Bound states of artificial atoms in open and closed waveguides

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I first met Andrzej Kossakowski at ICTP in Trieste, Italy, at a conference. I was too young to know who he was and understand the reach and importance of his contribution to Physics.

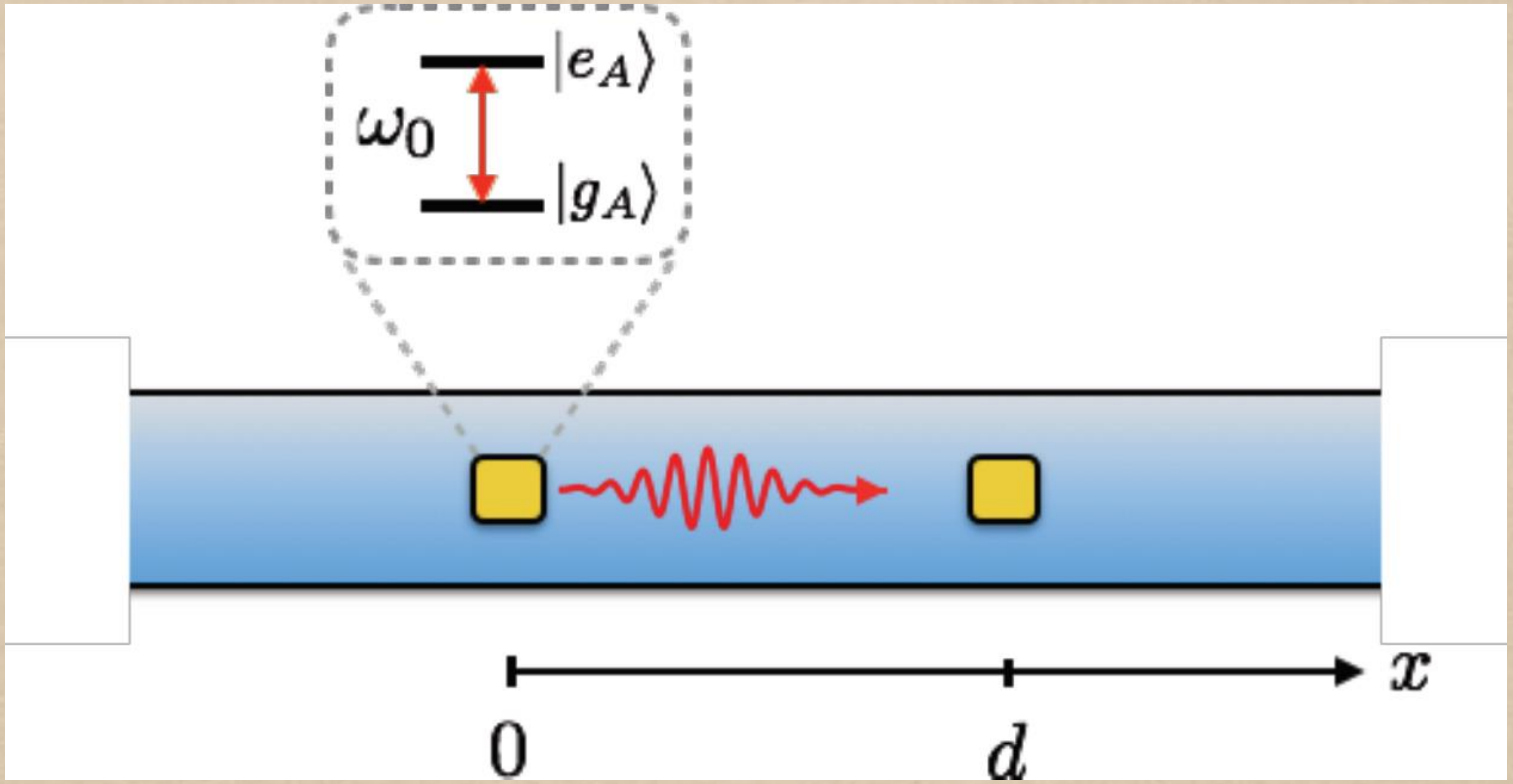
His hair was white, he had a jovial attitude that would contrast with his imposing figure. His laugh was loud, yet elegant. When he was calculating, he would isolate so much from the rest of the world that you felt you didn't had the right to disturb him. His favorite game was to solve (crack) differential equations. He loved examples.

During the years that followed, I had the fortune to meet Andrzej many times, and interact with him. I learned about complete positivity from him. He was able to read into people's mind. After explaining CP to me, he looked at me, paused for a few seconds, then added: no, there is no characterization of *positive* maps; why don't you work at it? I tried, but the problem was so difficult that it survived him.

Andrzej was a dominating figure in the realm of open quantum systems. His pupil (and my friend) Darek Chruscinski once told me: the GKLS equation [1, 2] stands next to the Schrödinger equation. I hesitated for a split second, then I got the point.

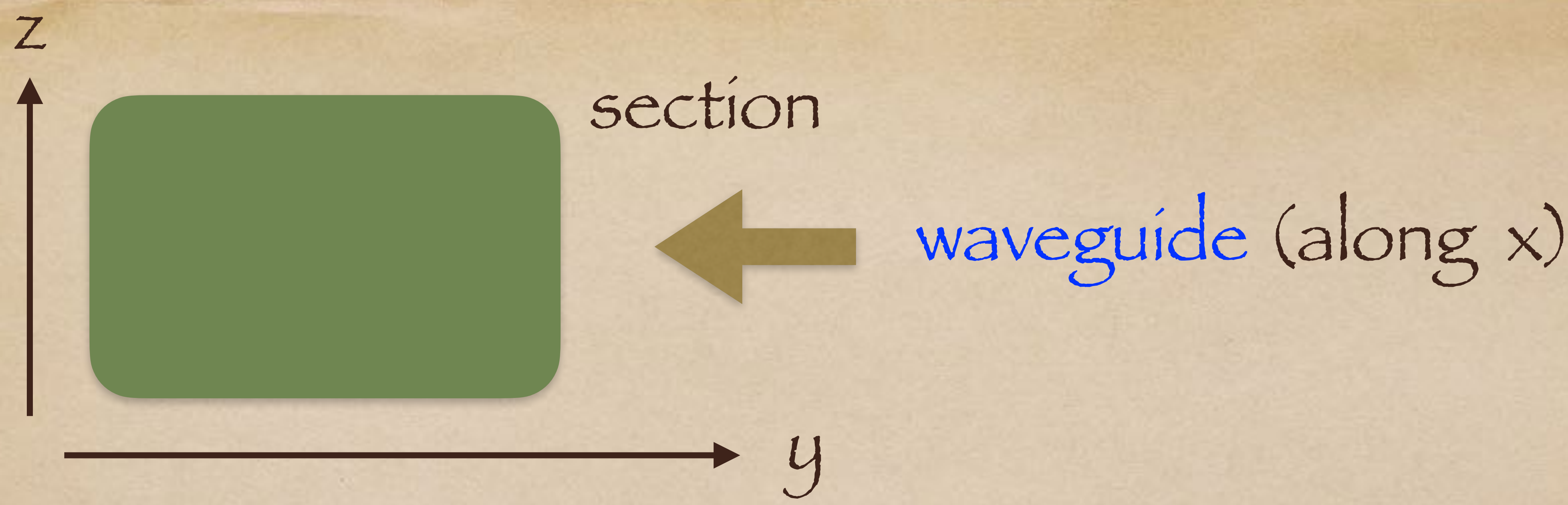
I like to remember Andrzej not only for his seminal contributions to physics, but also for his character and human side. I think that the best way to celebrate him is to resort to some personal recollections.

today's talk



1 or 2

two-level atoms in a waveguide, RWA



$$E_x|_S = 0 \qquad \text{and} \qquad \left.\frac{\partial B_x}{\partial n}\right|_S = 0$$

surface S
normal n

$$B_x = B_0 \cos\left(\frac{\pi y}{L_y}\right) e^{i(kx - \omega_{1,0}(k)t)},$$

TE$_{1,0}$ mode

$$B_y = -i\frac{kL_yB_0}{\pi} \sin\left(\frac{\pi y}{L_y}\right) e^{i(kx - \omega_{1,0}(k)t)},$$

$$E_z = i\frac{\omega_{1,0}(k)L_yB_0}{\pi} \sin\left(\frac{\pi y}{L_y}\right) e^{i(kx - \omega_{1,0}(k)t)}$$

$$A_z = \frac{L_y B_0}{\pi} \sin\left(\frac{\pi y}{L_y}\right) e^{i(kx - \omega_{1,0}(k)t)}$$

vector potential

$$\omega_{m,n}(k) = \sqrt{(vk)^2 + \omega_{m,n}(0)^2}$$

$$\omega_{m,n}(0) = v\left[\left(\frac{m\pi y}{L_y}\right)^2 + \left(\frac{n\pi z}{L_z}\right)^2\right]^{\frac{1}{2}}$$

atom + interaction

$$H_{\text{at}} = \frac{1}{2m_e}\left(\boldsymbol{p} - e\boldsymbol{A}^{(1,0)}(\boldsymbol{r})\right)^2 + V(\boldsymbol{r})$$

$$= H_{\text{at}}^0 - \frac{e}{m_e}\boldsymbol{p}\cdot\boldsymbol{A}^{(1,0)}(\boldsymbol{r}) + \frac{e^2}{2m_e}\left(\boldsymbol{A}^{(1,0)}(\boldsymbol{r})\right)^2$$

free Hamiltonian

$$H^{(1,0)} = \mathcal{E}_{el}^{(1,0)} + \mathcal{E}_{mag}^{(1,0)}$$

$$= \hbar v \int dk \sqrt{k^2 + \left(\frac{vM}{\hbar}\right)^2} a^\dagger(k) a(k).$$

interaction

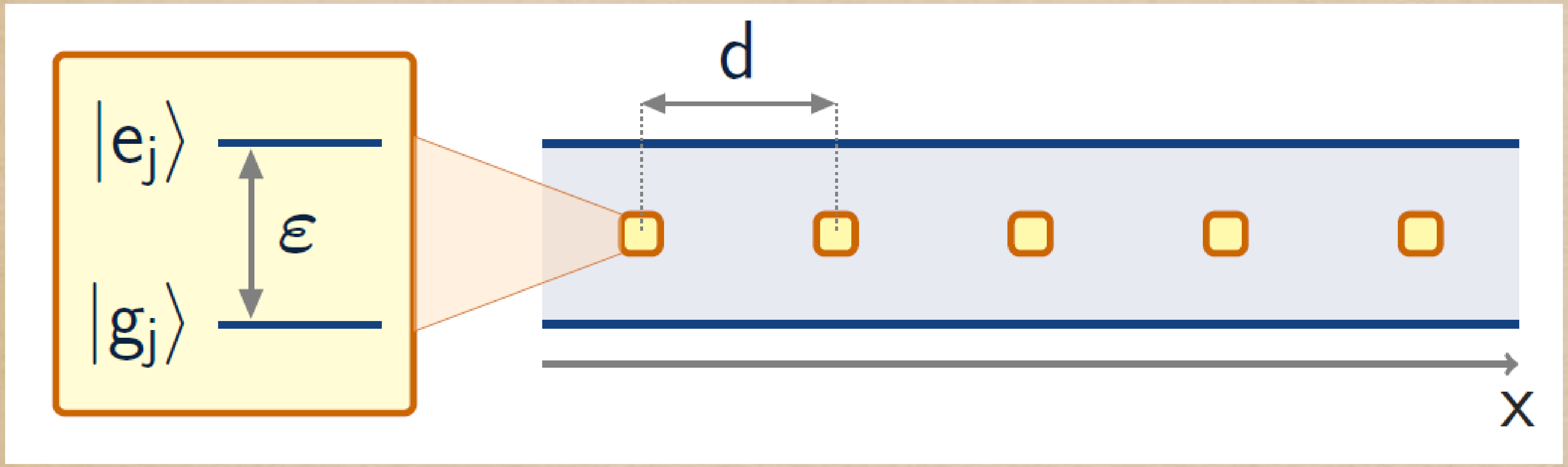
$$H_{\text{int}}^{(dip,RW)} = \omega_0 D_{eg} \left(\frac{\hbar}{2\pi\epsilon v L_y L_z}\right)^{\frac{1}{2}} \int \frac{dk}{(k^2 + (vM/\hbar)^2)^{1/4}}$$

$$\times \left[b(k)|e\rangle\langle g|e^{ikx_0} + b^\dagger(k)|g\rangle\langle e|e^{-ikx_0} \right].$$

atom A in $x_0 = 0$ and atom B in $x_0 = d$

needless to say,

a(ny) Hamiltonian is an approximation



there will ALWAYS be dissipative effects:

leakage out of the cavity, decoherence, dephasing

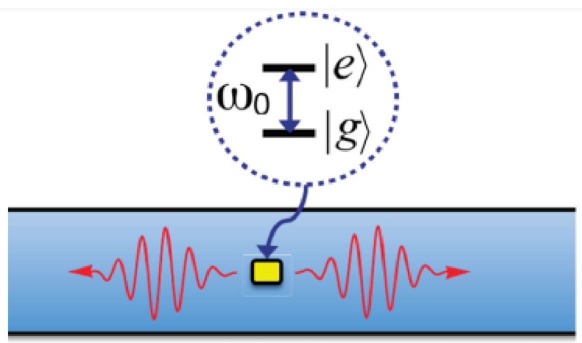
GKLS

but we consider Hamiltonian approach:

physical meaning

one atom in a (1D) waveguide

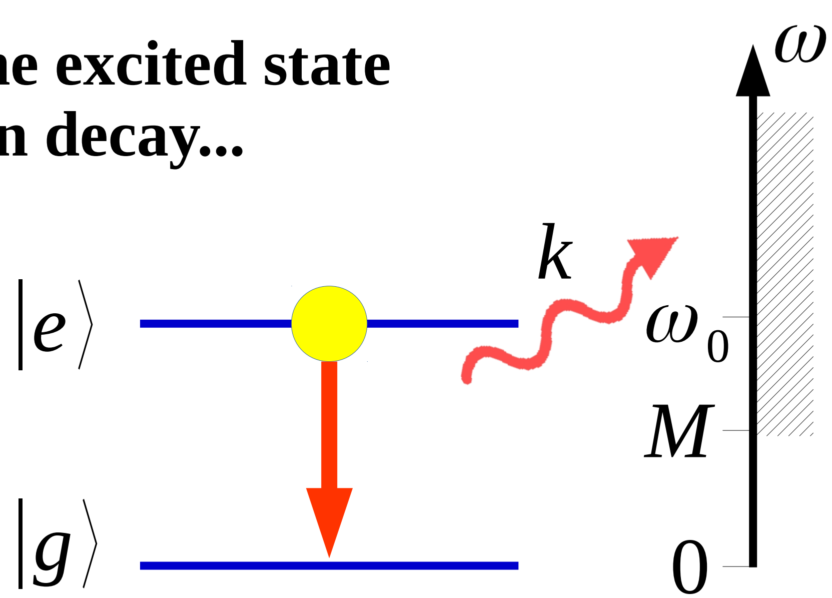
Coupling with the lowest-energy mode in a linear waveguide



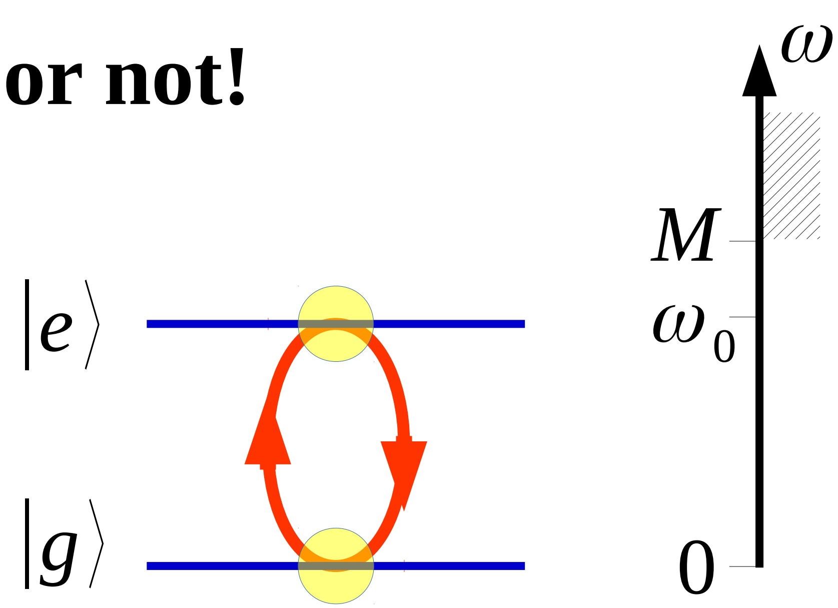
Dispersion relation (massive)

$$\omega(k) = \sqrt{k^2 + M^2}$$

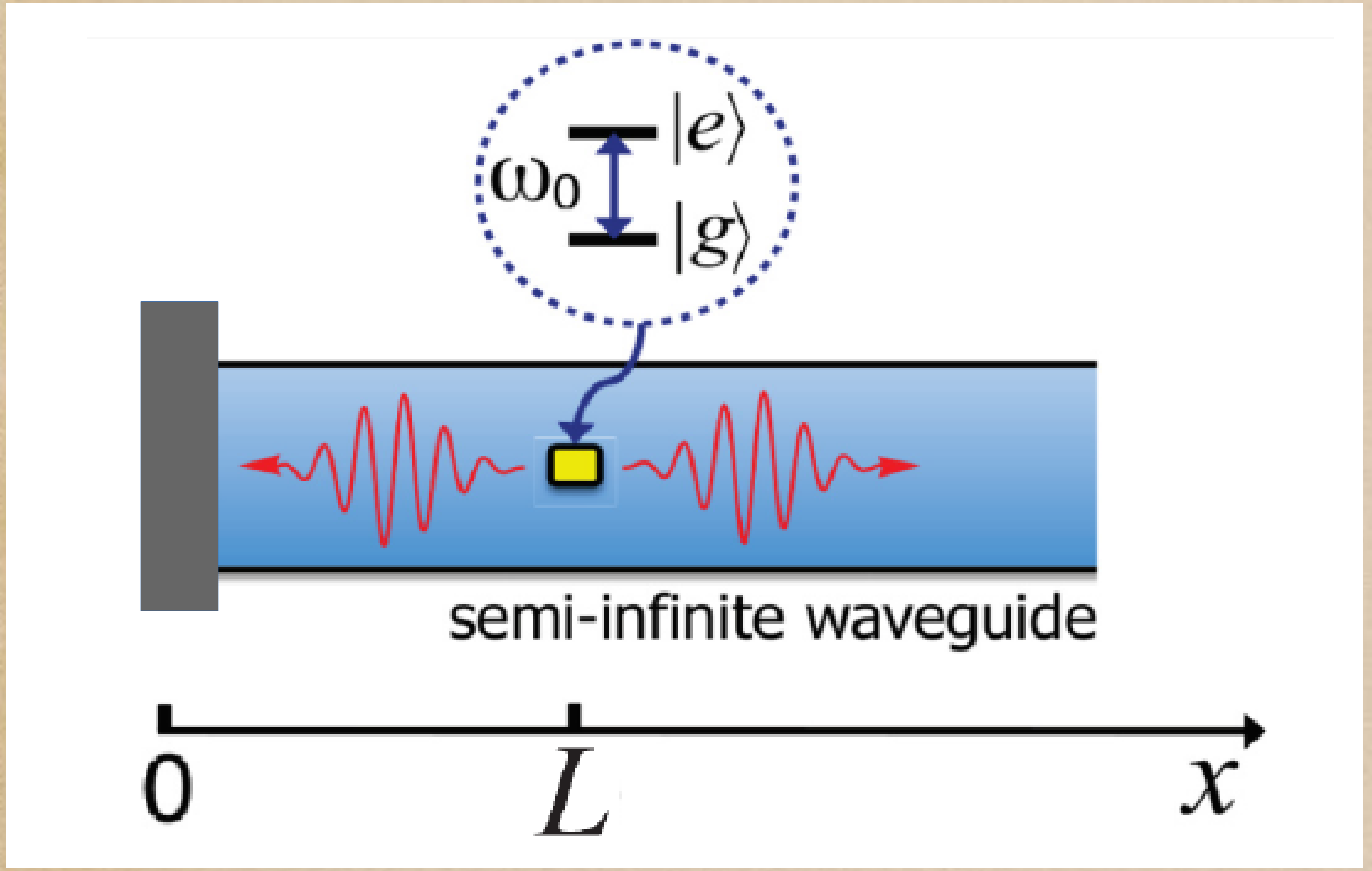
The excited state can decay...



...or not!

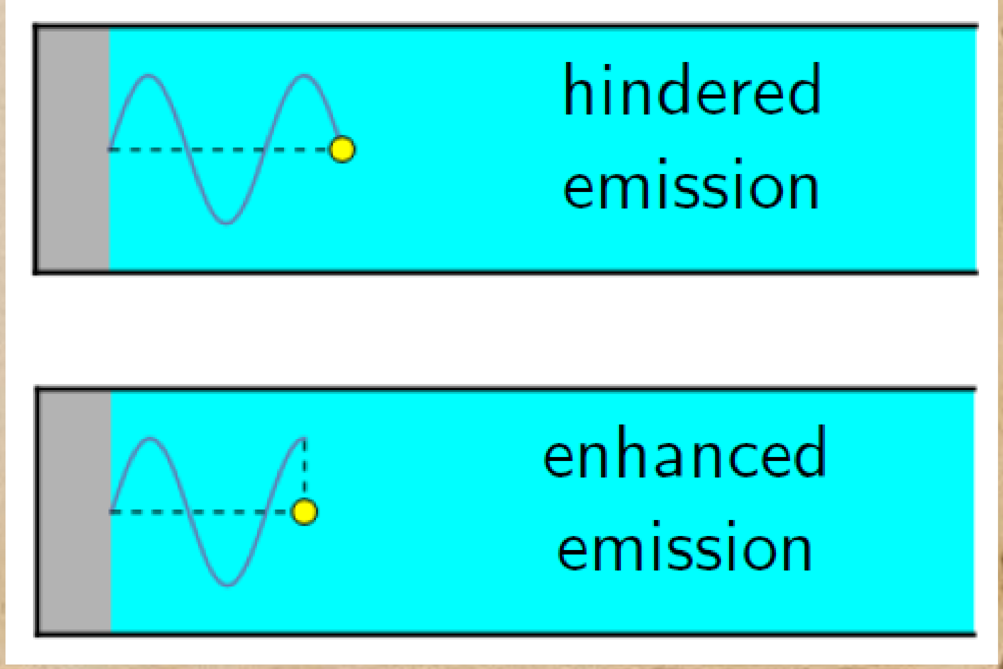


add a mirror



main ingredient:

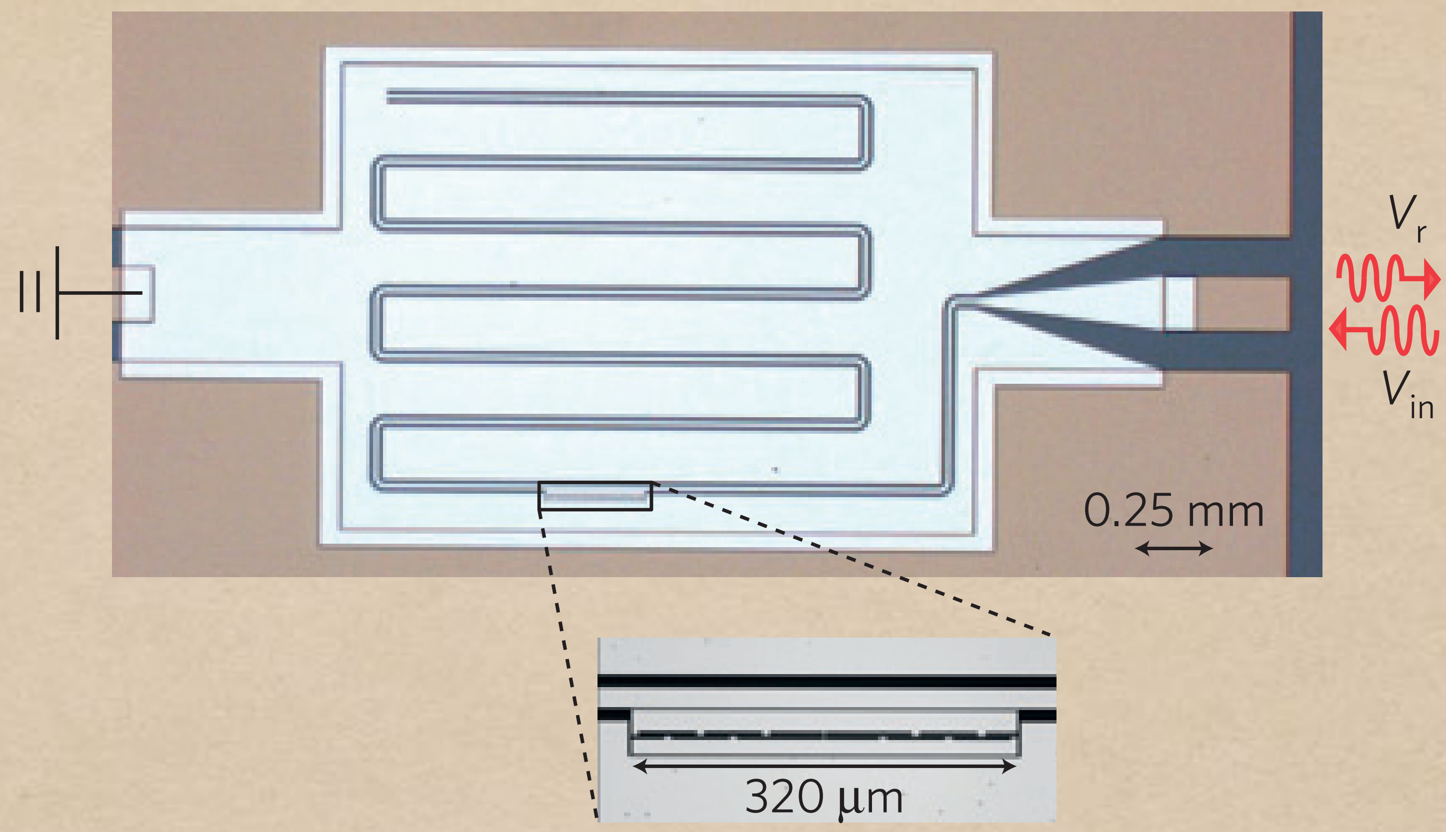
E=0 at mirror

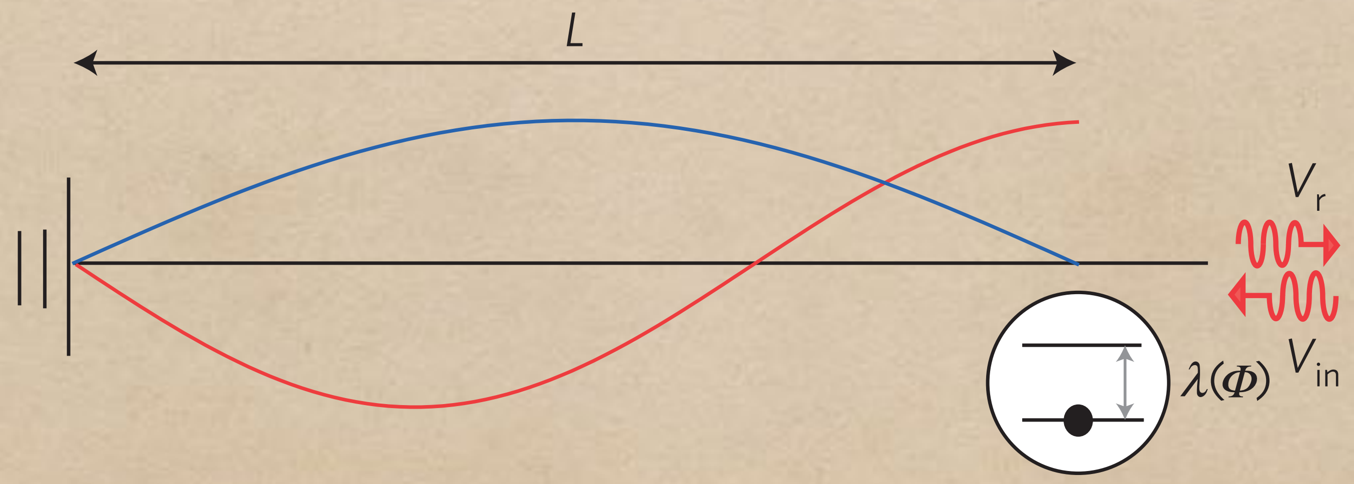


Fermi golden rule:

$$\gamma \propto \left|\langle g;\bar{k}|H_{\mathrm{int}}|e\rangle\right|^2 \propto \sin^2 \bar{k} L$$

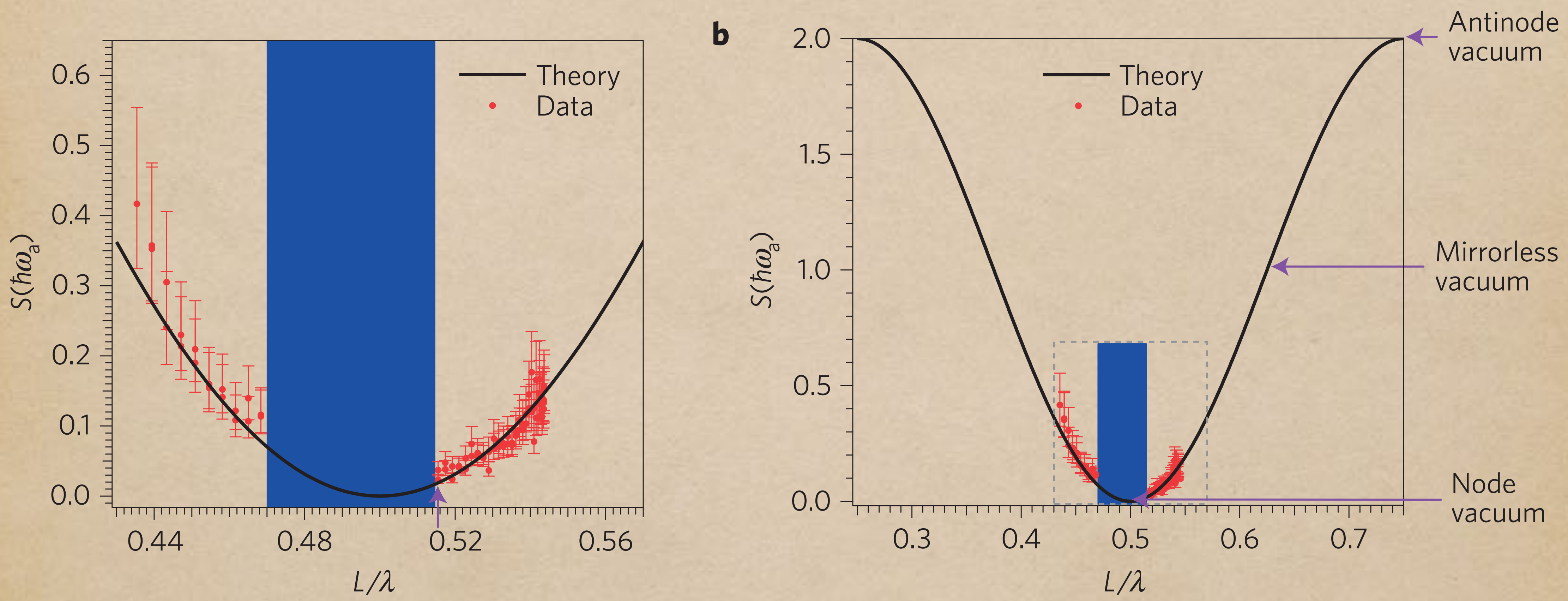
Hoi, Kockum, Tornberg, Pourkabirian, Johansson, Delsing, Wilson 2015





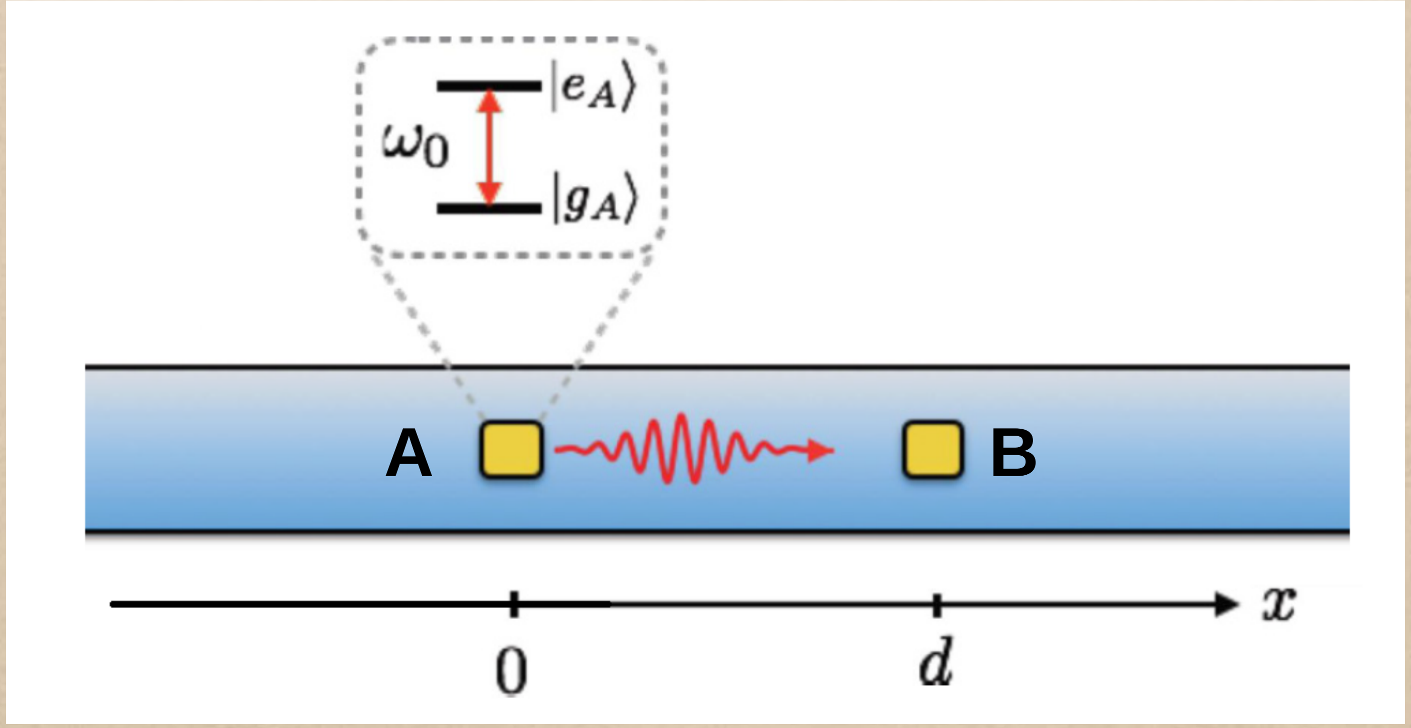
(Probing the quantum vacuum with an artificial atom in front of a mirror)

Hoi, Kockum, Tornberg, Pourkabirian, Johansson, Delsing, Wilson 2015



atom becomes "invisible" at 5.4 GHz

a pair of two-level (artificial) atoms in a waveguide



lowest energy mode, one-excitation sector

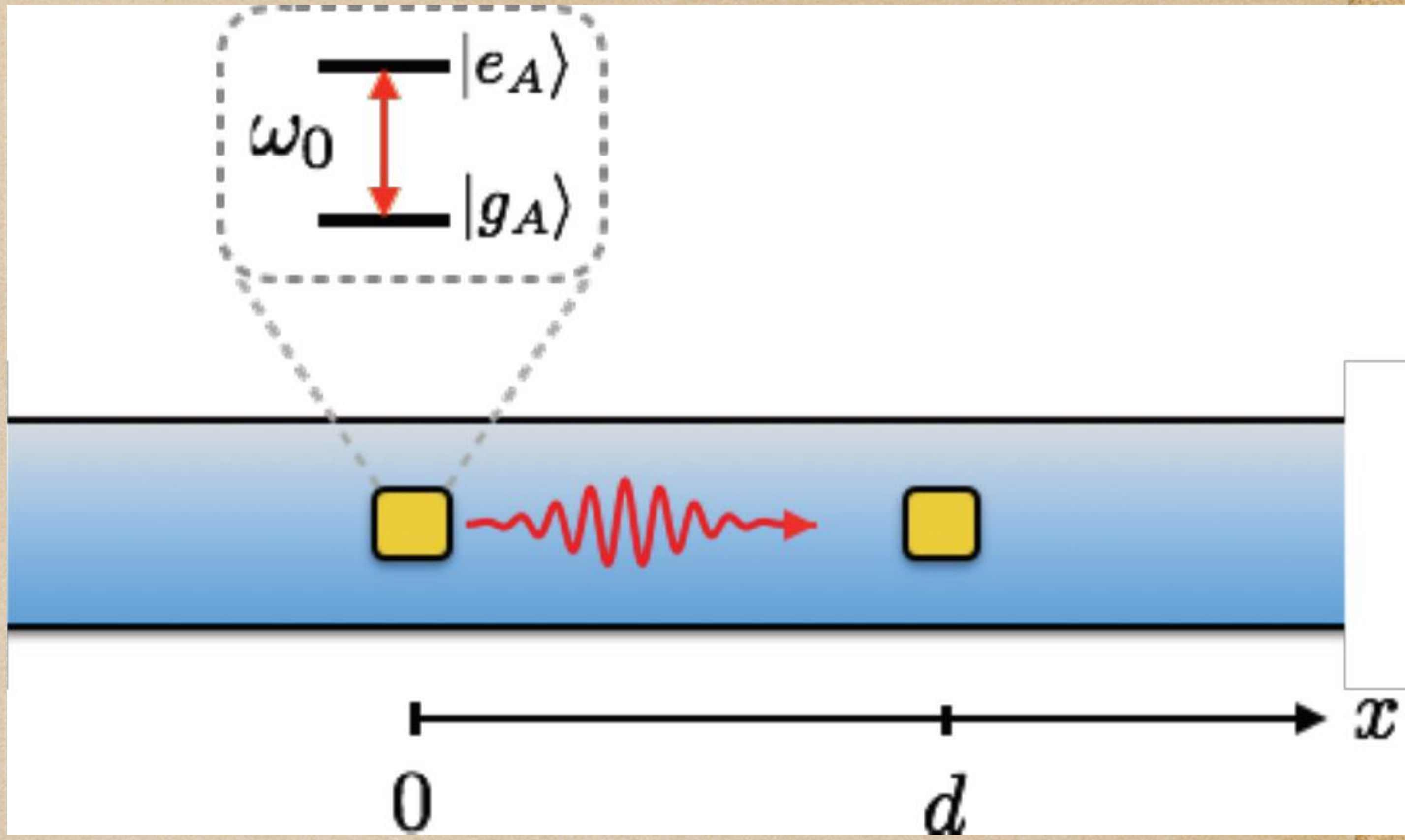
Shen, Fan (2005) + Gonzales-Tudela et al (2011)

System and Hamiltonian

No need to have mirror!

Atoms behave like "mirrors"

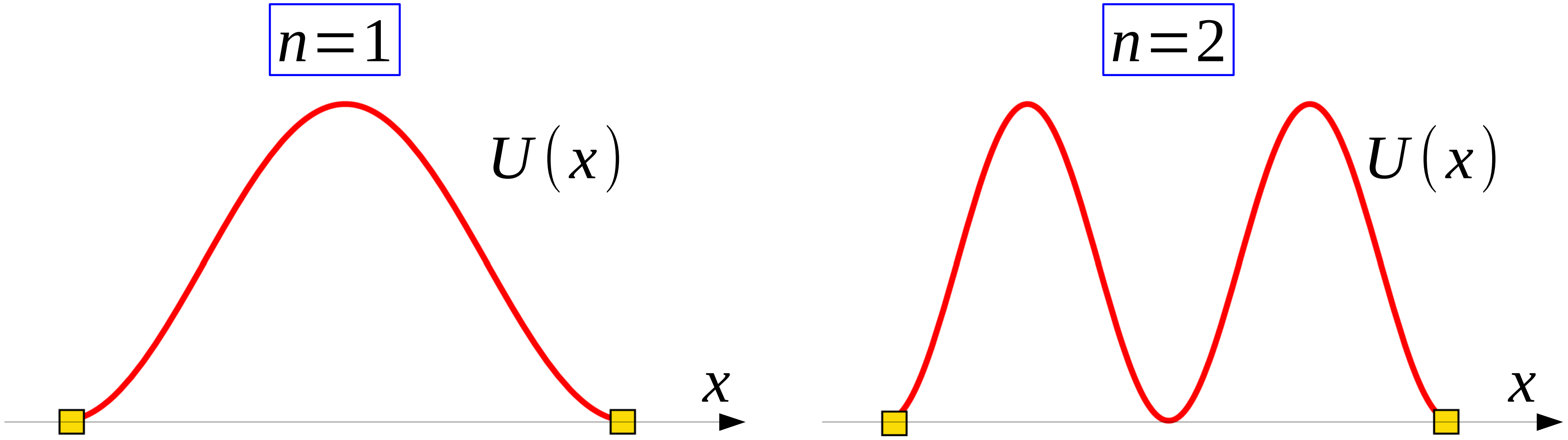
dynamical



$$
\begin{aligned}
H &= H_0 + \lambda V \\
&= \omega_0(|e_A\rangle\langle e_A| + |e_B\rangle\langle e_B|) + \int \mathrm{d}k\, \omega(k) b^\dagger(k) b(k) \\
&\quad + \lambda \int \frac{\mathrm{d}k}{\omega(k)^{1/2}} \Big[|e_A\rangle\langle g_A| b(k) + |g_A\rangle\langle e_A| b^\dagger(k) \\
&\qquad + |e_B\rangle\langle g_B| b(k) \mathrm{e}^{\mathrm{i}kd} + |g_B\rangle\langle e_B| b^\dagger(k) \mathrm{e}^{-\mathrm{i}kd} \Big],
\end{aligned}
$$

$$\omega(k) = \sqrt{k^2 + M^2}$$

$$M \propto L_y^{-1}$$



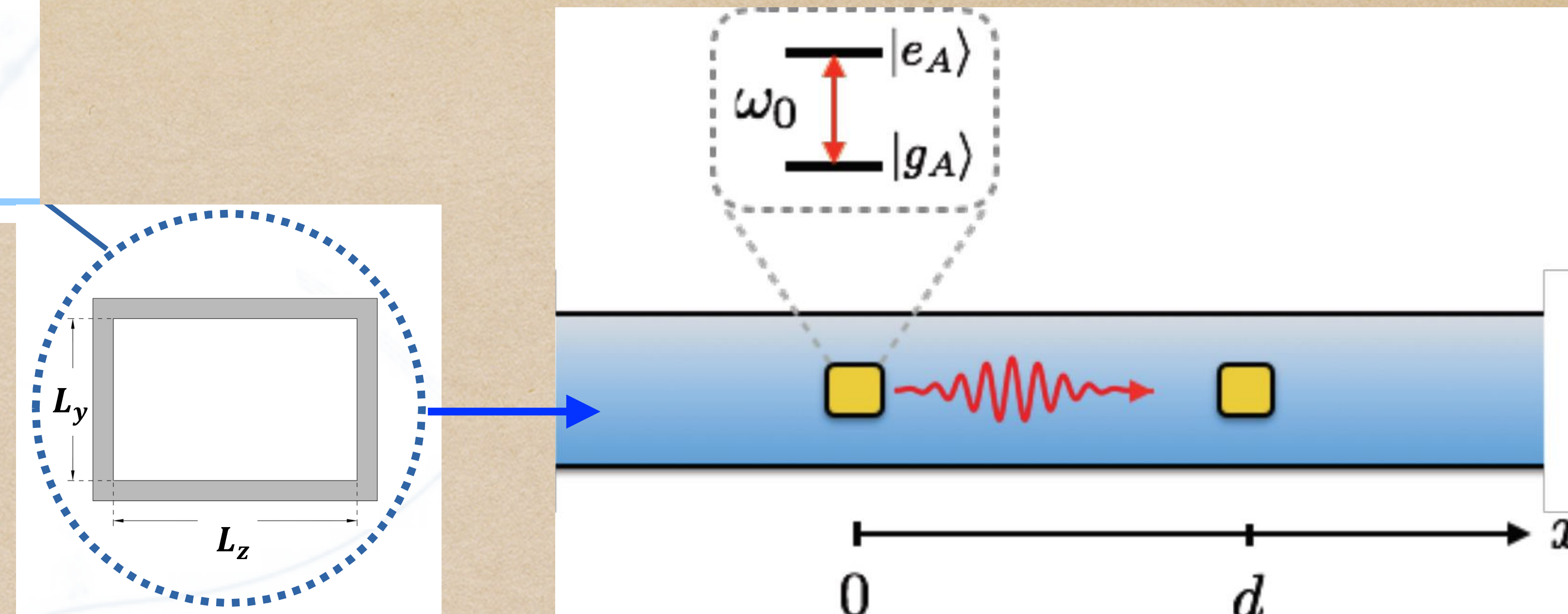
Field energy density

$$U(x) \propto \sin^2(\bar{k}\,x) \quad \text{if } x \in [0, d_n]$$

$$U(x) \simeq 0 \quad \text{elsewhere}$$

atoms behave like dynamical mirrors

$$\omega_{nm}(k) = \sqrt{\frac{k^2}{\mu\epsilon} + M_{nm}^2}$$

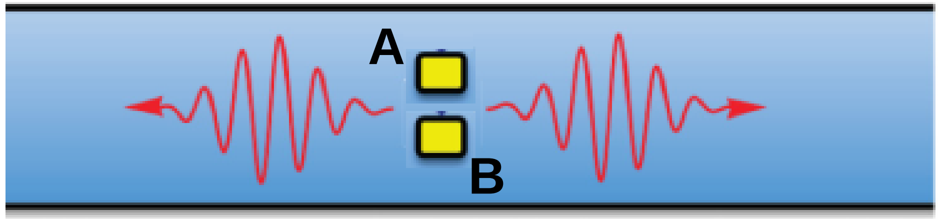


$\text{TE}_{1,0}$ mode; role of boundary conditions

$$|\psi\rangle = \left(c_A|e_A, g_B\rangle + c_B|g_A, e_B\rangle\right) \otimes |\text{vac}\rangle + |g_A, g_B\rangle \otimes |\varphi\rangle$$

$$d_n = \frac{n\pi}{\bar{k}}, \quad \text{with} \quad \bar{k} := \sqrt{\left(\omega_0 + \frac{2\lambda^2}{M}\right)^2 - M^2}.$$

observation: dark state of an atomic pair (identical but distinguishable atoms)



The one-excitation antisymmetric state

$$|\Psi^{(-)}\rangle = \frac{|e_A, g_B\rangle - |g_A, e_B\rangle}{\sqrt{2}}$$

decouples from the interaction

$$H_{\text{int}}|\Psi^{(-)}\rangle = 0$$

The one-excitation symmetric state

$$|\Psi^{(+)}\rangle = \frac{|e_A, g_B\rangle + |g_A, e_B\rangle}{\sqrt{2}}$$

decays faster than a free atom

$$\gamma^{(+)} = 2\gamma_{\text{free}}$$

General wavefunction in the sector

$$|\psi\rangle = \left(c_A|e_A, g_B\rangle + c_B|g_A, e_B\rangle\right)|\text{vac}\rangle + |g_A, g_b\rangle|1\text{ photon}\rangle$$

Bound states $H|\psi\rangle = E|\psi\rangle$ with $\langle\psi|\psi\rangle = 1$

The eigenvalue equation $H|\psi\rangle = E|\psi\rangle = \sqrt{\bar{k}^2 + M^2}|\psi\rangle$

can be satisfied by a normalizable state only if:

i) $c_A + e^{\pm i\bar{k}d} c_B = 0$

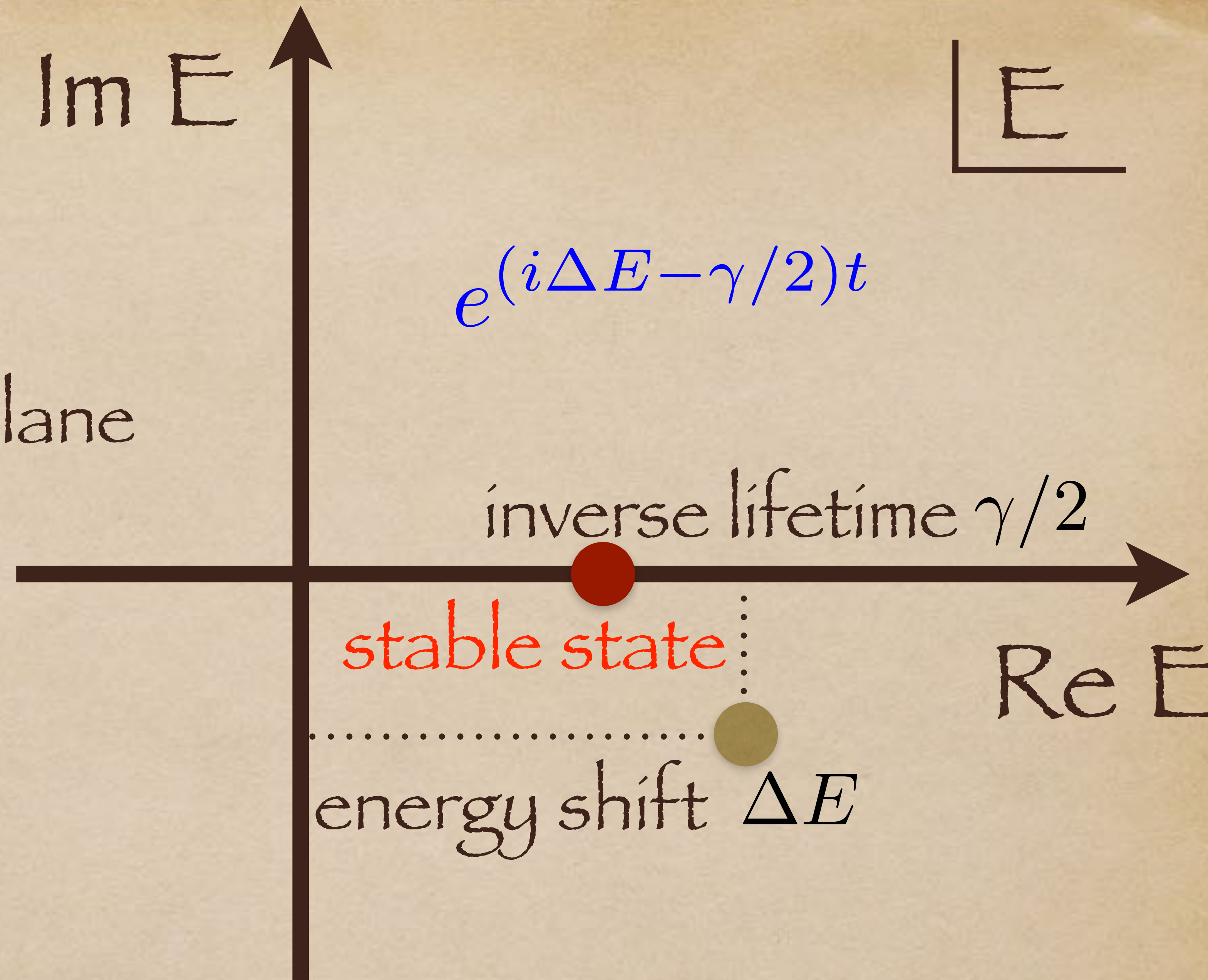
ii) $E = \omega_o + \int dk \frac{\lambda^2}{\sqrt{k^2+M^2}} \frac{1 - e^{i(\bar{k}-k)d}}{E - \sqrt{k^2+M^2}}$ has real solutions

iff

$$c_A = (-1)^{n+1} c_B, \qquad d = d_n = \frac{n\pi}{\bar{k}} \quad (n \in \mathbb{Z}_+)$$

fast tutorial

complex energy plane

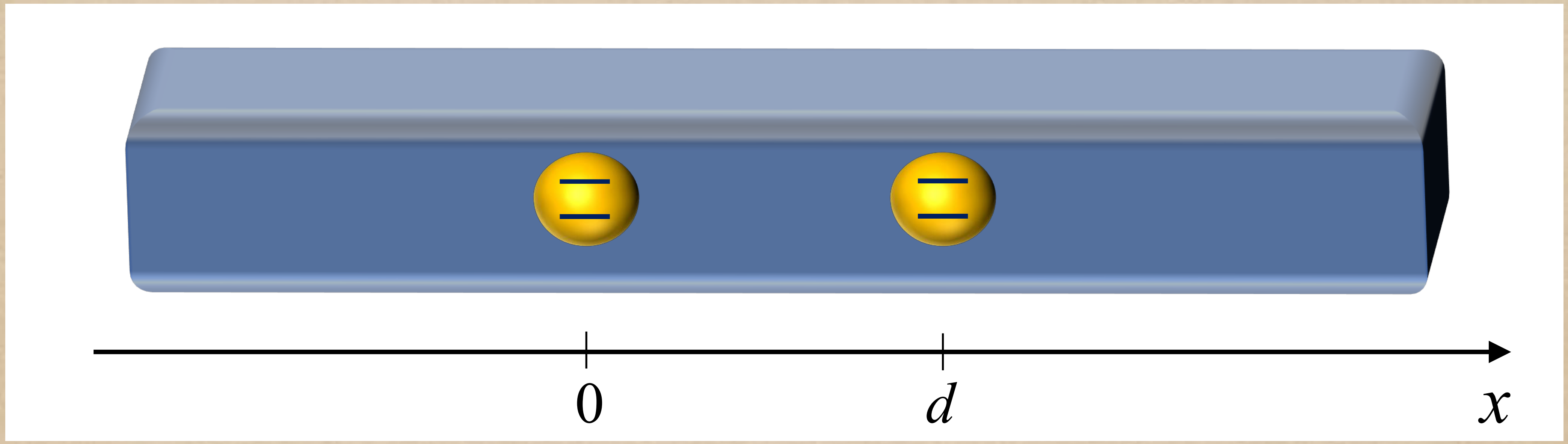


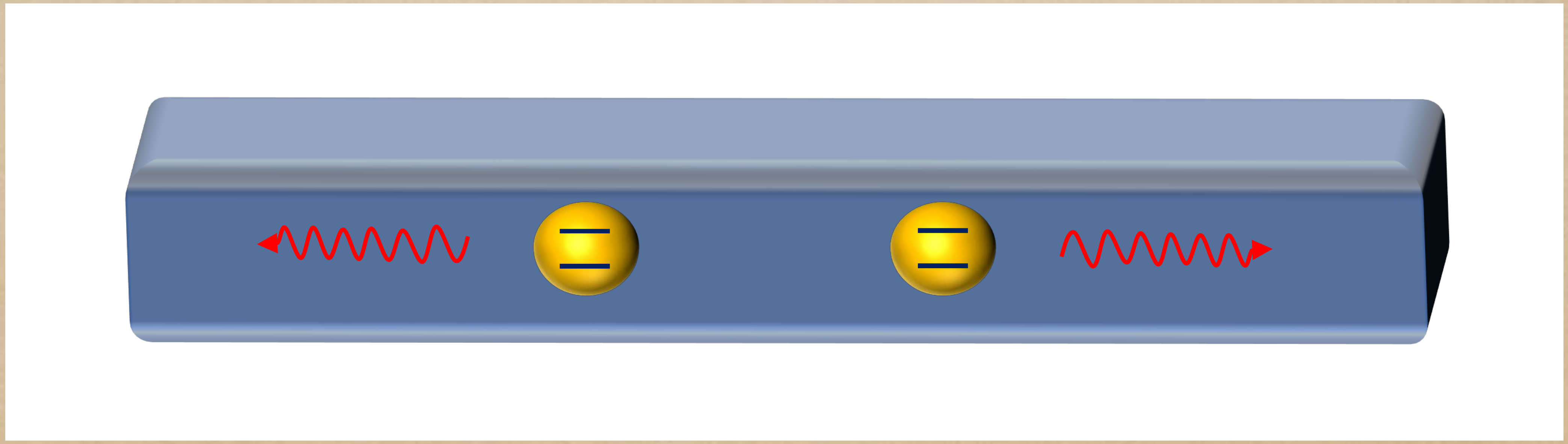
$\underline{|\,E}$

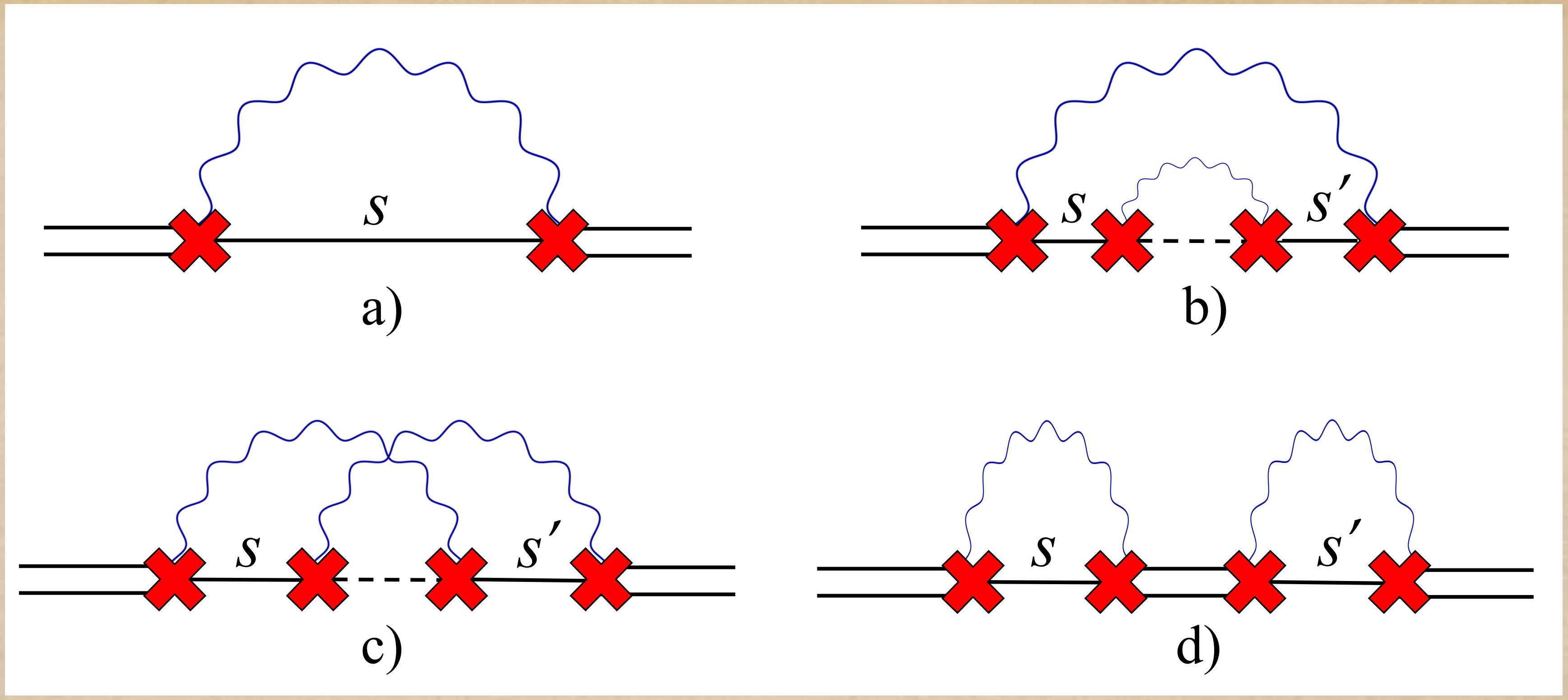
Schwinger (simple poles);

Araki et al (proof of Fermi "Golden rule")

two-excitation sector

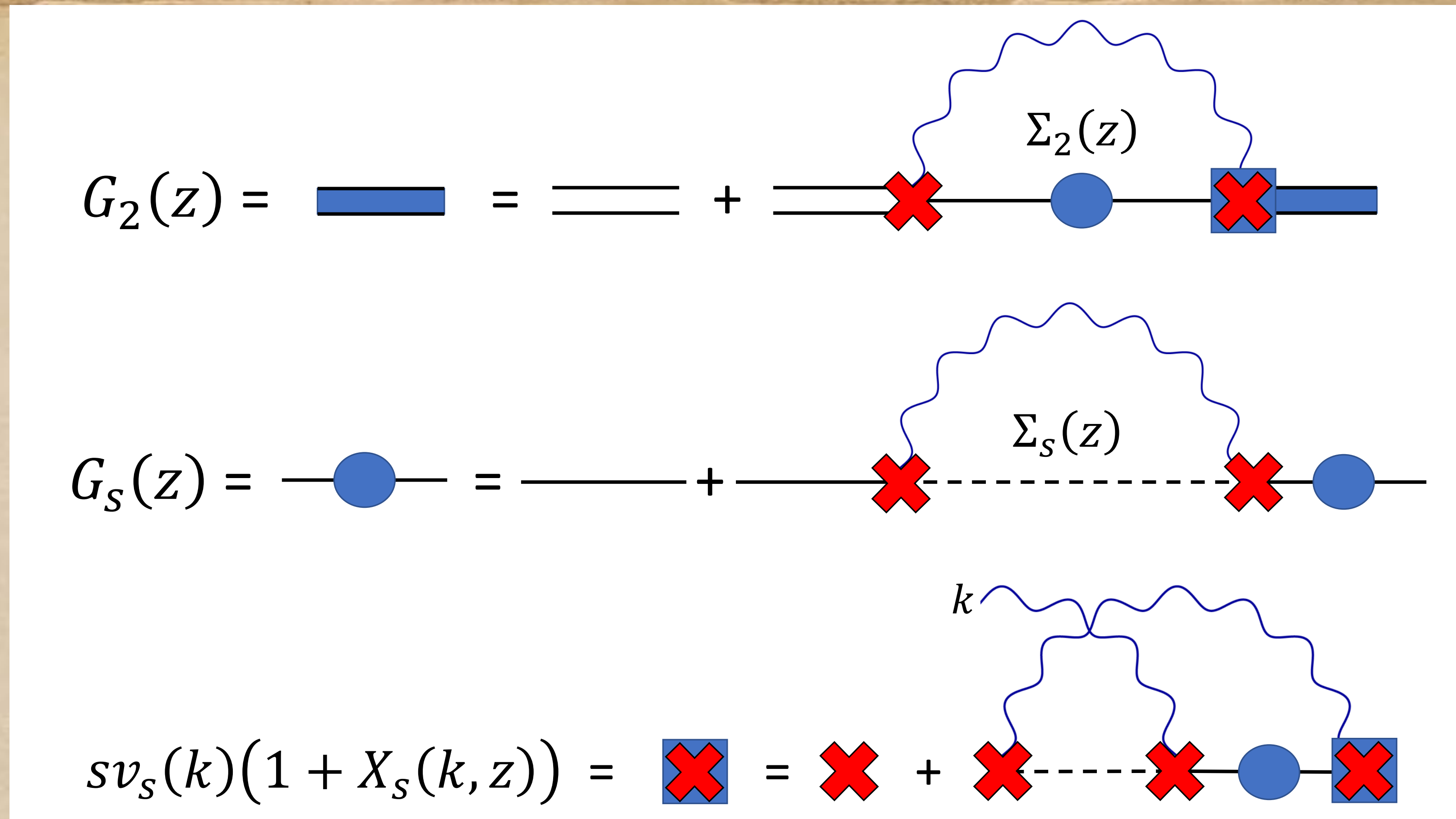


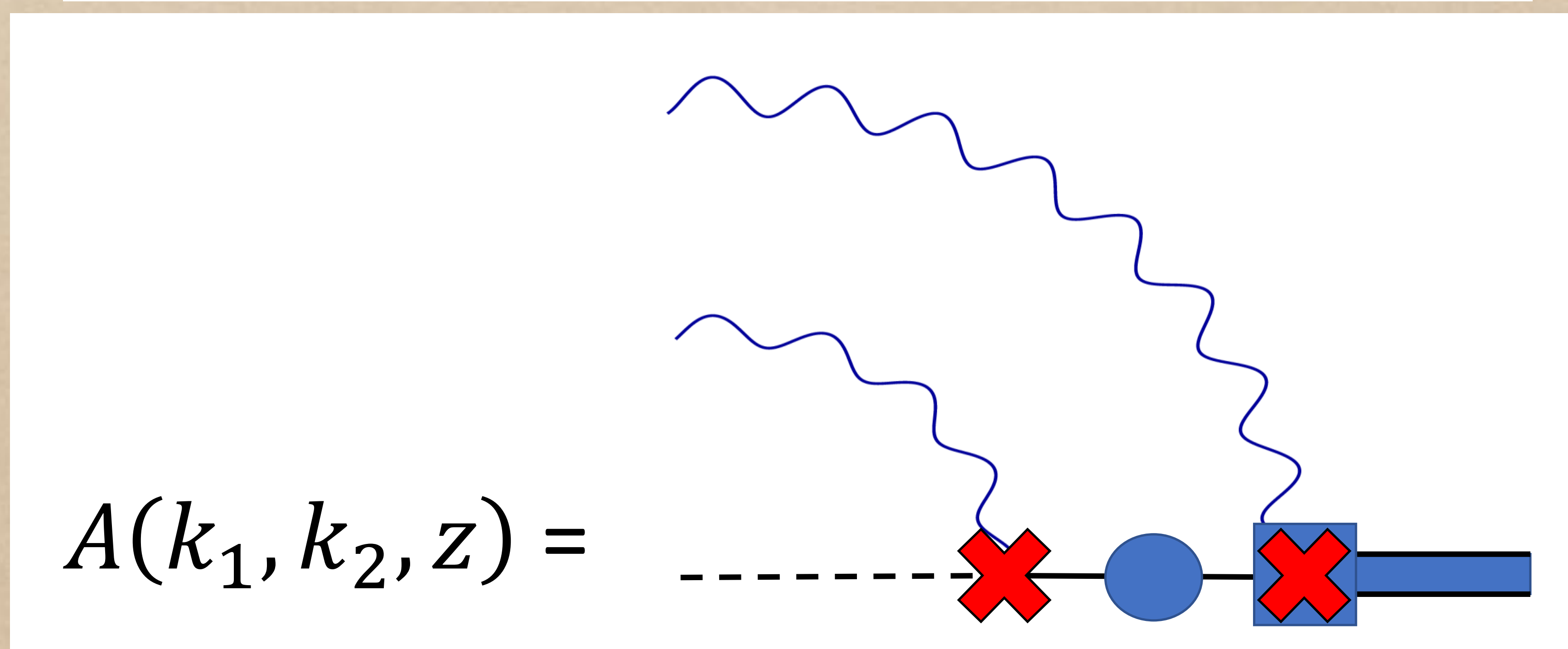




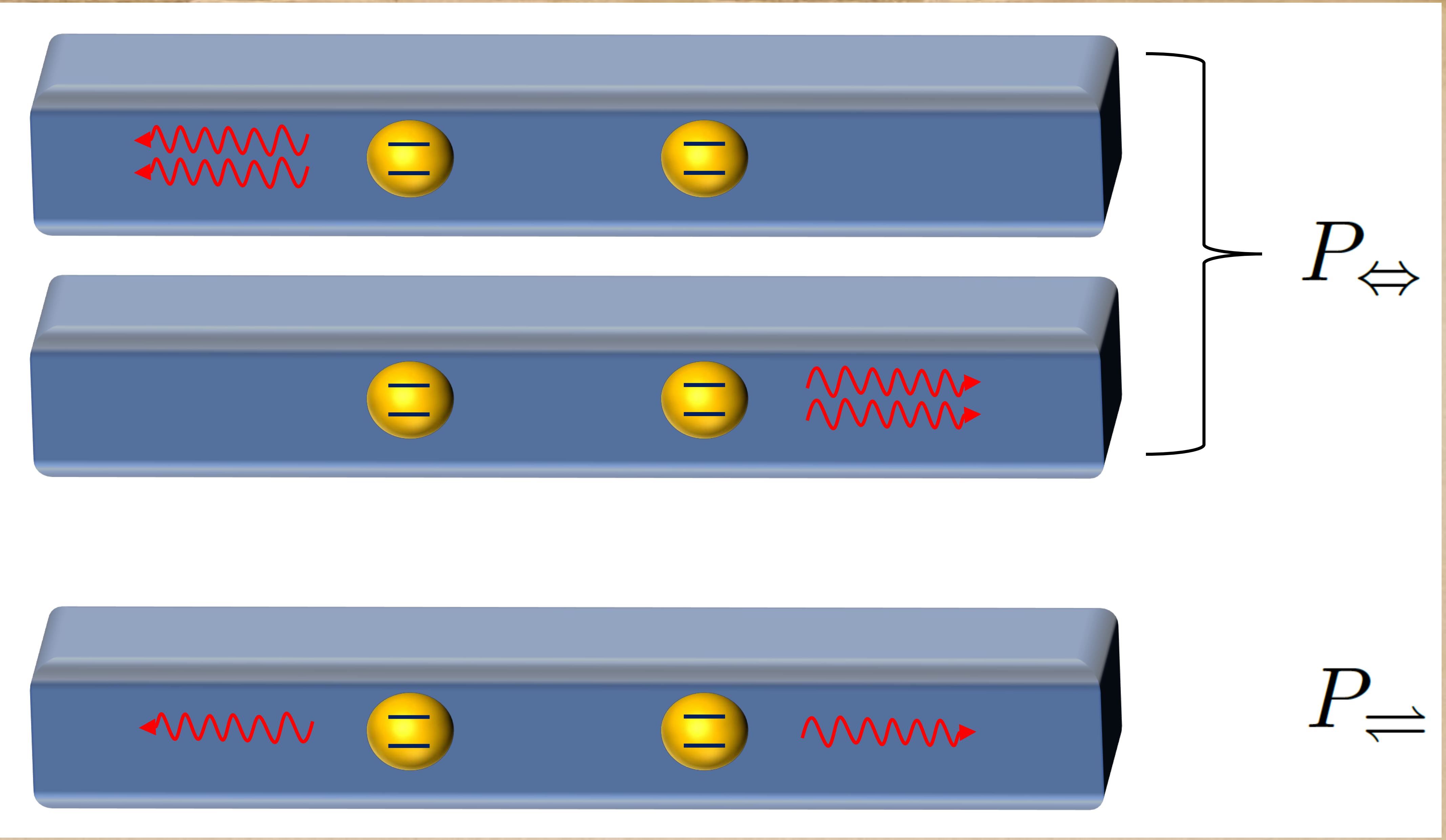
analysis is more complicated

renormalization procedure is involved



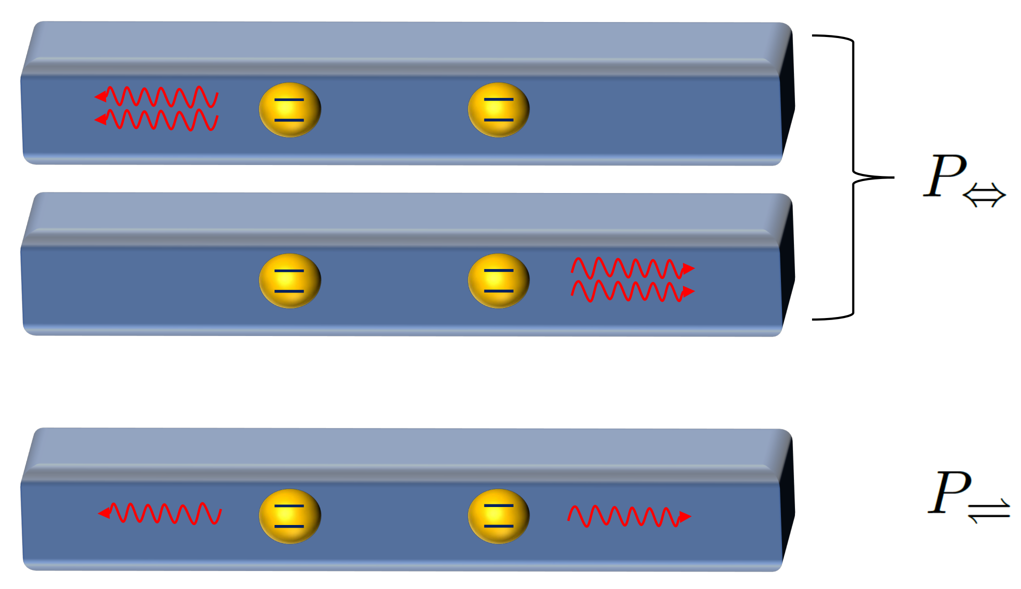


re-sum diagrams and renormalize

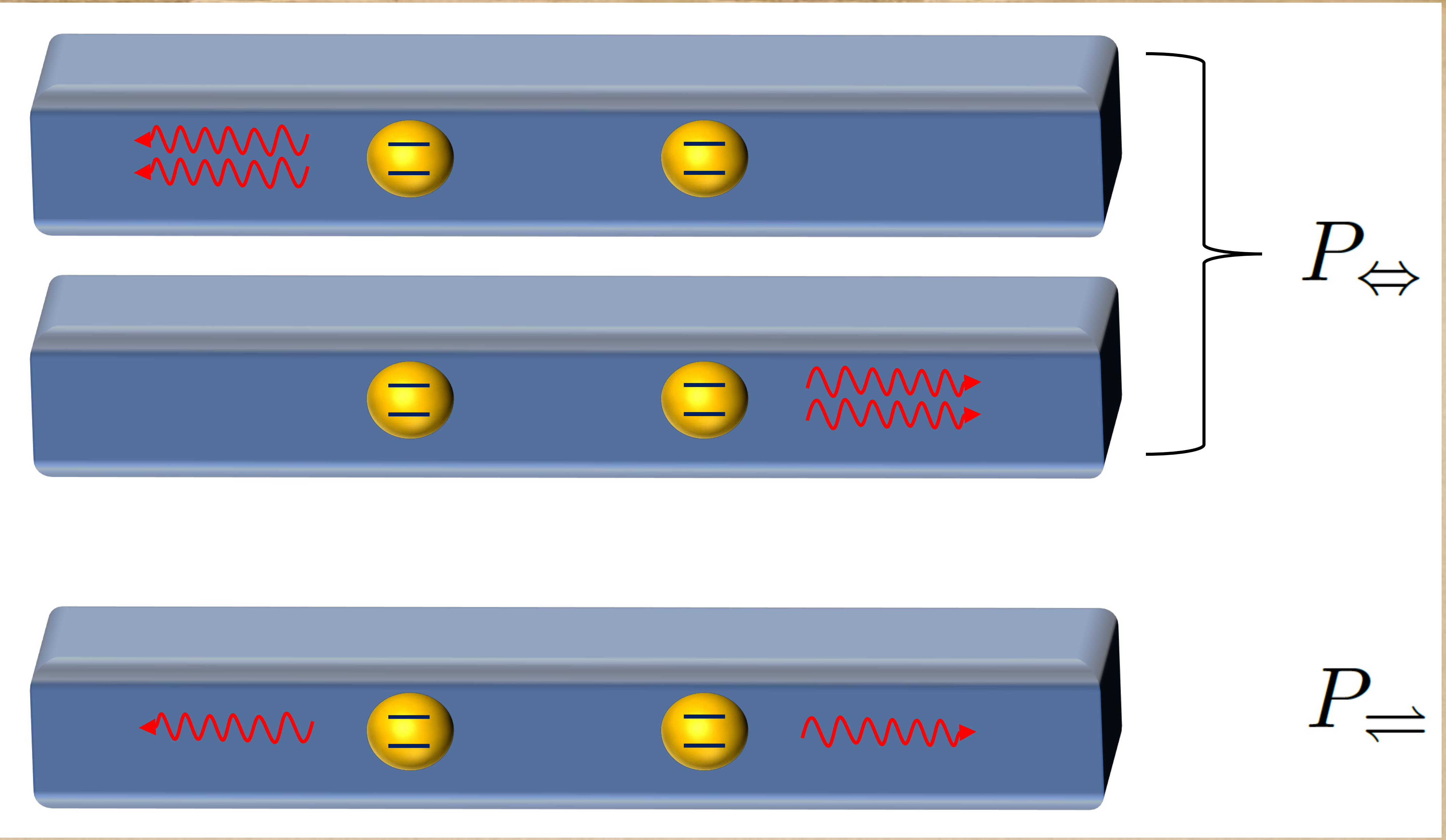


$$P(k_1, k_2) = \lim_{t \to \infty} |A(k_1, k_2, t)|^2$$

$$P(k_1,k_2) = 8\Big\{ 2\cos^4\left(\frac{k_0 d}{2}\right)|F_+(k_1-k_0,\pm k_2-k_0)|^2$$
$$+ 2\sin^4\left(\frac{k_0 d}{2}\right)|F_-(k_1-k_0,\pm k_2-k_0)|^2$$
$$\pm \sin^2(k_0 d)\mathrm{Re}\Big[F_+^*(k_1-k_0,\pm k_2-k_0)$$
$$\times F_-(k_1-k_0,\pm k_2-k_0)\Big]\Big\}$$



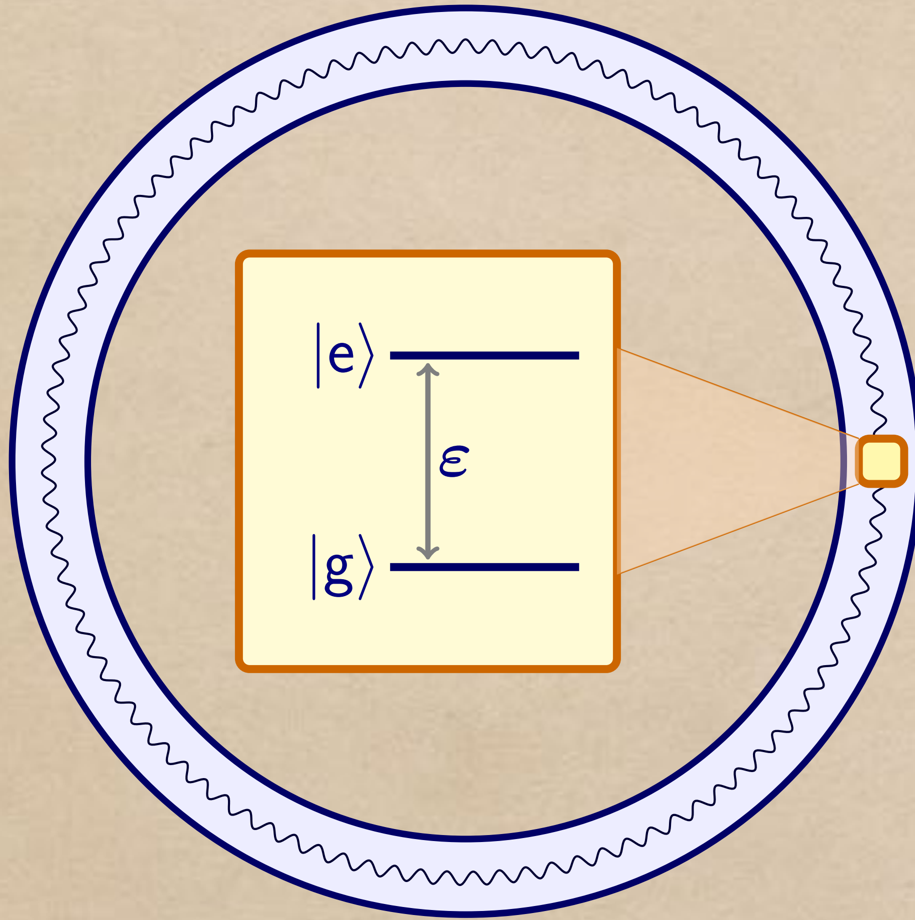
$$P_{\Leftrightarrow} = \frac{1}{2}\int_{D_{\Leftrightarrow}} dk_1 dk_2 P(k_1,k_2)$$
$$= \frac{1}{2}\left(1 + \frac{\sin^2(k_0 d)}{1+\sin^2(k_0 d)}\right)$$
$$P_{\rightleftharpoons} = \frac{1}{2}\int_{D_{\rightleftharpoons}} dk_1 dk_2 P(k_1,k_2)$$
$$= \frac{1}{2}\left(1 - \frac{\sin^2(k_0 d)}{1+\sin^2(k_0 d)}\right)$$

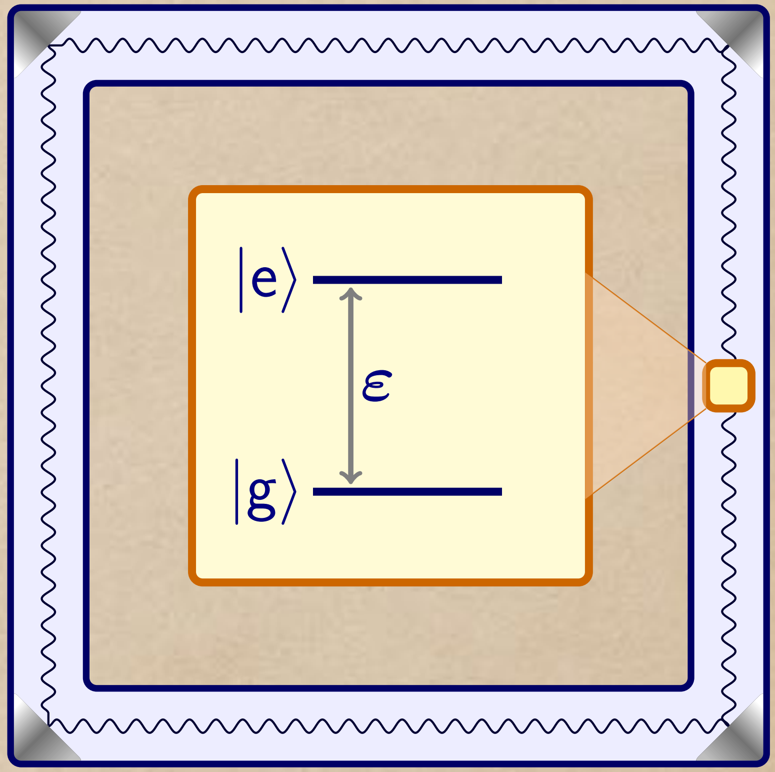


$$R(\lambda)\Big|_{\lambda\to 0} = \frac{P_{\Leftrightarrow}}{P_{\rightleftharpoons}} = 1 + 2\sin^2(k_0 d)$$

$$\overset{\sin(k_0 d)=\pm 1}{=} 3$$

closed geometry





STATE

$$|\Psi\rangle = a\,|e\rangle \otimes |\mathrm{vac}\rangle + |g\rangle \otimes \sum_k \xi_k b_k^\dagger\,|\mathrm{vac}\rangle$$

$$\xi(x) = \sqrt{\frac{2\pi}{L}} \sum_k \xi_k \mathrm{e}^{\frac{2\pi \mathrm{i} k x}{L}}$$

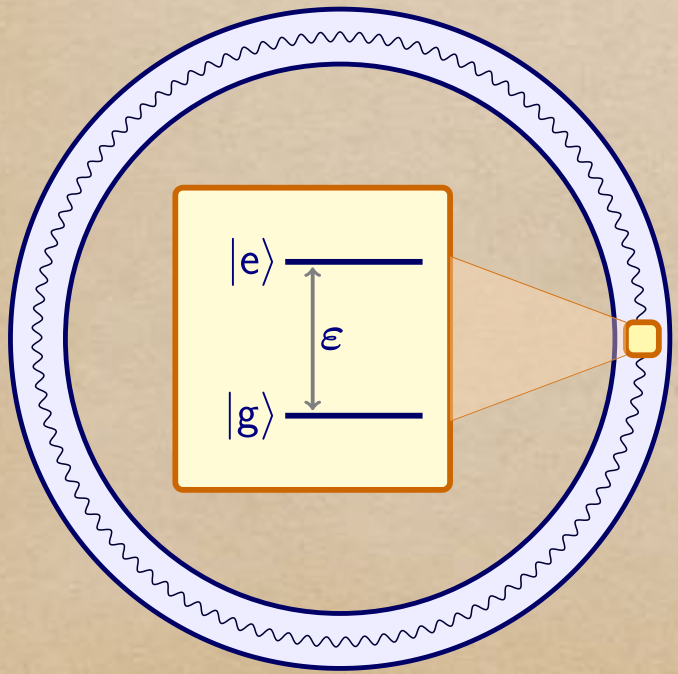
$$|a|^2 + \int_{-L/2}^{L/2} |\xi(x)|^2\,\mathrm{d}x = 1$$

Hamiltonian

$$H_0 = \varepsilon\,\sigma^+\sigma^- + \sum_{k=-\infty}^{\infty} \omega_k b_k^\dagger b_k$$

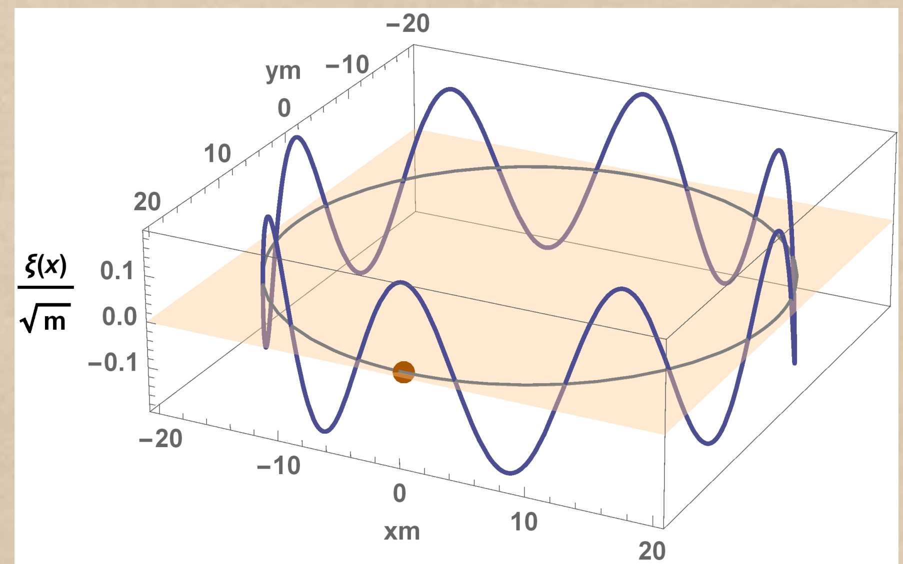
$$H_{\rm int} = \sum_{k=-\infty}^{\infty} F_k \left(\sigma^+ b_k + \sigma^- b_k^\dagger\right)$$

$$F_k = \sqrt{\frac{\gamma}{L\omega_k}}$$

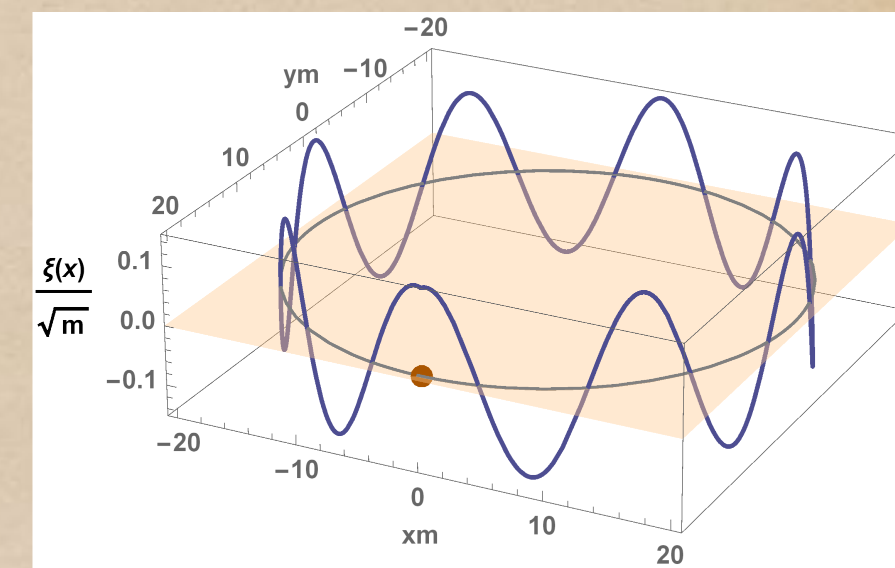


$$\Sigma(E) = \sum_k \frac{F_k^2}{E - \omega_k}$$ self energy

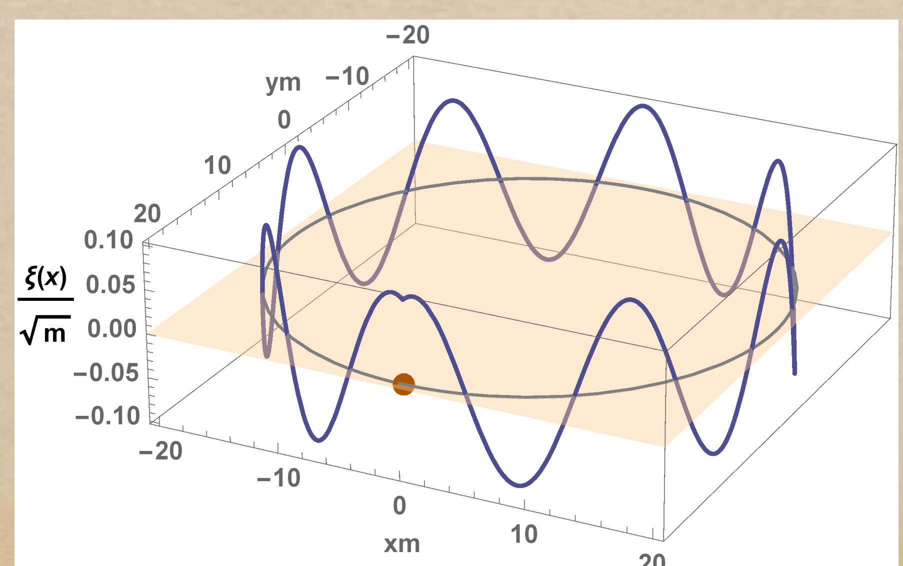
$$E - \varepsilon - \Sigma(E) = 0$$ bound states



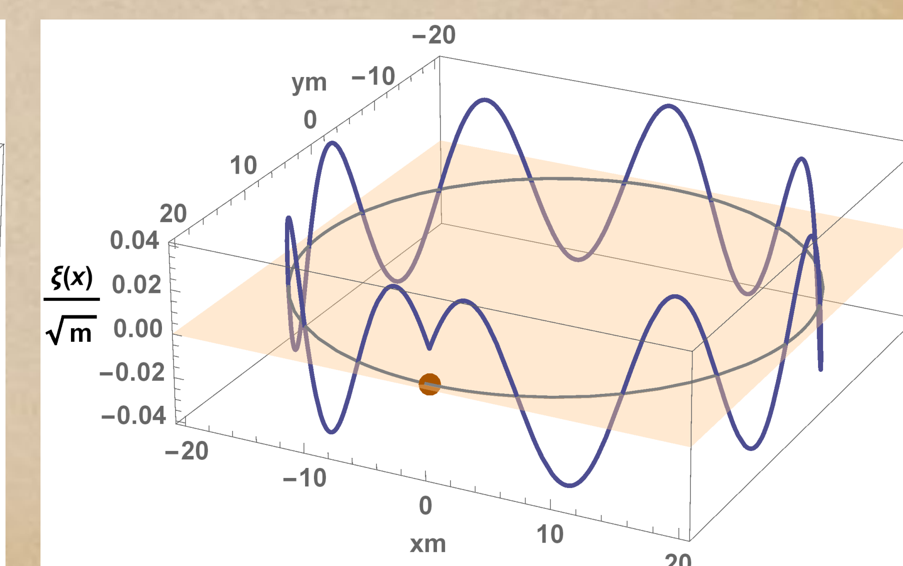
(a) $|a|^2 = 0$, $\delta E/m = 0$;



(b) $|a|^2 = 0.40$, $\delta E/m = 10^{-3}$;



(c) $|a|^2 = 0.72$, $\delta E/m = 2 \cdot 10^{-3}$;



(d) $|a|^2 = 0.96$, $\delta E/m = 5 \cdot 10^{-3}$;

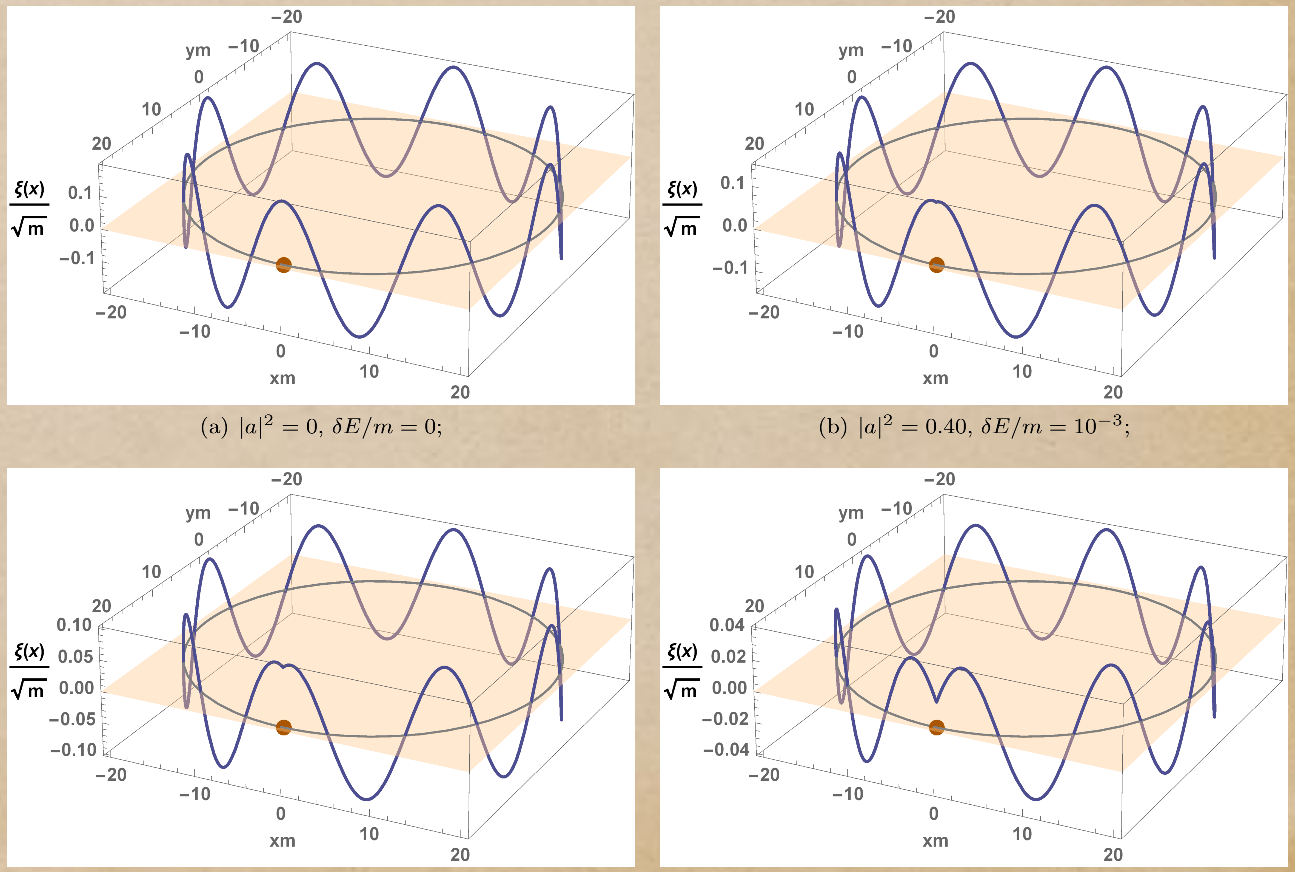
$$\Sigma(E) = \gamma\left(\frac{\cot(q(E)L/2)}{q(E)}\theta(E) + \beta_0(E)\right)$$

non perturbative

$$\xi_1(x) \;=\; \frac{\sqrt{2\pi\gamma E}}{q(E)}\Big[\cot\left(q(E)L/2\right)\,\cos\left(q(E)x\right) + \sin\left(q(E)|x|\right)\Big]\theta(E) + \eta(x)$$

reminder:

$$|a|^2 + \int_{-L/2}^{L/2} |\xi(x)|^2\,\mathrm{d}x = 1$$



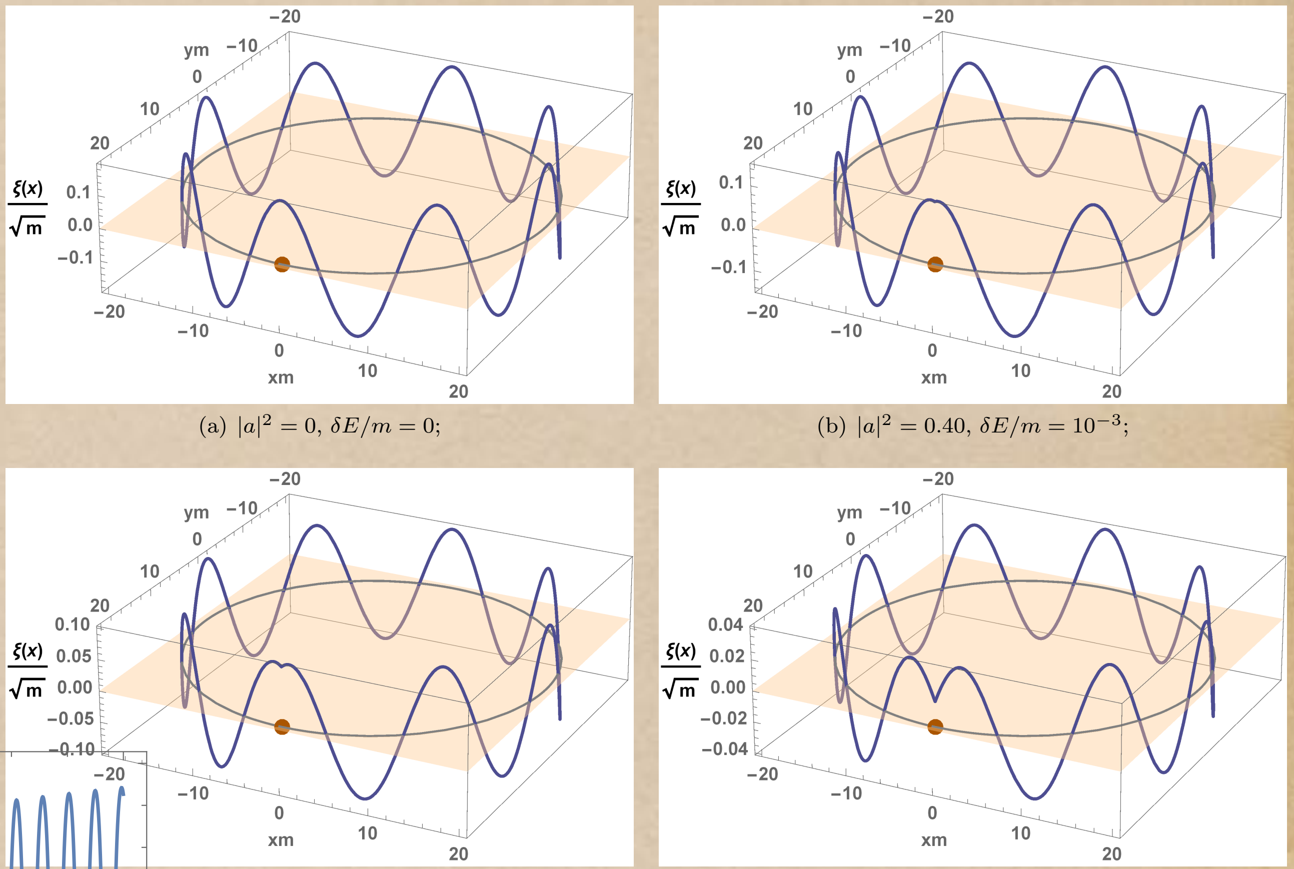
(a) $|a|^2 = 0$, $\delta E/m = 0$;

(b) $|a|^2 = 0.40$, $\delta E/m = 10^{-3}$;

(c) $|a|^2 = 0.72$, $\delta E/m = 2\cdot 10^{-3}$;

(d) $|a|^2 = 0.96$, $\delta E/m = 5\cdot 10^{-3}$;

reminder: $$|a|^2 + \int_{-L/2}^{L/2} |\xi(x)|^2 \, \mathrm{d}x = 1$$



(a) $|a|^2 = 0$, $\delta E/m = 0$;

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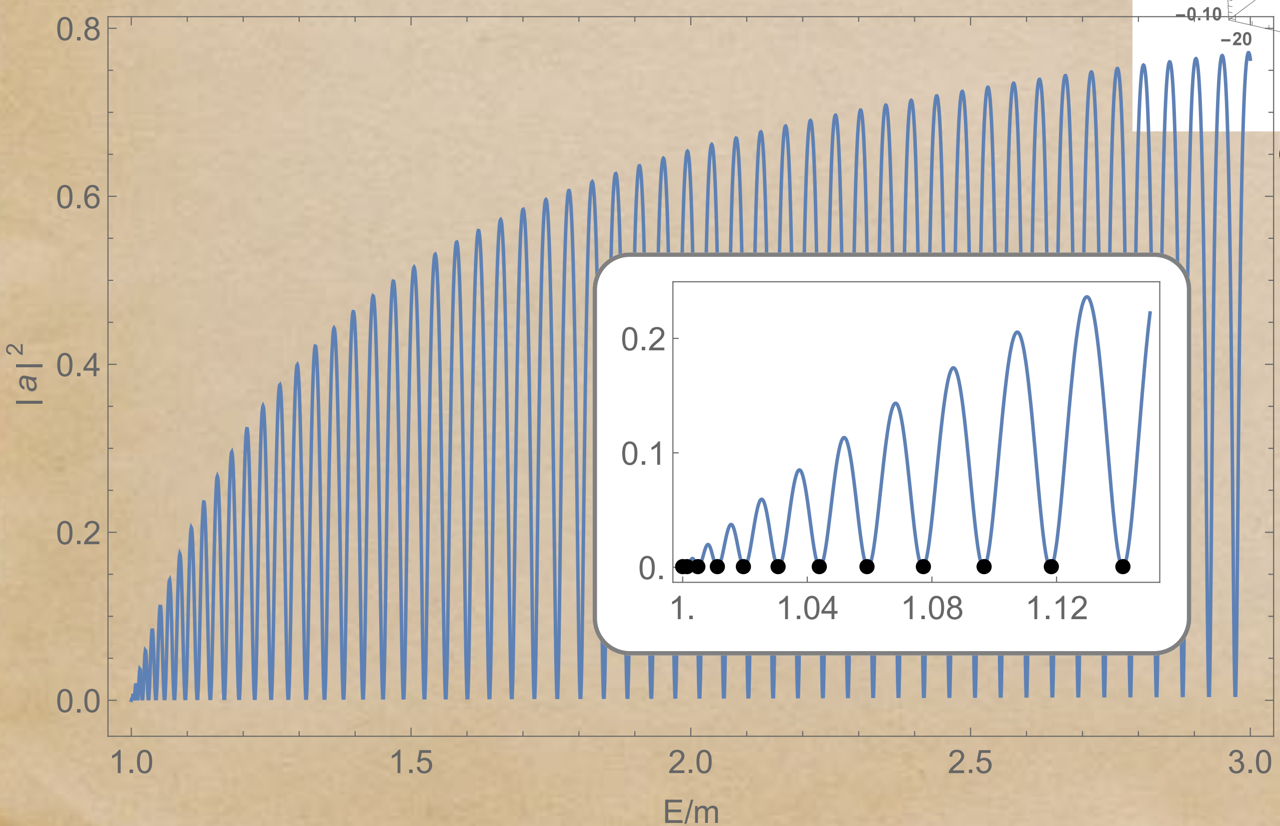
(d) $|a|^2 = 0.96$, $\delta E/m = 5 \cdot 10^{-3}$;



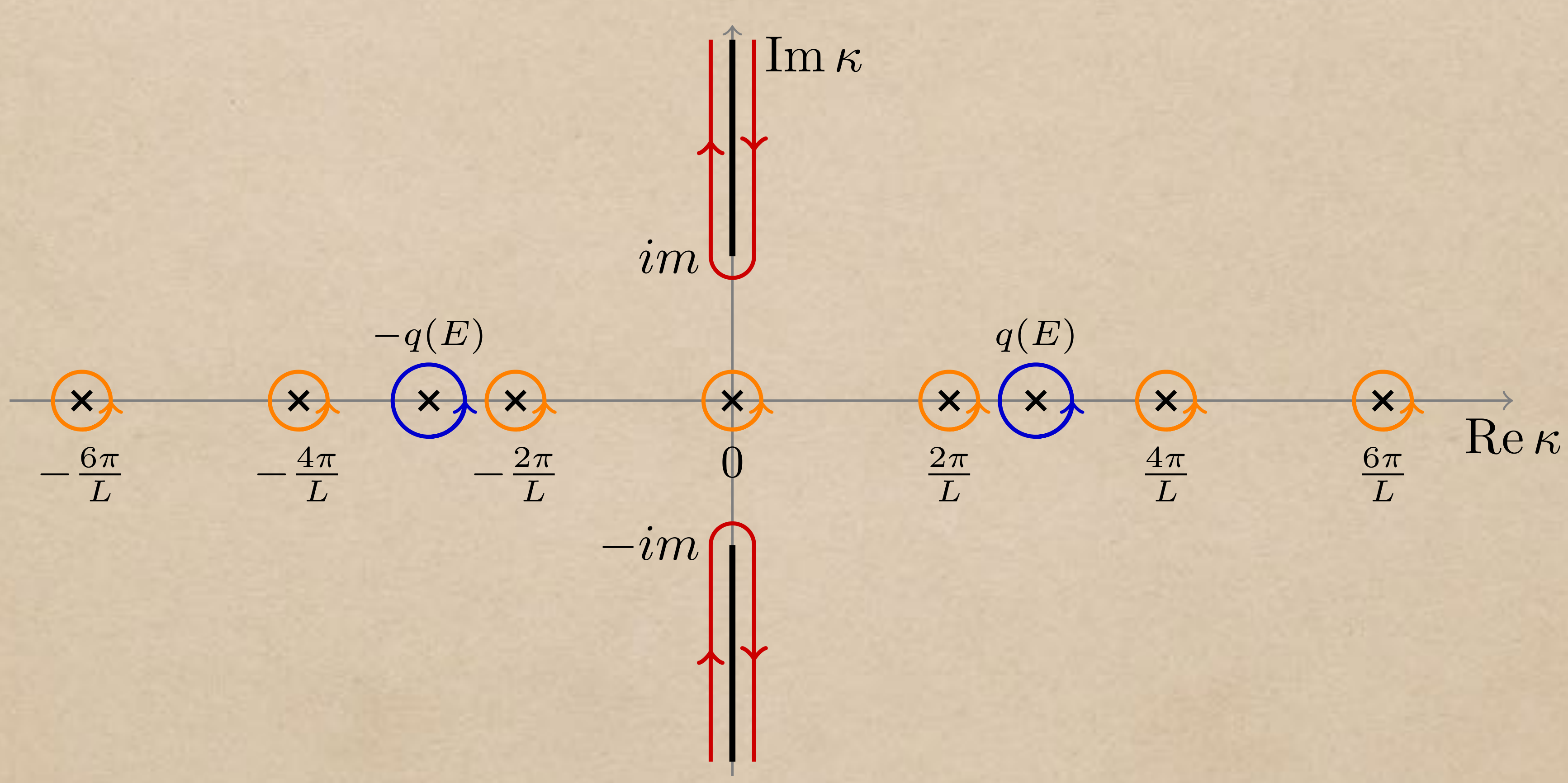
TWO emitters

$$H_{\text{int}} = \sum_{k=-\infty}^{\infty} \sum_{\alpha=1,2} F_k \left(\mathrm{e}^{\frac{2\pi \mathrm{i} k x_\alpha}{L}} \sigma_\alpha^+ b_k + \mathrm{e}^{\frac{-2\pi \mathrm{i} k x_\alpha}{L}} \sigma_\alpha^- b_k^\dagger \right)$$

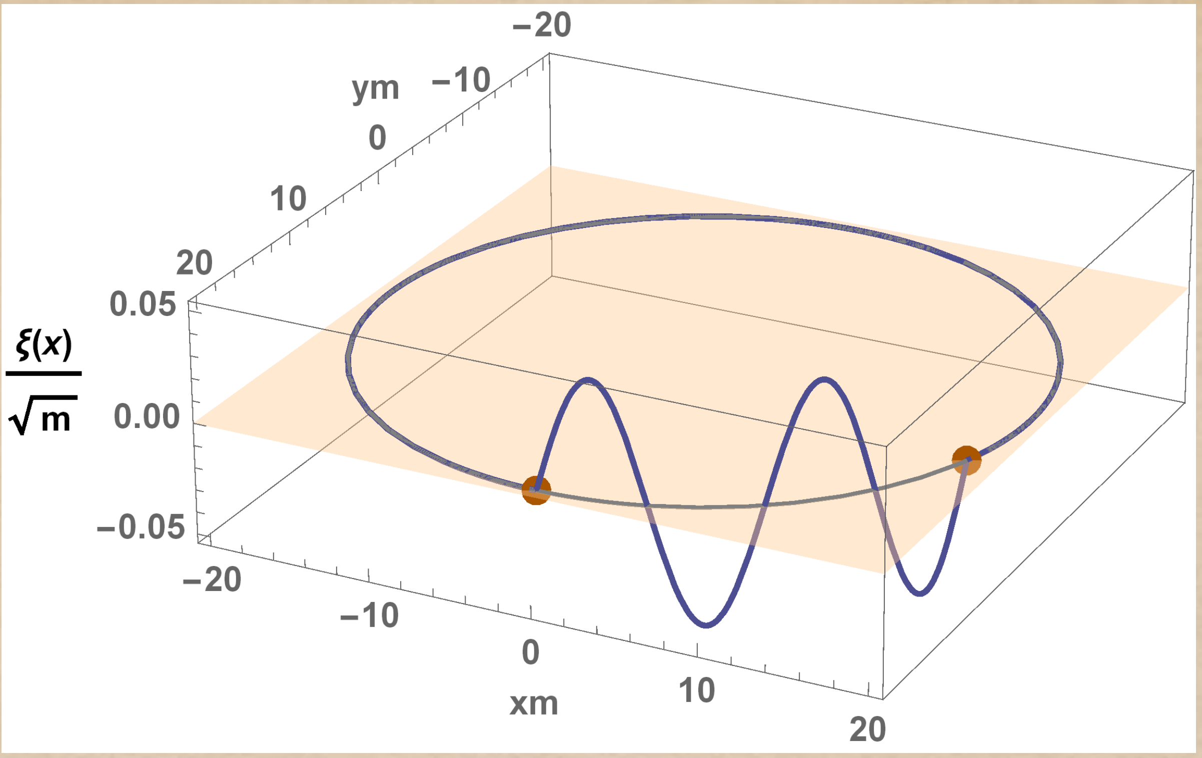
$$|\Psi\rangle = \sum_{\alpha=1,2} a_\alpha \sigma_\alpha^+ |G\rangle \otimes |\text{vac}\rangle + |G\rangle \otimes \sum_k \xi_k b_k^\dagger |\text{vac}\rangle$$

$$|G\rangle = |g_1\rangle \otimes |g_2\rangle$$

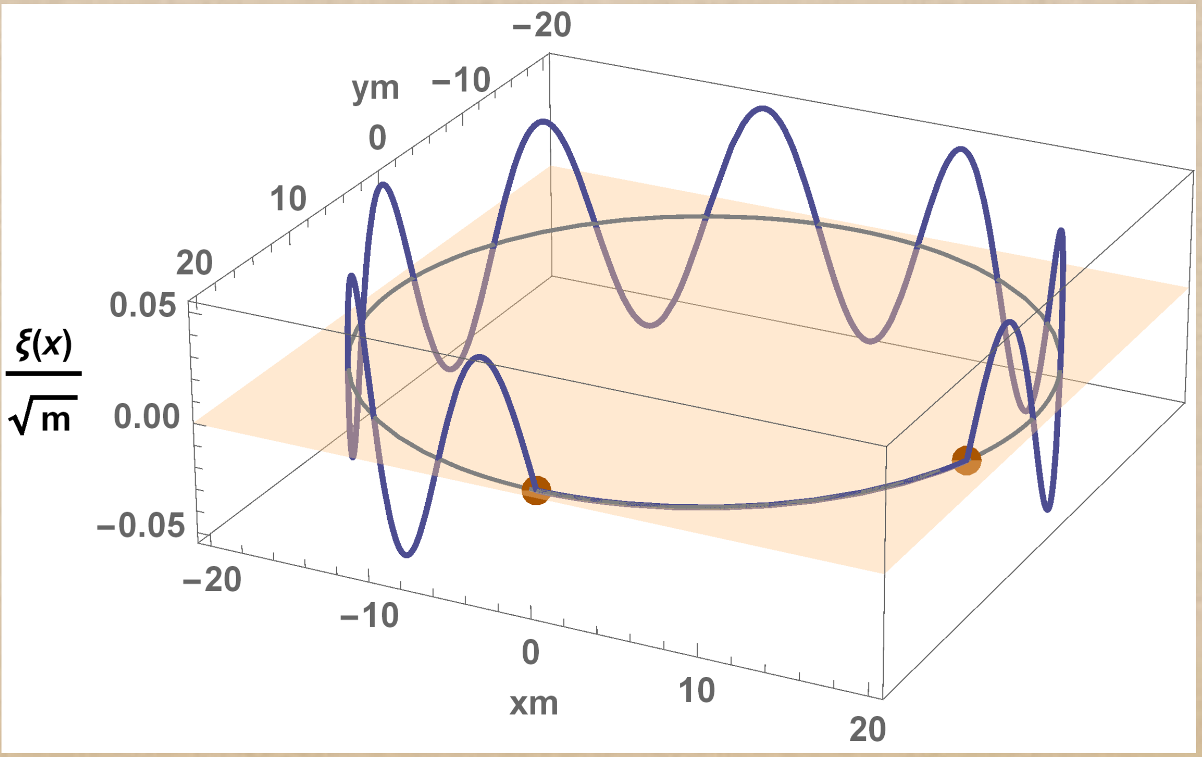
$$\left[(E-\varepsilon)\mathbb{1}-\Sigma(E)\right]\boldsymbol{a}=0$$



(qubit)



(a) $a_1 + a_2 = 0,\ |a_1|^2 + |a_2|^2 = 0.966,\ E = E_4$



(b) $a_1 - a_2 = 0,\ |a1|^2 + |a2|^2 = 0.934,\ E = \tilde{E}_{15}$

many ideas

- Calajo, Ciccarello, Chang, Rabl, PRA 2016, PRA 2017
  bound states, slow light, 1D photonic waveguide, strong coupling,
- Jaako, Xiang, Garcia-Ripoll, Rabl, PRA 2016
  ultra-strong coupling)
- Zheng, Gauthier, Baranger, PRL 2013
  photon-photon interactions
- Paulisch, Kimble, Gonzalez-Tudela, NJP 2016
  atomic degrees of freedom
- Rosario Hamann, Muller, Jerger, Zanner, Combes, Pletyukhov, Weides, Stace, Fedorov PRL 2018
  Nonreciprocity, "diod"
- Yudson: two-photons

Thank you