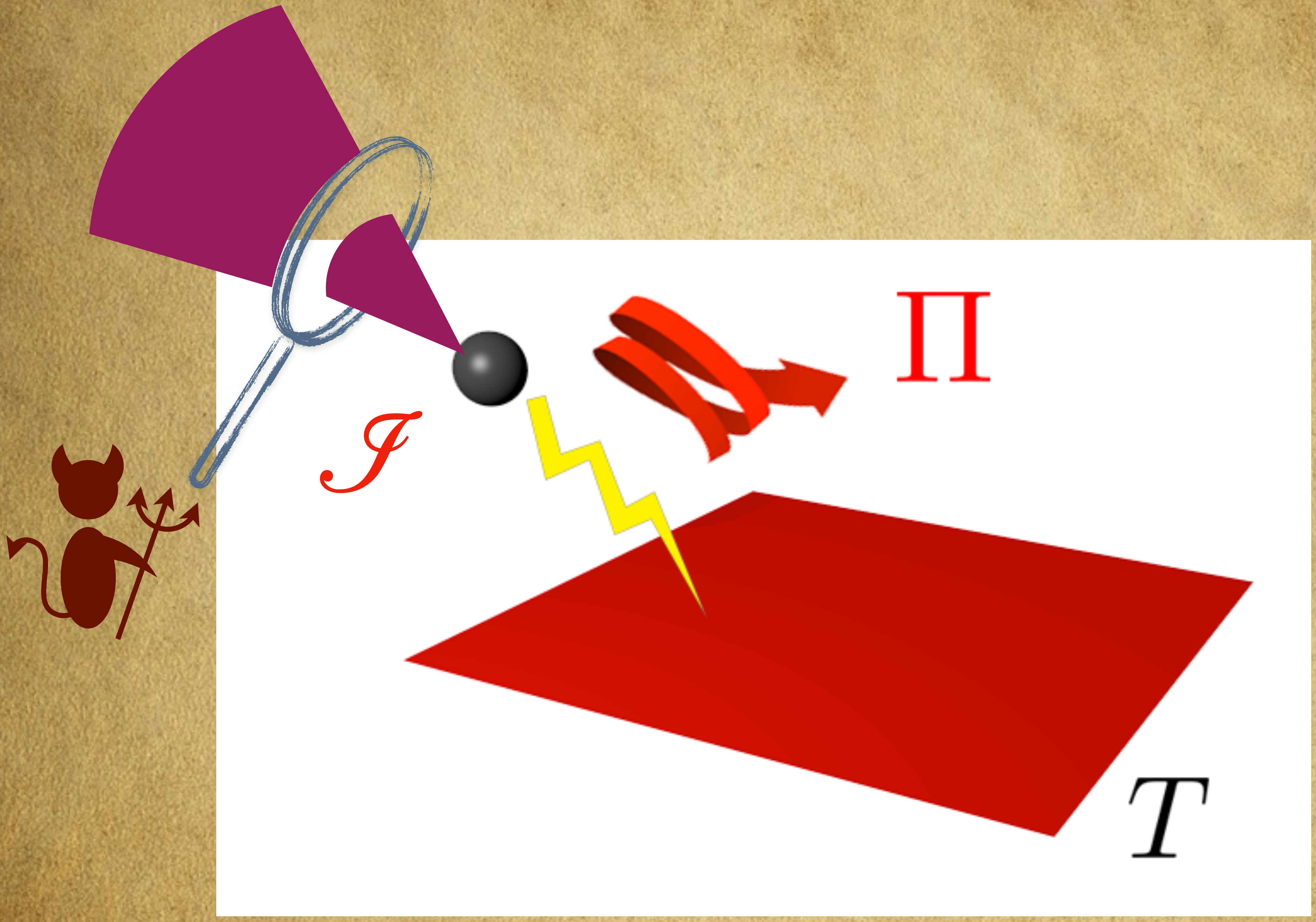
What happens if you plug in the effects of measuring?

Don't look yet!!!



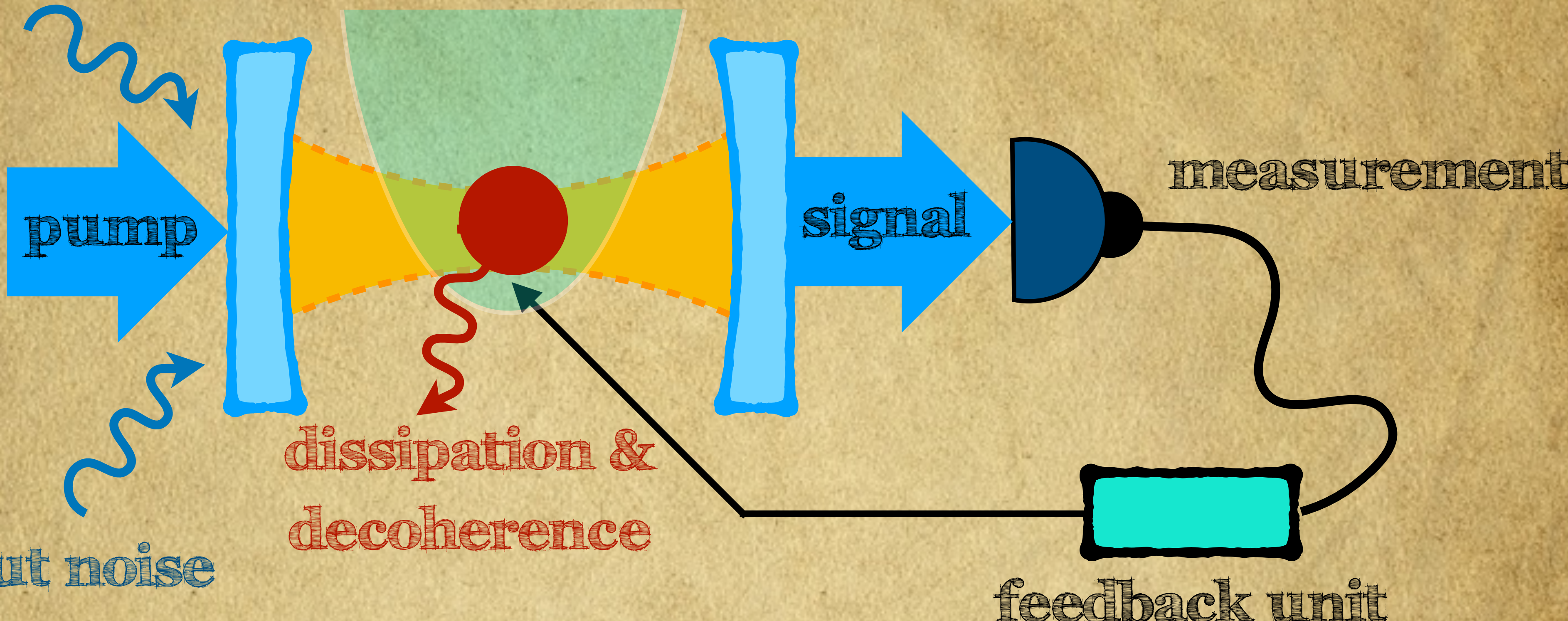
A Belenchia, L Mancino, G T Landi, and M Paternostro,
Nature Quantum Information 6, 97 (2019)

..now open your eyes..



A Belenchia, L Mancino, G T Landi, and M Paternostro,
Nature Quantum Information 6, 97 (2019)

Let's fix the ideas



Now restrict the framework to quadratic evolution and Gaussian states & measurements

A Belenchia, M Paternostro, and G T Landi, arXiv:2105.12518 (2021)

A Belenchia, et al., Nature Quantum Information 6, 97 (2019)

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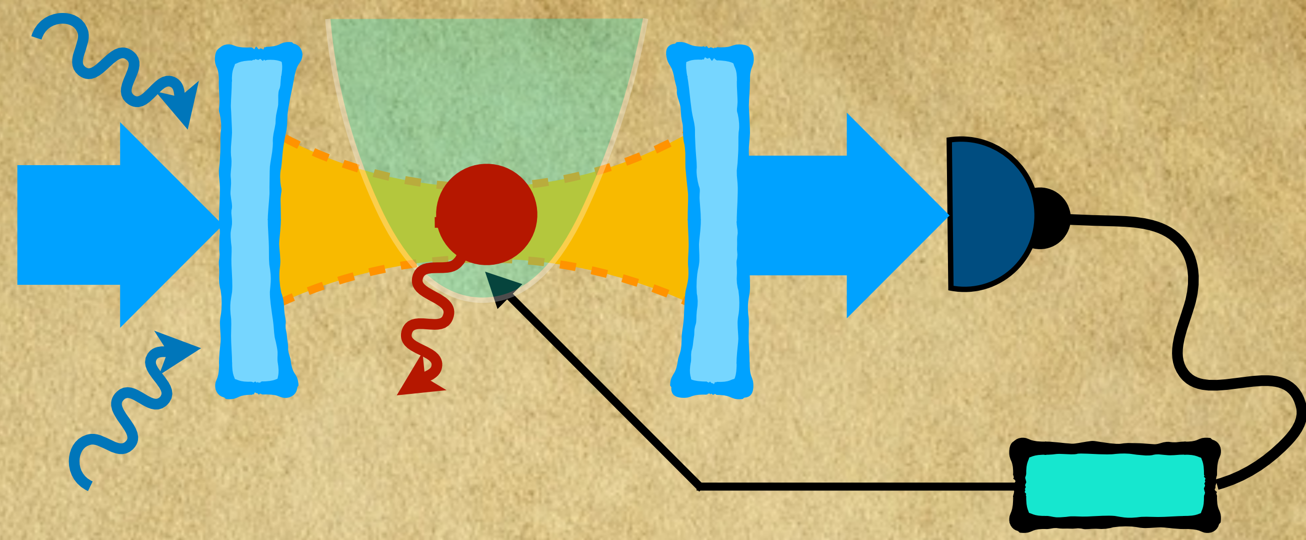
General formalism



Stochastic master equation

$$d\rho = -i[\hat{H},\rho]dt + \sum_k \mathscr{D}[\hat{c}_k](\rho)dt + \sum_k \sqrt{\eta_k}\mathscr{H}[\hat{c}_k](\rho)dw_k$$

Un-monitored dynamics

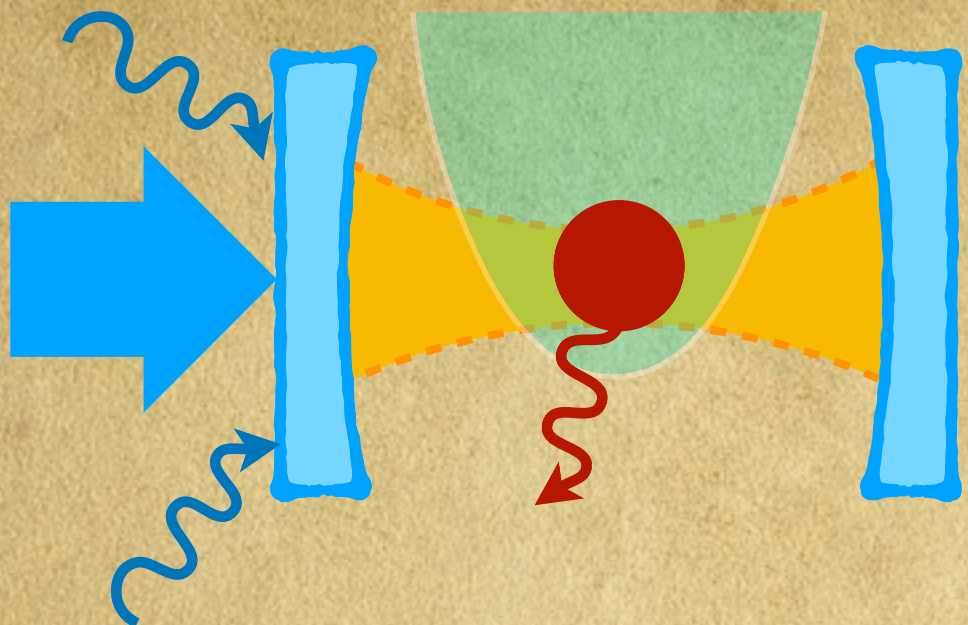
Stochastic terms

$$\mathscr{H}[\hat{c}]\rho = \hat{c}\rho + \rho\hat{c}^\dagger - \langle \hat{c} + \hat{c}^\dagger \rangle \rho$$

A Belenchia, M Paternostro, and G T Landi, arXiv:2105.12518 (2021)

M. G. Genoni, L. Lami, and A. Serafini, Contemp. Phys. 57, 331 (2016)

Un-Conditioned Gaussian dynamics



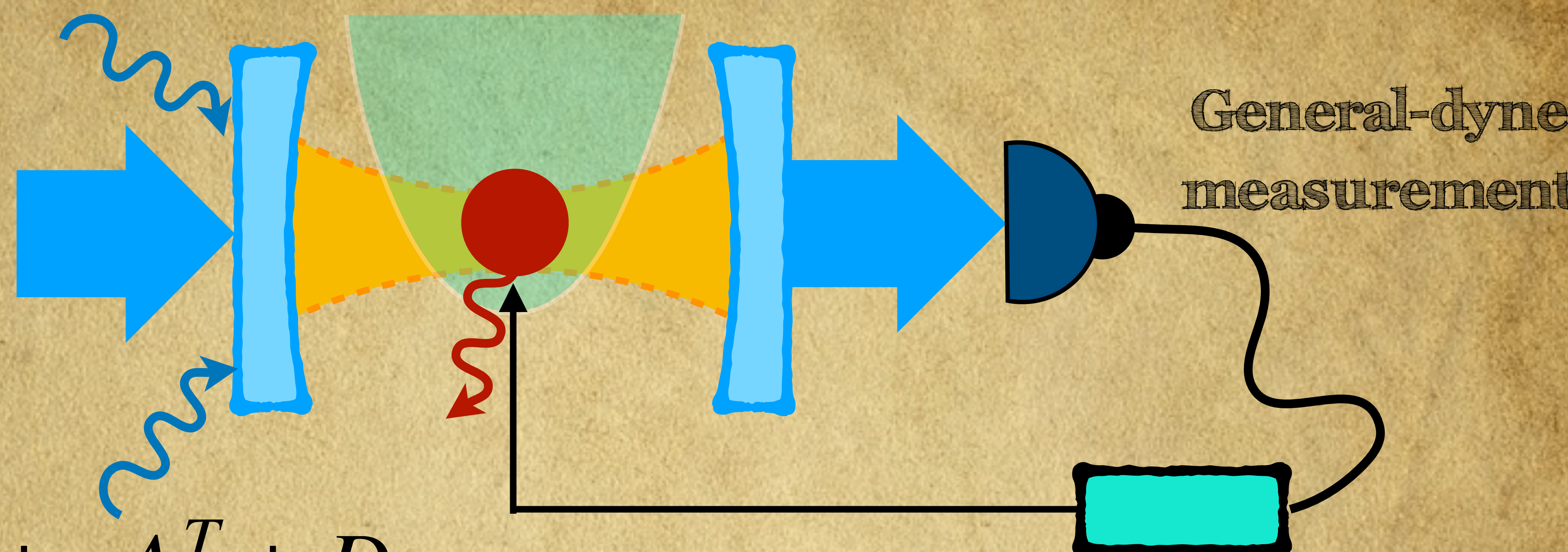
$$\dot{\sigma} = A\sigma + \sigma A^T + D$$

$$d\mathbf{x} = (A\mathbf{x} + \mathbf{b})\, dt$$

A Belenchia, M Paternostro, and G T Landi, arXiv:2105.12518 (2021)

A Belenchia, et al., Nature Quantum Information 6, 97 (2019)

Conditioned Gaussian dynamics



$$\dot{\sigma} = A\sigma + \sigma A^T + D$$

$$\Downarrow$$

$$\dot{\sigma} = \tilde{A}\sigma + \sigma\tilde{A}^T + \tilde{D} - \sigma B B^T \sigma = A\sigma + \sigma A^T + D - \chi(\sigma)$$

contains terms depending on the measurement

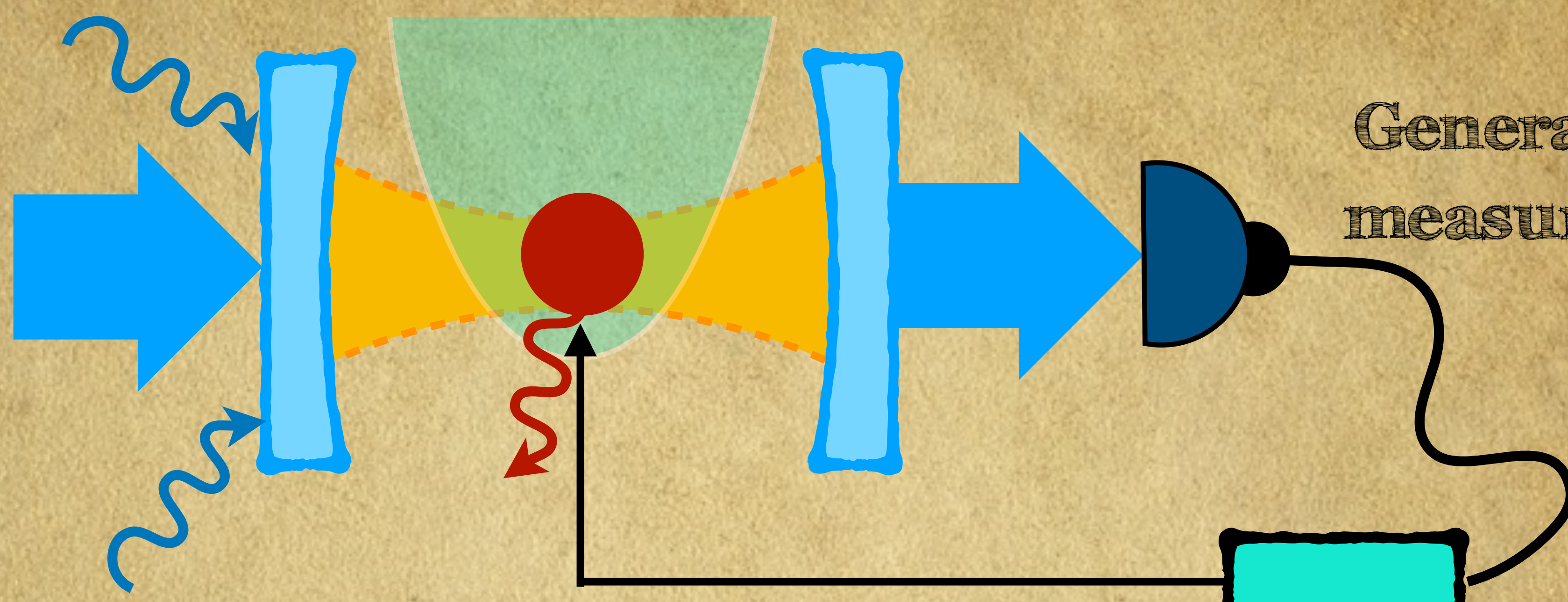
$$d\mathbf{x} = (A\mathbf{x} + \mathbf{b})\,dt$$

$$\Downarrow$$

$$d\bar{\mathbf{x}} = (A\bar{\mathbf{x}} + \mathbf{b})dt + \mu(\sigma)d\mathbf{w}$$

A Belenchia, M Paternostro, and G T Landi, arXiv:2105.12518 (2021)
A Belenchia, et al., Nature Quantum Information 6, 97 (2019)

Conditioned Gaussian dynamics



$$dS = d\Phi_{\bar{\mathbf{x}}} + d\Sigma_{\bar{\mathbf{x}}}$$

deterministic
(only depends on CM)

stochastic
(depend also on
1st moments)

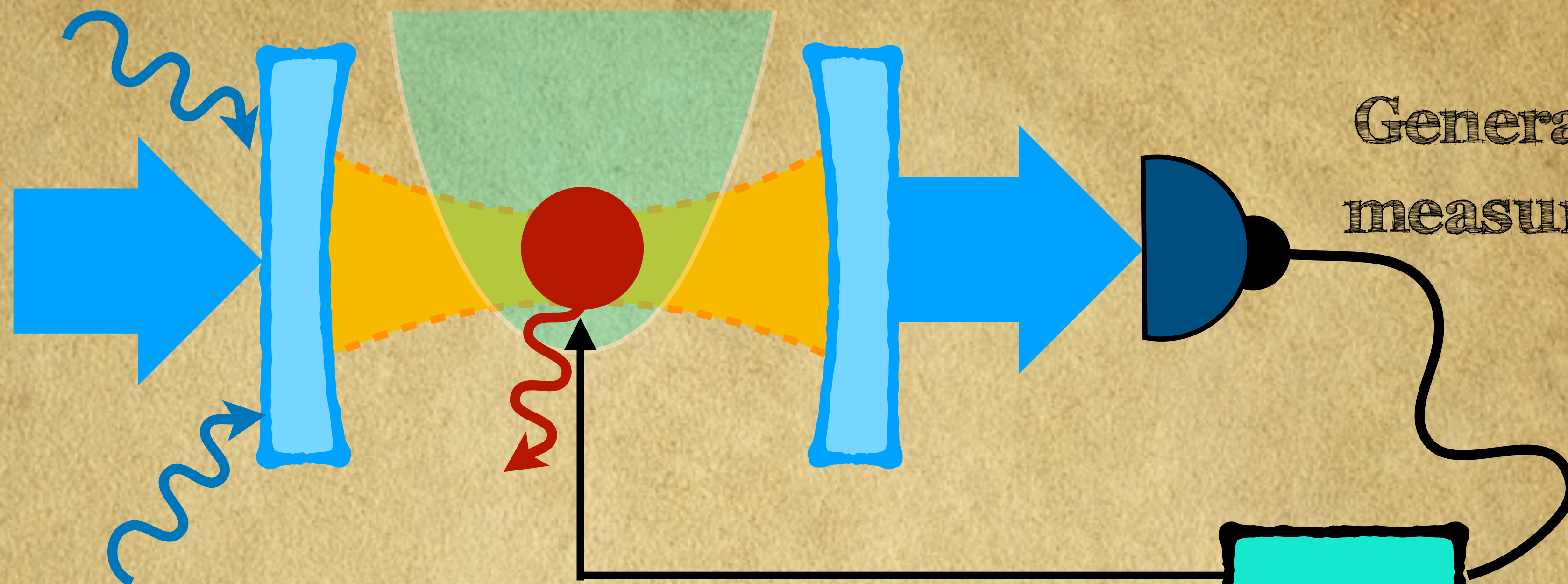
$$\phi = \mathbb{E}\left[d\Phi_{\bar{\mathbf{x}}}/dt\right]$$

$$\Pi = \mathbb{E}\left[d\Sigma_{\bar{\mathbf{x}}}/dt\right]$$

A Belenchia, M Paternostro, and G T Landi, arXiv:2105.12518 (2021)

A Belenchia, et al., Nature Quantum Information 6, 97 (2019)

Conditioned Gaussian dynamics



$$dS = dS_{\rm uc} + \dot{\mathscr{I}}\, dt$$

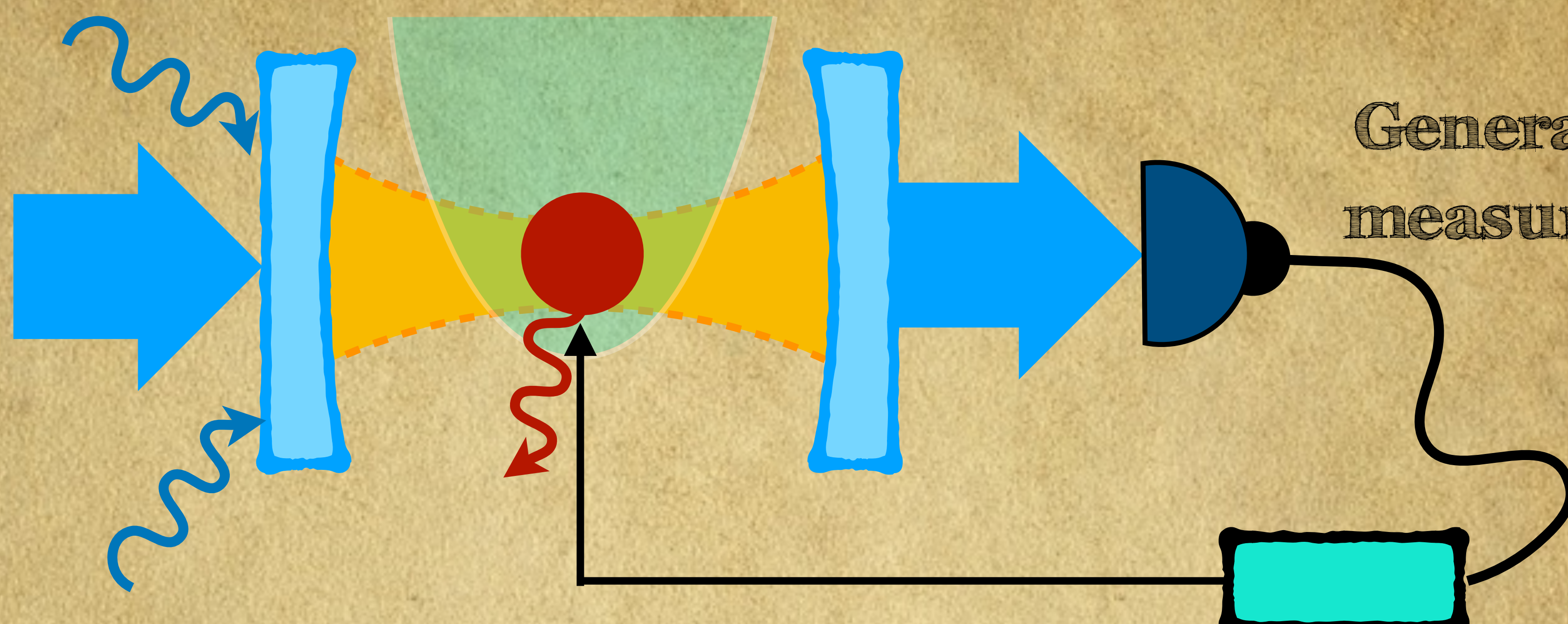
$$\phi = \mathbb{E}\left[d\Phi_{\bar{\mathbf{x}}}/dt\right]$$

$$\dot{\mathscr{I}} = \frac{1}{2}\mathrm{Tr}[\sigma^{-1}\mathrm{D} - \sigma^{-1}\chi(\sigma)] - \frac{1}{2}\mathrm{Tr}[\sigma_{\rm uc}^{-1}\mathrm{D}]$$

$$\Pi = \mathbb{E}\left[d\Sigma_{\bar{\mathbf{x}}}/dt\right]$$

A Belenchia, M Paternostro, and G T Landi, arXiv:2105.12518 (2021)
A Belenchia, et al., Nature Quantum Information 6, 97 (2019)

Conditioned Gaussian dynamics



$$dS = dS_{\text{uc}} + \dot{\mathcal{J}}\,dt$$

$$\Pi_c(t) = \Pi_{uc}(t) + \dot{\mathcal{J}}$$

$$\phi = \mathbb{E}\left[d\Phi_{\bar{\mathbf{x}}}/dt\right]$$

$$\Pi = \mathbb{E}\left[d\Sigma_{\bar{\mathbf{x}}}/dt\right]$$

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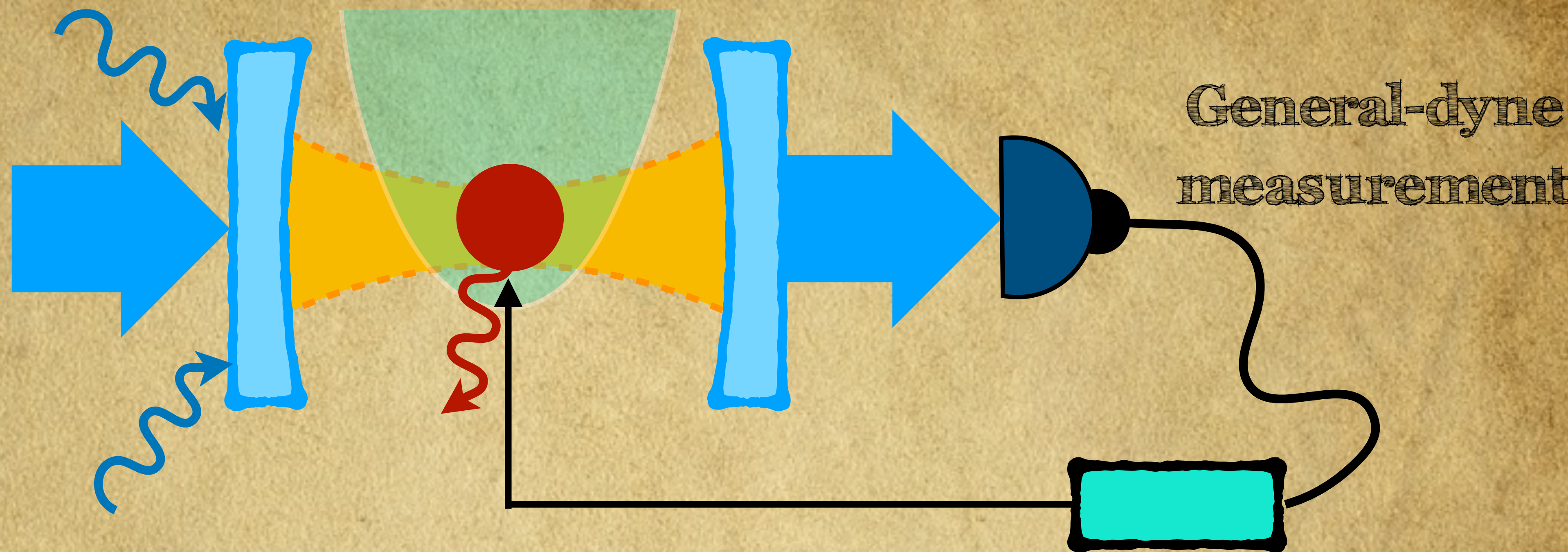


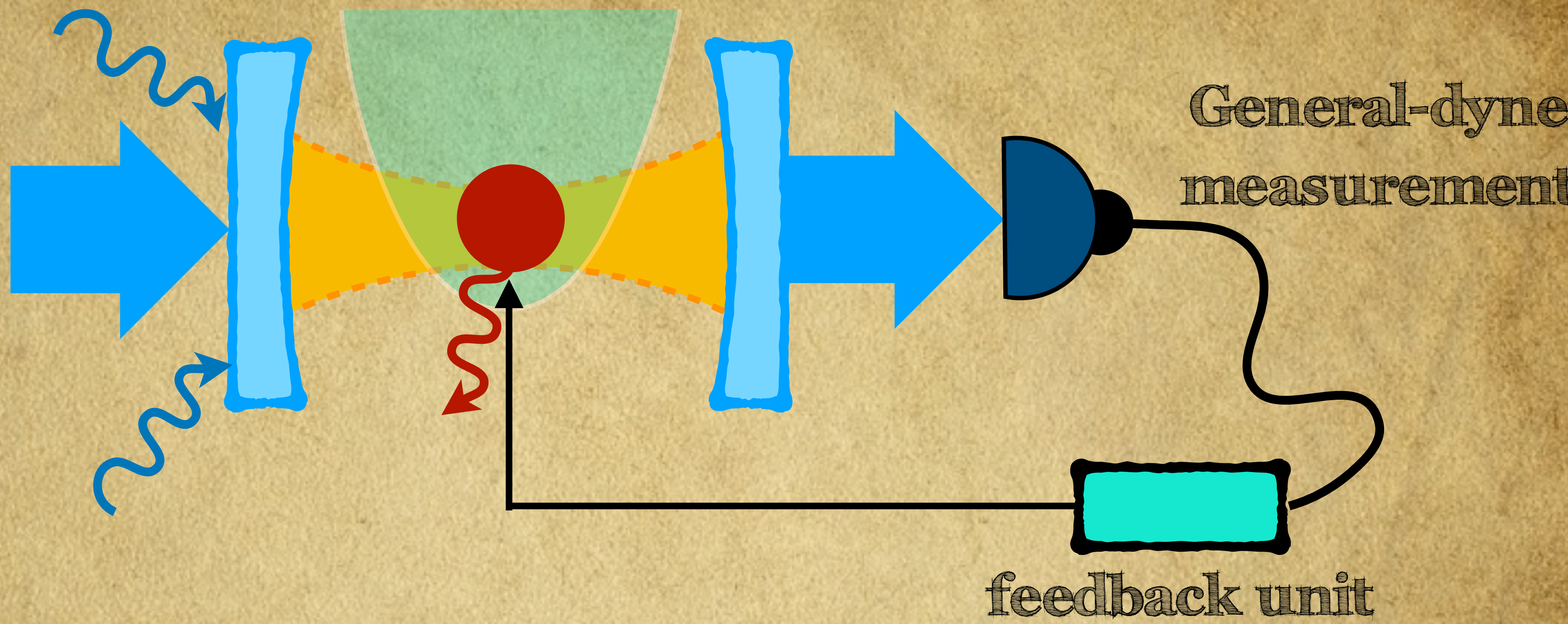
$\Pi_{uc}(t) \geq 0$ **second law for un-conditioned dynamics**

$$\Pi_c(t) = \Pi_{uc}(t) + \dot{\mathcal{J}}$$

A Belenchia, M Paternostro, and G T Landi, arXiv:2105.12518 (2021)

A Belenchia, et al., Nature Quantum Information 6, 97 (2019)

Conditioned Gaussian dynamics



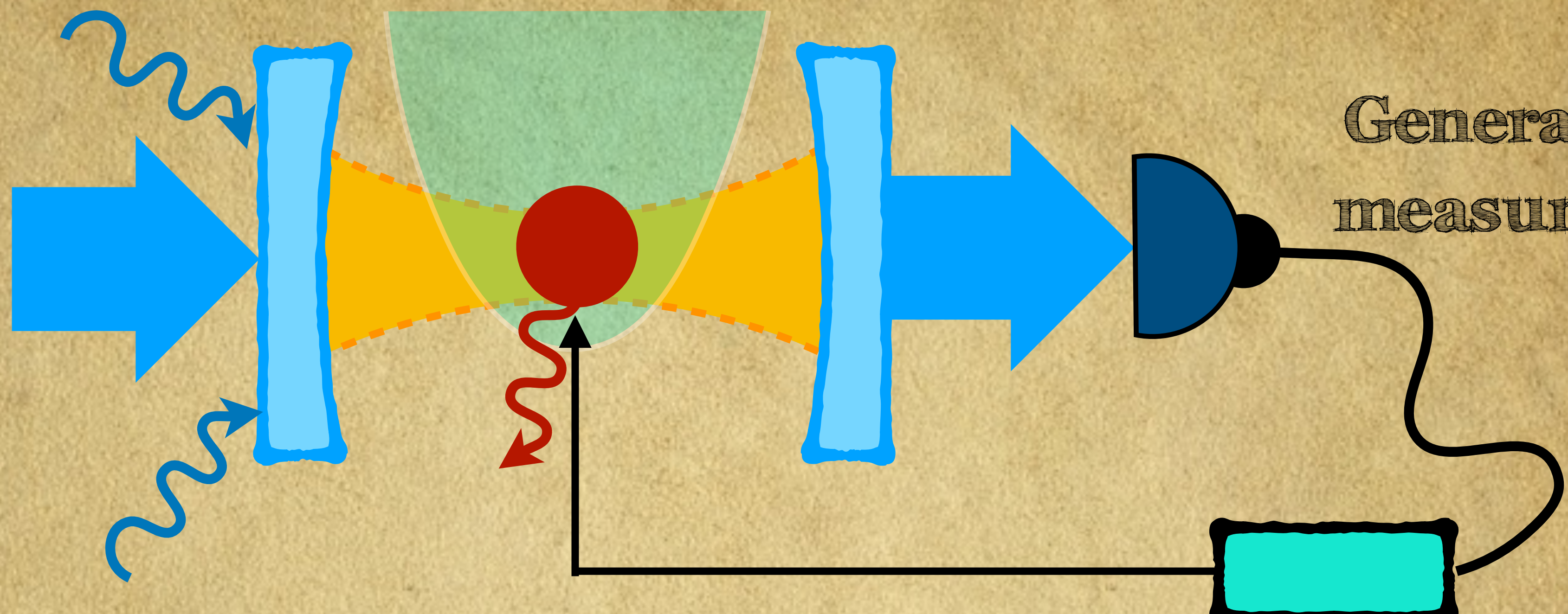
$\Pi_c(t) \geq \dot{\mathcal{J}}$ second law for conditioned dynamics

$$\Pi_c(t) = \Pi_{uc}(t) + \dot{\mathcal{J}}$$

A Belenchia, M Paternostro, and G T Landi, arXiv:2105.12518 (2021)

A Belenchia, et al., Nature Quantum Information 6, 97 (2019)

Conditioned Gaussian dynamics



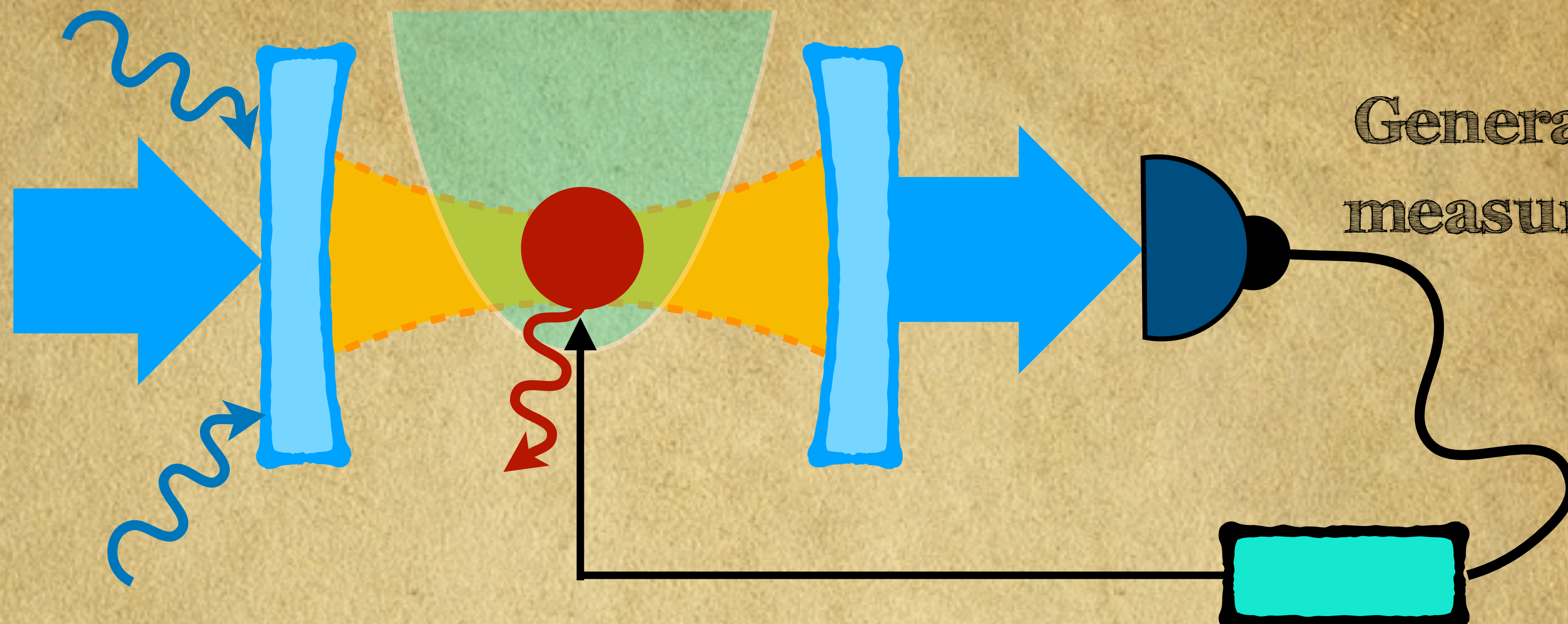
$\Sigma_c \geq \mathcal{I}$ integral second law for conditioned dynamics

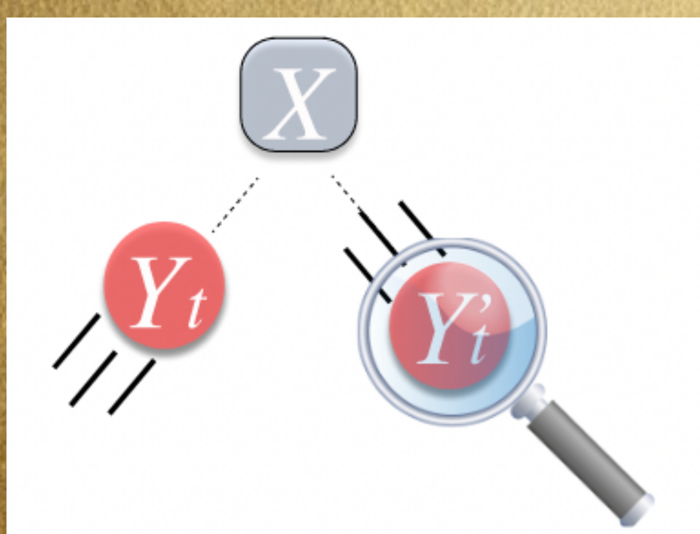
$$\Pi_c(t) = \Pi_{uc}(t) + \dot{\mathcal{I}}$$

Observable (SEE PART 3)

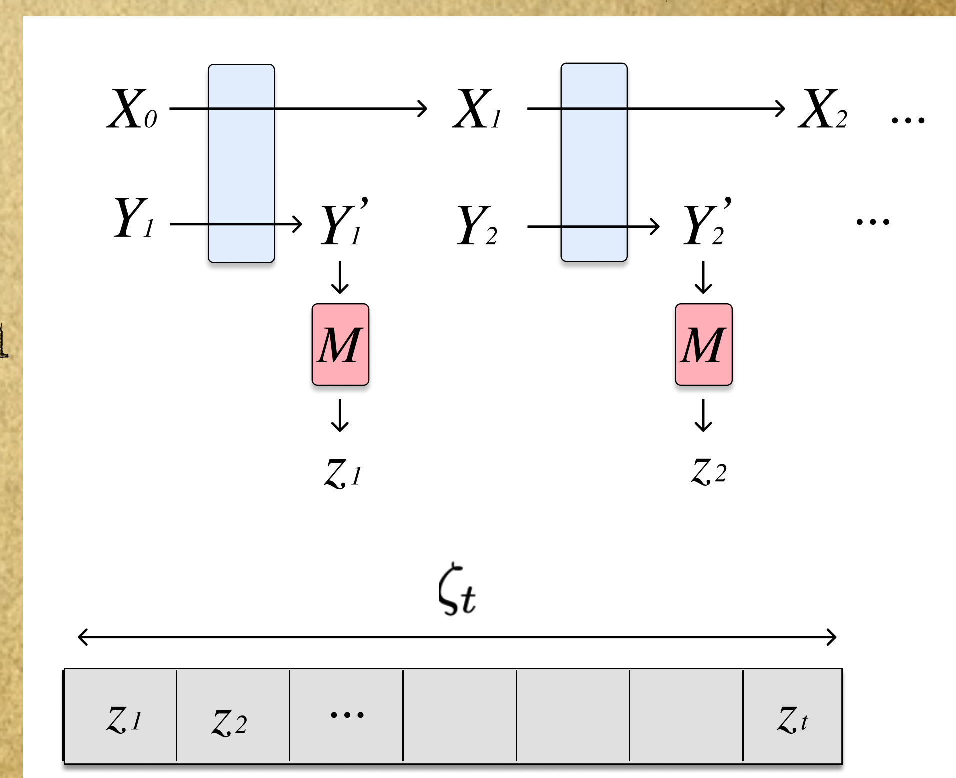
A Belenchia, M Paternostro, and G T Landi, arXiv:2105.12518 (2021)
A Belenchia, et al., Nature Quantum Information 6, 97 (2019)

Generalising it





Interpretation through collisional model



G T Landi, M Paternostro, and A Belenchia, arXiv:2103.06247 (2021)

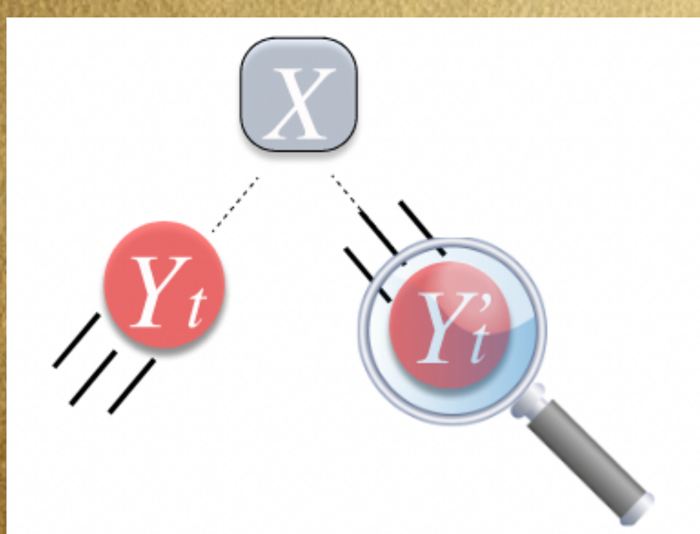
Can we generalise?

$$P(\zeta_t) = \mathrm{tr}_{XY_1\ldots Y_t}\left\{M_{z_t}\ldots M_{z_1}\rho_{XY_1\ldots Y_t}M_{z_1}^\dagger\ldots M_{z_t}^\dagger\right\}$$

$$\rho_{XY_1\ldots Y_t} = \left(\Pi_{k=1}^t U_k\right)\left(\rho_{X_0}\bigotimes_{j=1}^{t}\rho_{Y_j}\right)\left(\Pi_{k=1}^t U_k\right)^\dagger$$

$$\rho_{X_t|\zeta_t} = \frac{1}{P(\zeta_t)}\mathrm{tr}_{Y_1\ldots Y_t}\left\{\left(\Pi_{k=1}^t M_{z_k}\right)\rho_{XY_1\ldots Y_t}\left(\Pi_{k=1}^t M_{z_k}\right)^\dagger\right\}$$

Conditional state



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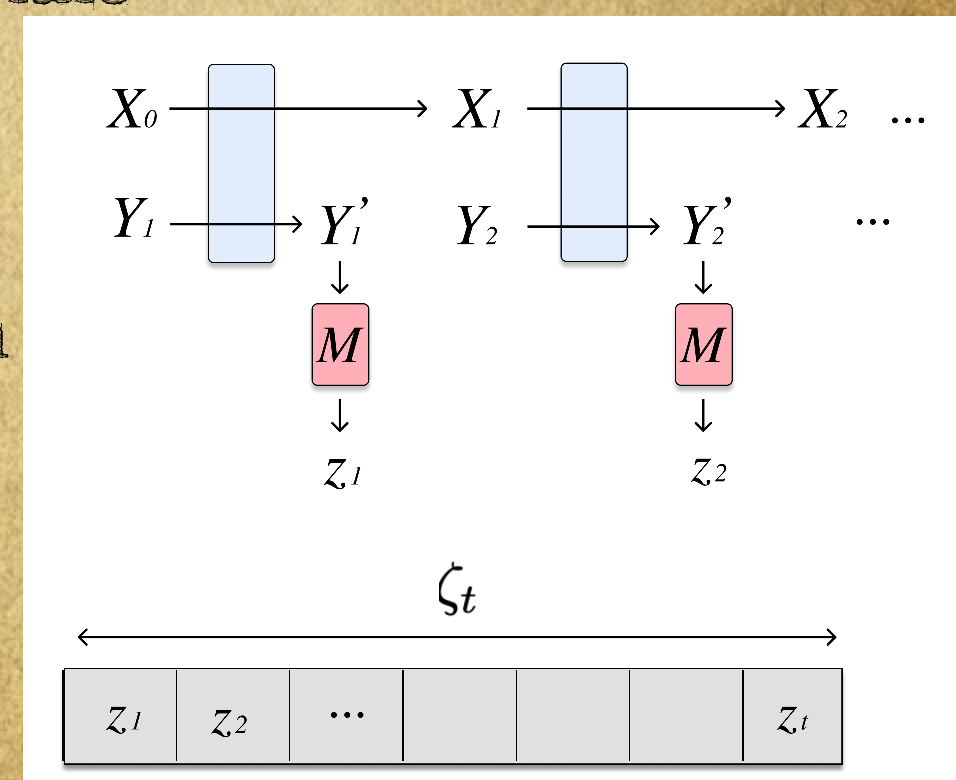


G T Landi, M Paternostro, and A Belenchia, arXiv:2103.06247 (2021)

Can we generalise?

Information rate

$$\Delta I_t := I(X_t : \zeta_t) - I(X_{t-1} : \zeta_{t-1})$$

can take any sign

Holevo information: info on X contained in ζ_t

$$I(X_t : \zeta_t) := S(X_t) - S(X_t|\zeta_t)$$

strictly positive

$$\Delta I_t = G_t - L_t$$

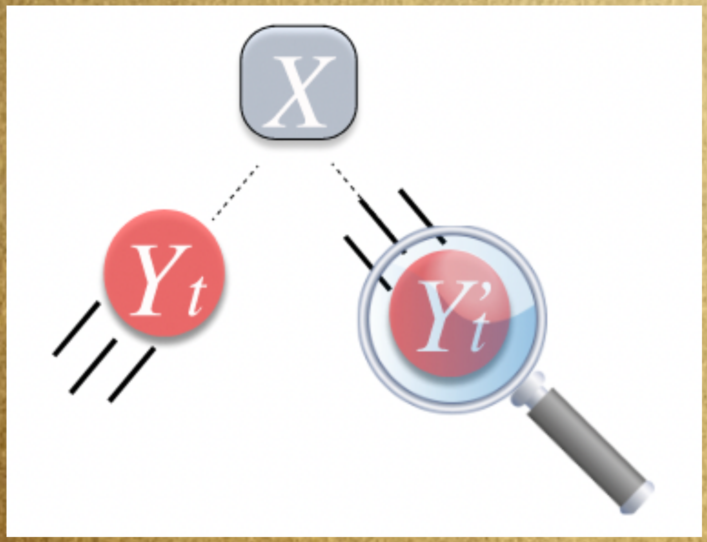
differential information gain

$$I(X_t : \zeta_t) - I(X_t : \zeta_{t-1}) = S(X_t|\zeta_{t-1}) - S(X_t|\zeta_t)$$

information loss

$$L_t := I(X_{t-1} : \zeta_{t-1}) - I(X_t : \zeta_{t-1})$$

non-negative



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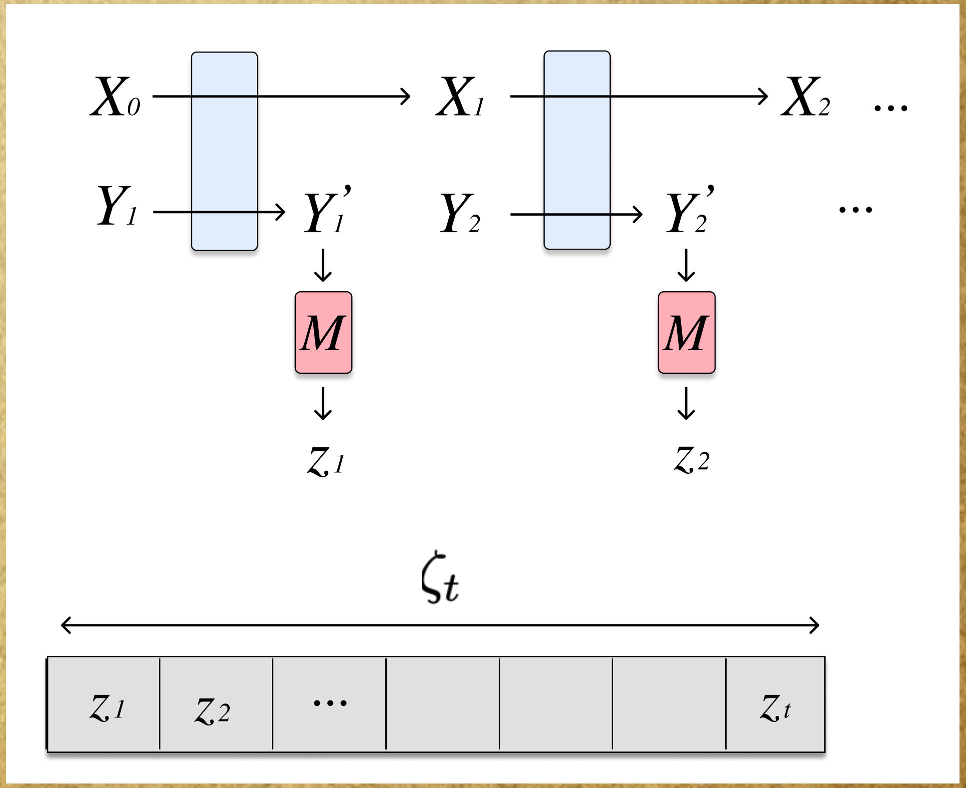

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can take any sign

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$$I(X_t : \zeta_t) := S(X_t) - S(X_t|\zeta_t)$$

strictly positive

$$\Delta I_t = G_t - L_t \xrightarrow{\text{steady state}} \Delta I_\infty = 0 \quad \text{with} \quad G_\infty = L_\infty \neq 0$$

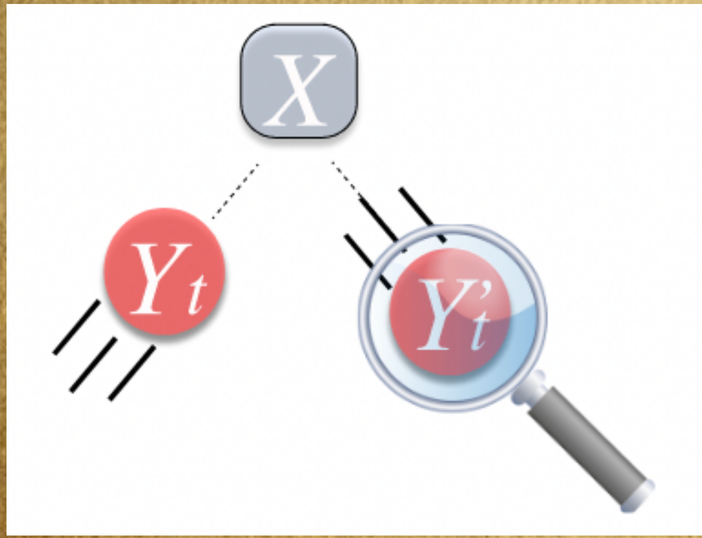
Informational steady state:

balance between gain & loss

measurements sustain the state

$$\Delta\Sigma_t^u = S(X_t) - S(X_{t-1}) + \Delta\Phi_t^u$$

$$\Delta\Sigma_t^c = S(X_t|\zeta_t) - S(X_{t-1}|\zeta_{t-1}) + \Delta\Phi_t^c$$



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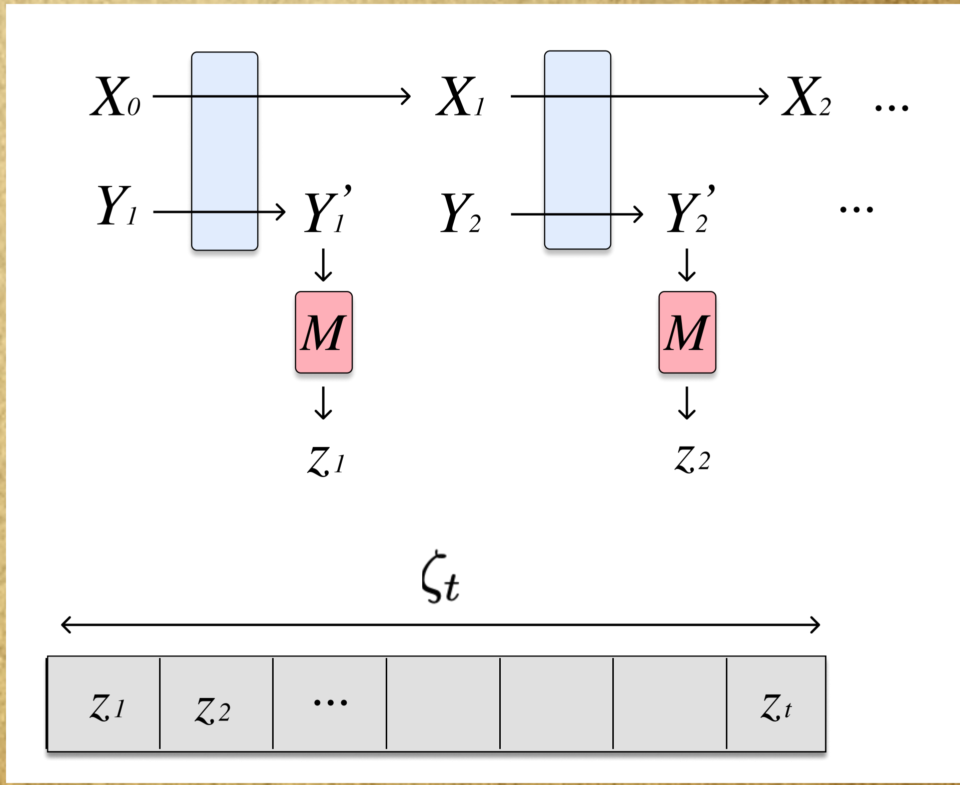

G T Landi, M Paternostro, and A Belenchia, arXiv:2103.06247 (2021)

Can we generalise?

Conditioning on the outcomes is subjective (I decide to read outcomes or not…)

No influence on flux of entropy into the ancillae

$$\Delta\Phi_t^c = \Delta\Phi_t^u$$

$$\Delta\Sigma_t^c = \Delta\Sigma_t^u + \Delta I_t$$

$$\Delta\Sigma_t^u = S(X_t) - S(X_{t-1}) + \Delta\Phi_t^u$$

$$\Delta\Sigma_t^c = S(X_t|\zeta_t) - S(X_{t-1}|\zeta_{t-1}) + \Delta\Phi_t^c$$

Informational steady state:

balance between gain & loss

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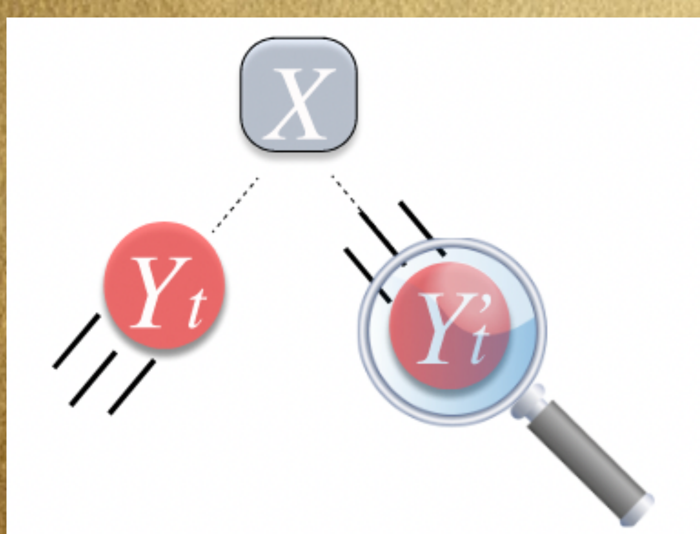

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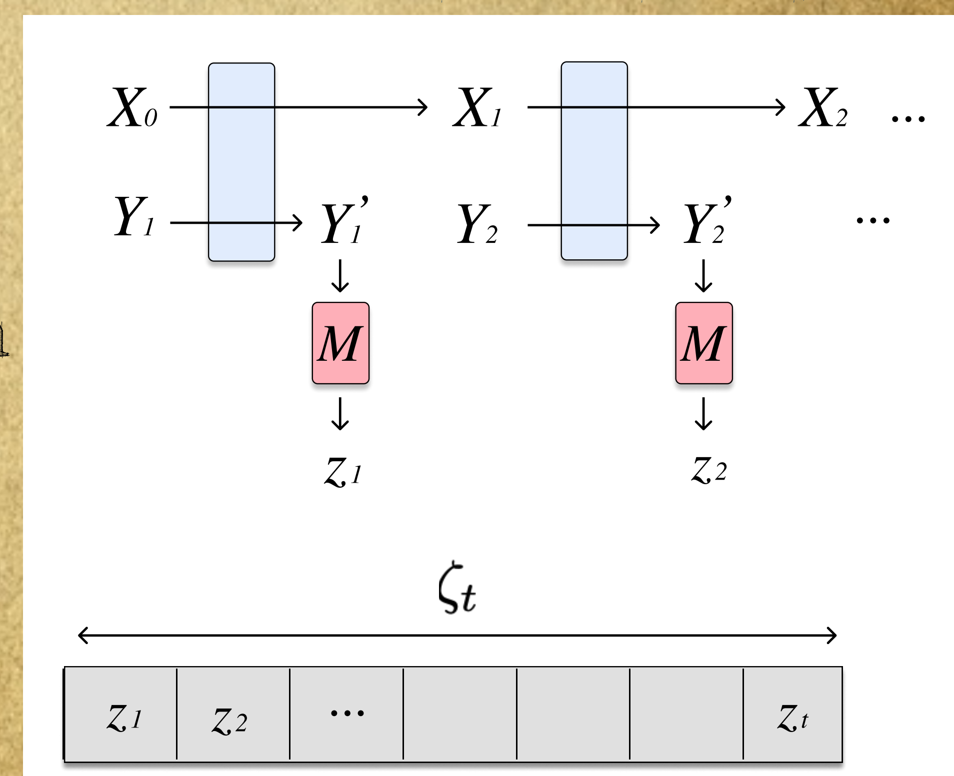
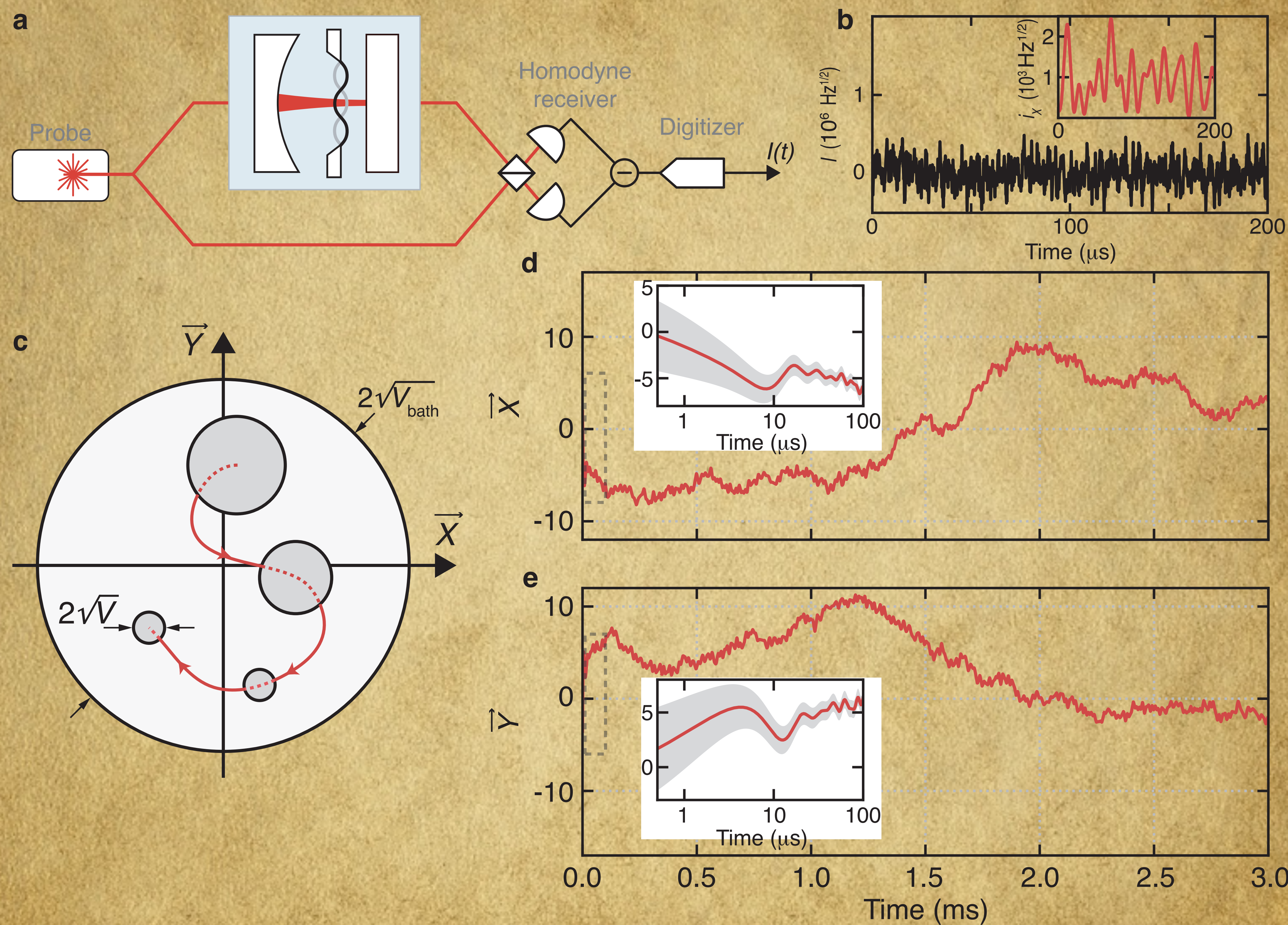


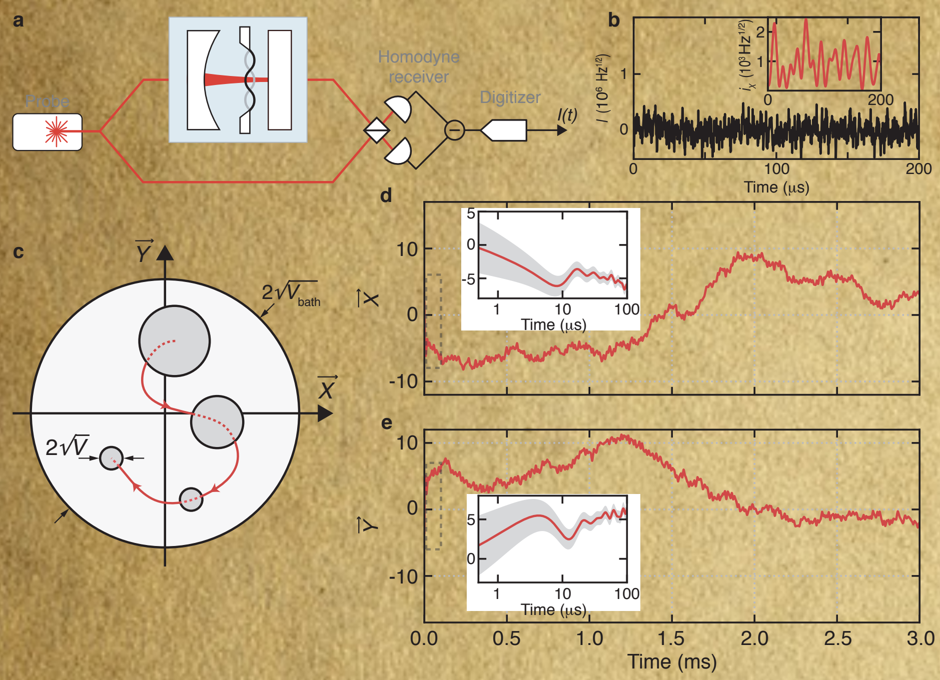
G T Landi, M Paternostro, and A Belenchia, arXiv:2103.06247 (2021)

Observing trajectories of mechanical systems



M. Rossi, D. Mason, J. Chen, and A. Schliesser, Phys. Rev. Lett. 123, 163601 (2019)

Observing trajectories of mechanical systems



$$d\mathbf{r}(t) = -\frac{\Gamma_m}{2}\mathbf{r}dt + \sqrt{4\eta_{\text{det}}\Gamma_{\text{qba}}V(t)d\mathbf{W}},$$

$$\dot{V}(t) = \Gamma_m(V_{\text{uc}} - V(t)) - 4\eta_{\text{det}}\Gamma_{qba}V(t)^2$$

dynamics of the experiment

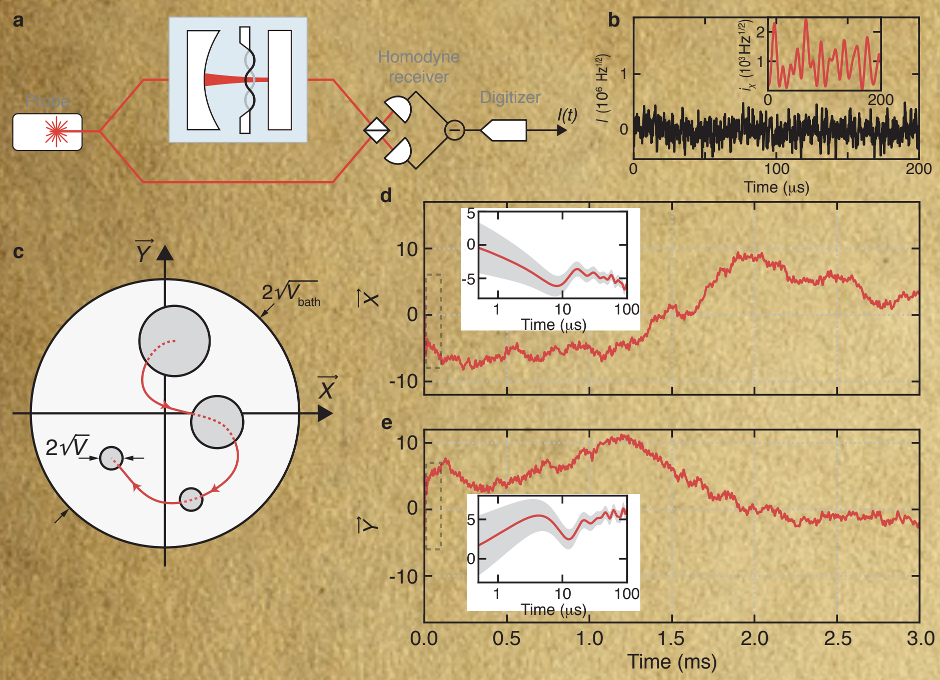
$$d\mathbf{r}(t) = A\mathbf{r}(t)dt + (V(t)C^T + \Gamma^T)d\mathbf{W},$$

$$\dot{V}(t) = AV(t) + V(t)A^T + D - \underbrace{(V(t)C^T + \Gamma^T)(CV(t) + \Gamma^T)}_{\chi(V(t))}$$

theoretical counterpart

M. Rossi, L. Mancino, G. T. Landi, M. Paternostro, A. Schliesser, and A. Belenchia, Phys. Rev. Lett. 125, 080601 (2020)

Observing trajectories of mechanical systems



Initial state: equilibrium state at environment temperature

↓

Steady state of the unconditional dynamics: NESS very close to equilibrium

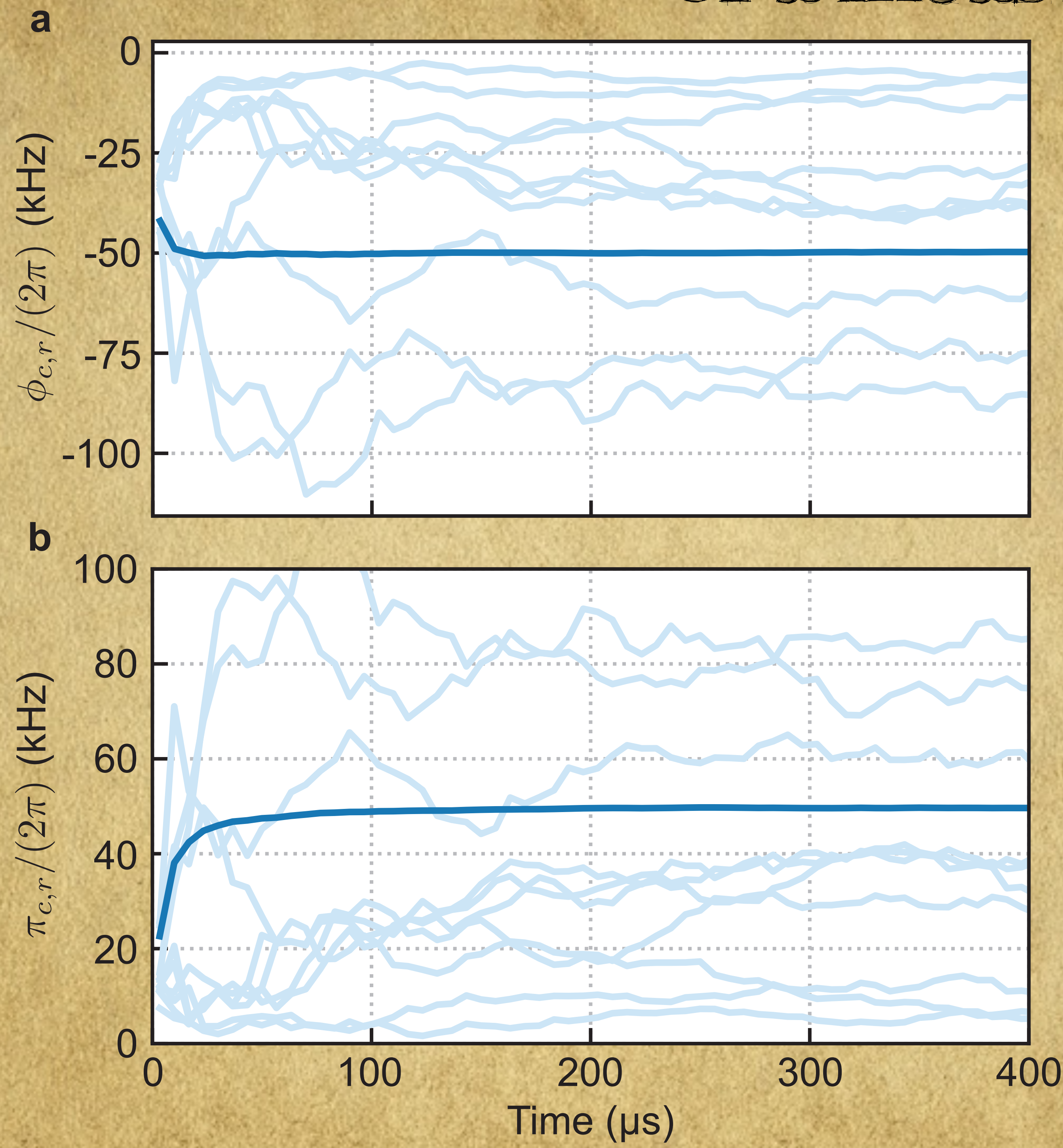
$$\Pi_{uc}(t) = \Gamma_m \left[V_{\rm uc}/(n_{\rm th}+1/2) - 1 \right] + 4\Gamma_{\rm qba} V_{\rm uc}$$

$$\Pi_{uc}(t) = \text{const}\,.$$ and $$\Pi_c(t) = \dot{\mathcal{J}} + \text{const}\,.$$

$$\dot{\mathcal{J}} = \Gamma_{\rm m}\left(V_{\rm uc}/V(t) - 1\right) - 4\eta_{\rm det}\Gamma_{\rm qba} V(t)$$

M. Rossi, L. Mancino, G. T. Landi, M. Paternostro, A. Schliesser, and A. Belenchia, Phys. Rev. Lett. 125, 080601 (2020)

Observing entropy production rates of a measured system



M. Rossi, L. Mancino, G. T. Landi, M. Paternostro, A. Schliesser, and A. Belenchia, Phys. Rev. Lett. 125, 080601 (2020)

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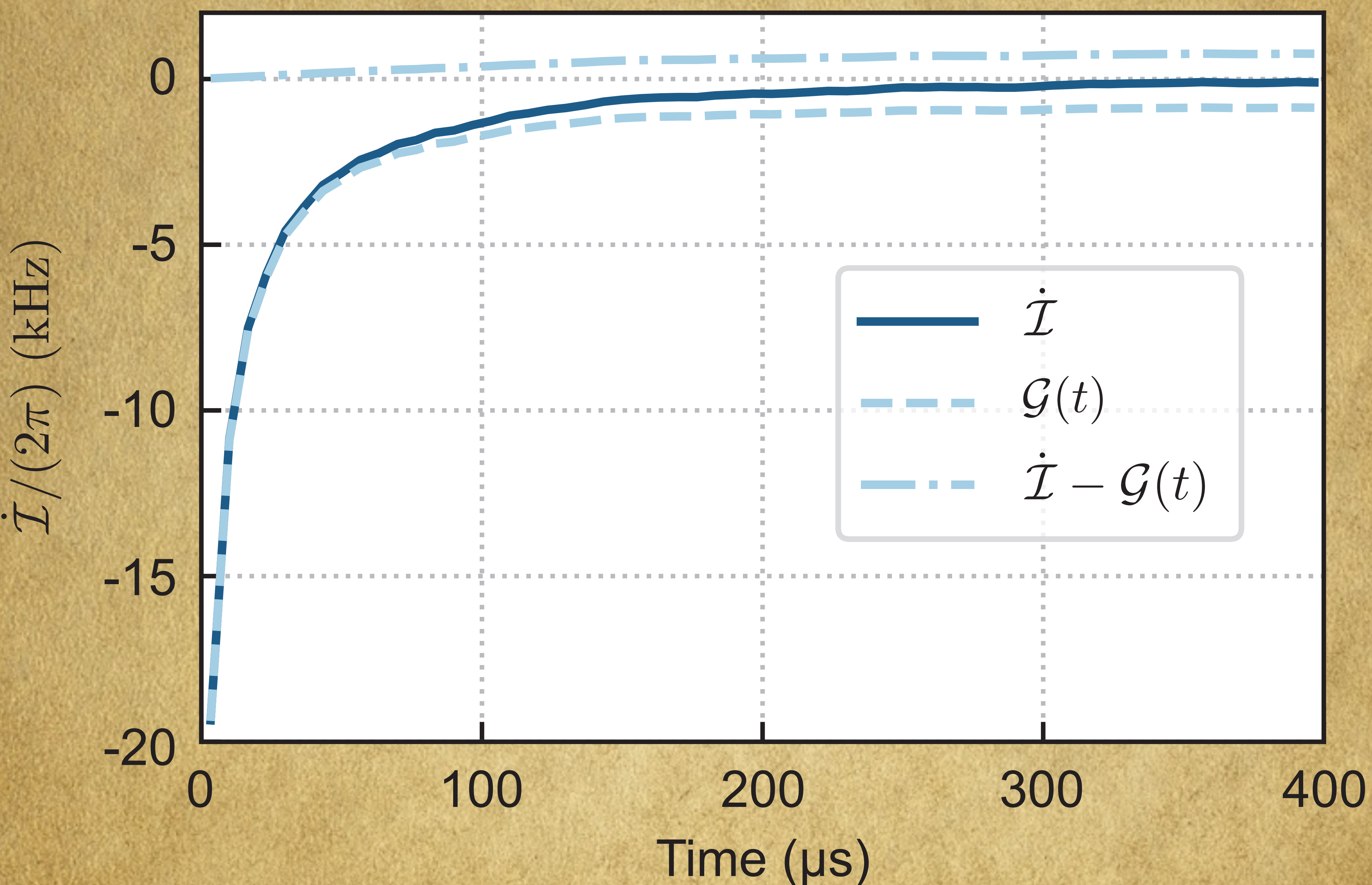

M. Rossi, L. Mancino, G. T. Landi, M. Paternostro, A. Schliesser, and A. Belenchia, Phys. Rev. Lett. 125, 080601 (2020)

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The Belfast crew

Bread on tables..

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