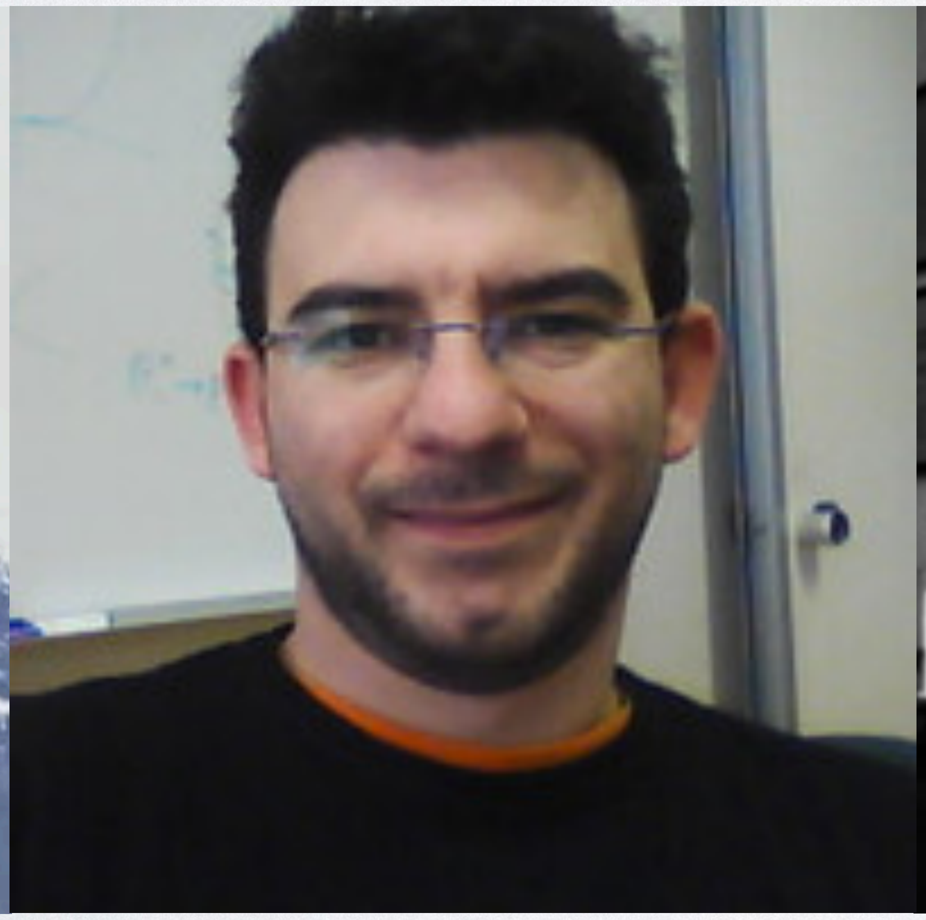
UNIVERSITÀ DEGLI STUDI DI PALERMO

DiFC

Quantum correlations in $\mathcal{PT}$-symmetric systems

Federico Roccati[1,*], Salvatore Lorenzo[1], G Massimo Palma[1,2], Gabriel T Landi[3], Matteo Brunelli[4] and Francesco Ciccarello[1,2]

Quantum Sci. Technol. **6** (2021) 025005



HERMITIAN HAMILTONIANS $\mathcal{H} = \mathcal{H}^\dagger$

$$\mathcal{H} = \begin{bmatrix} \omega_1 & g \\ g^* & \omega_2 \end{bmatrix}$$

- real eigenvalues unitary time evolutions
- orthogonal eigenstates

$$|\psi(t)\rangle \to e^{-i\varepsilon t/\hbar}|\psi(0)\rangle$$

NON HERMITIAN HAMILTONIANS $\mathcal{H} \neq \mathcal{H}^\dagger$

$$\mathcal{H} = \begin{bmatrix} \omega_1 - i\gamma_1 & g \\ g & \omega_2 - i\gamma_2 \end{bmatrix}$$

- complex eigenvalues
- non-unitary time evolutions

phenomenological description of energy linewidths, decay processes, (Weisskopf, Gamow)

$$|\psi(t)\rangle \to e^{-i(\varepsilon - i\gamma)t/\hbar}|\psi(0)\rangle$$

GAIN- LOSS PT-SYMMETRIC HAMILTONIANS

$$H = \begin{pmatrix} i\gamma & g \\ g & -i\gamma \end{pmatrix}$$

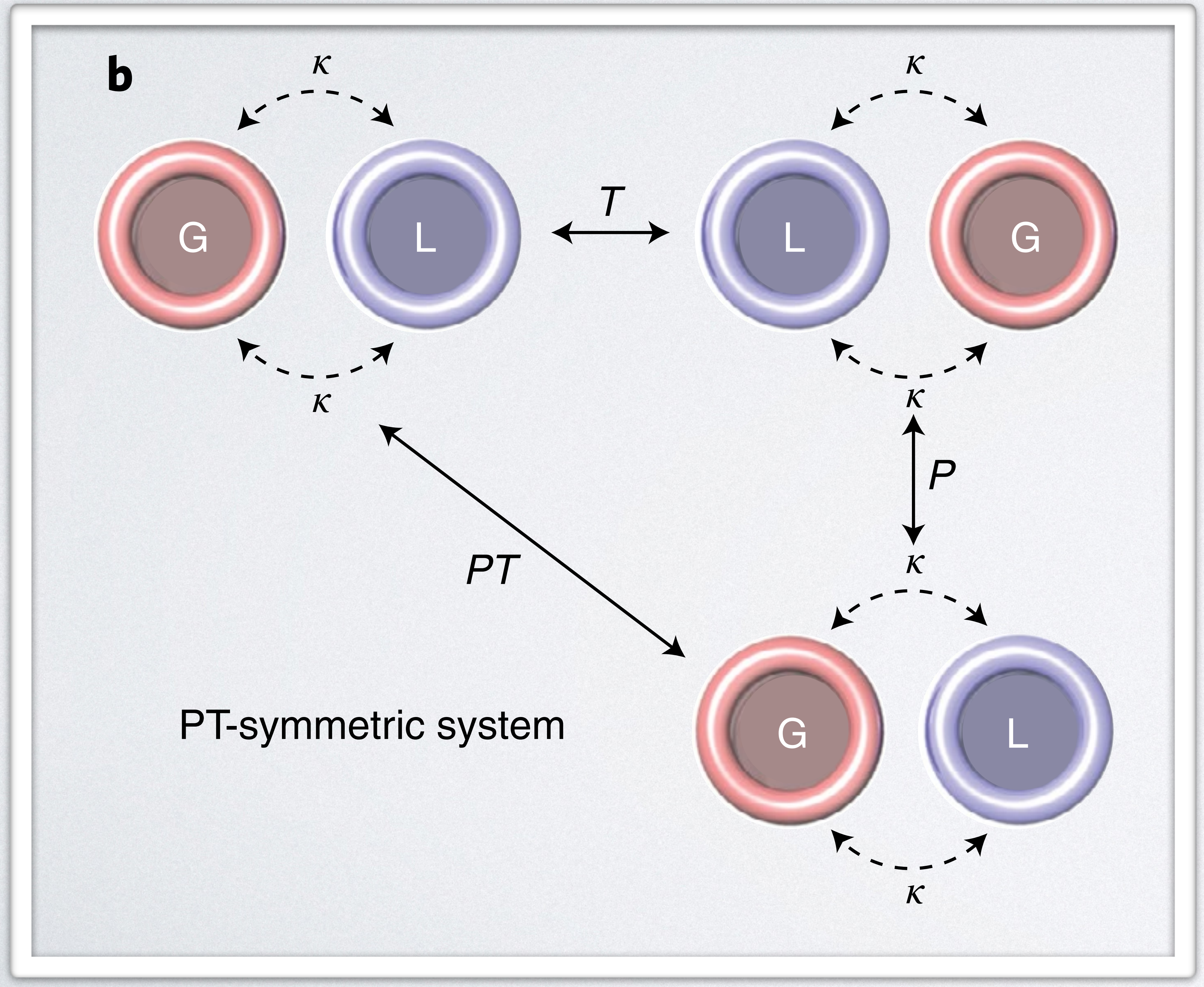
γ = gain/loss

g = excitation exchange

[PT,H] =0

P = parity

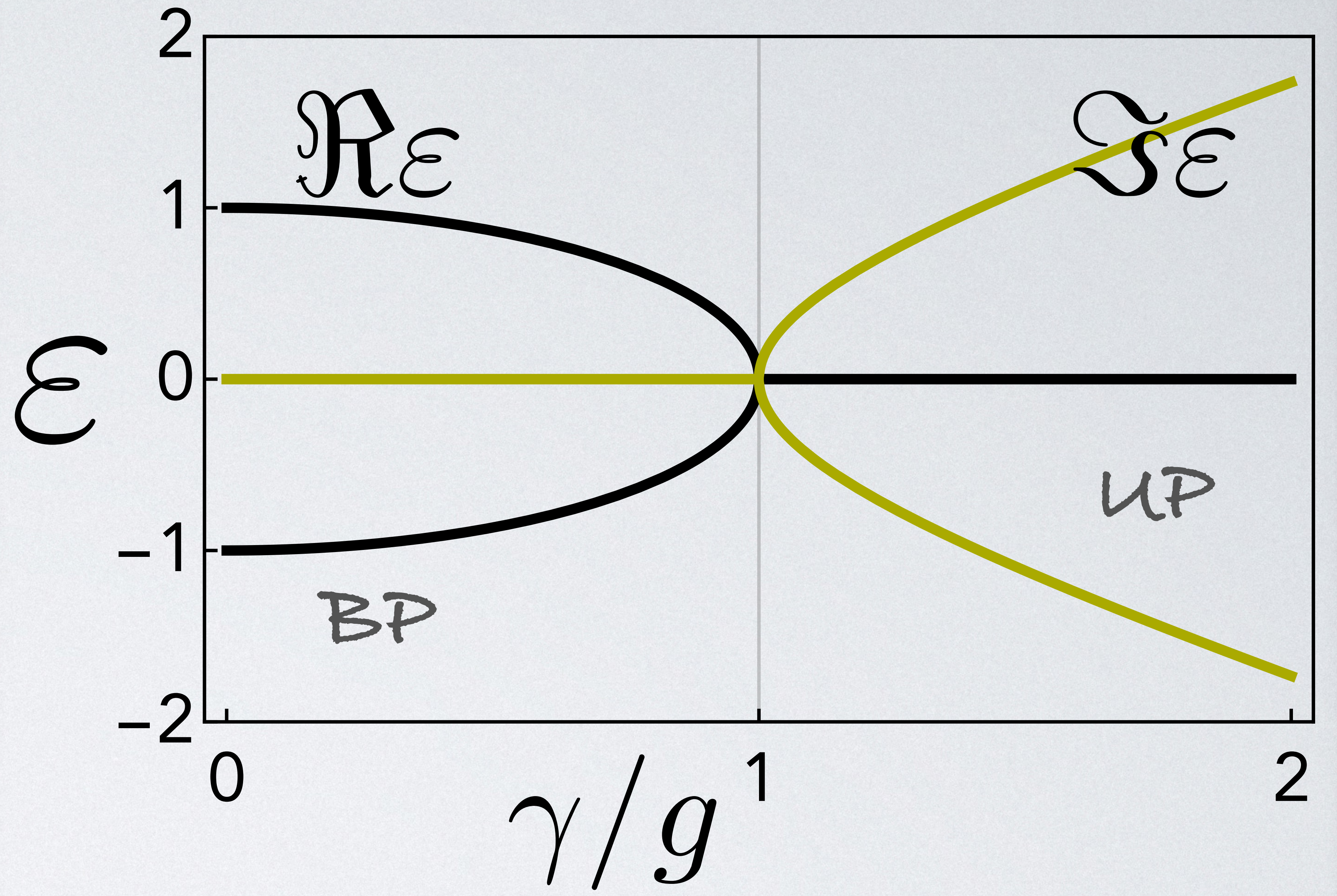
T= time reversal



$$P : i \rightarrow i,\ \hat{x} \rightarrow -\hat{x},\ \hat{p} \rightarrow -\hat{p},$$
$$T : i \rightarrow -i,\ \hat{x} \rightarrow \hat{x},\ \hat{p} \rightarrow -\hat{p}$$

EXCEPTIONAL POINTS AND BROKEN SYMMETRY

$$H = \begin{pmatrix} i\gamma & g \\ g & -i\gamma \end{pmatrix} \qquad \varepsilon = \pm\sqrt{g^2 - \gamma^2}$$



$g > \gamma$

$\mathcal{PT}$ symmetric phase

real eigenvalues

Exceptional Point (EP) $g = \gamma$

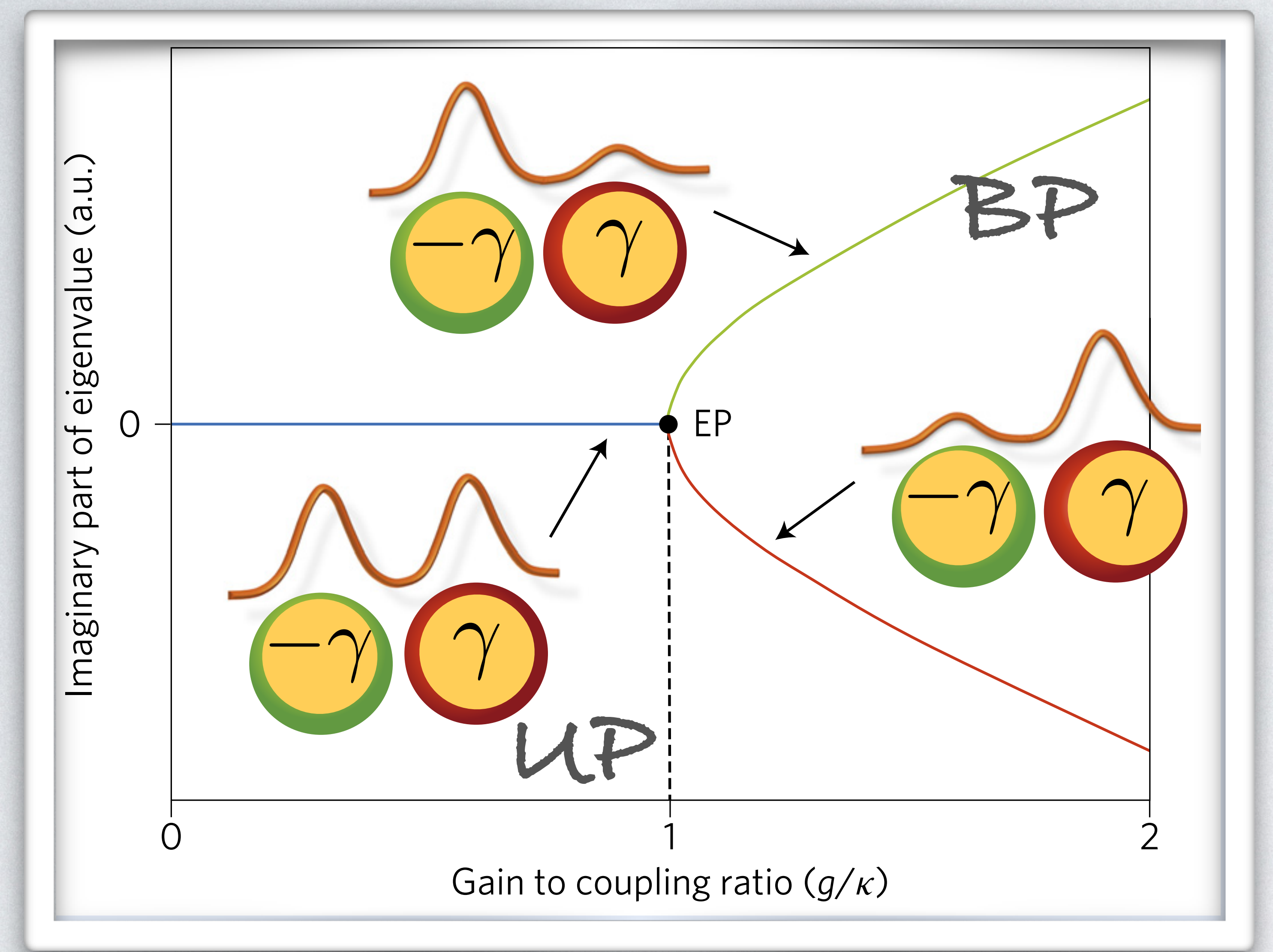
the eigenvalues AND the eigenstates coalesce

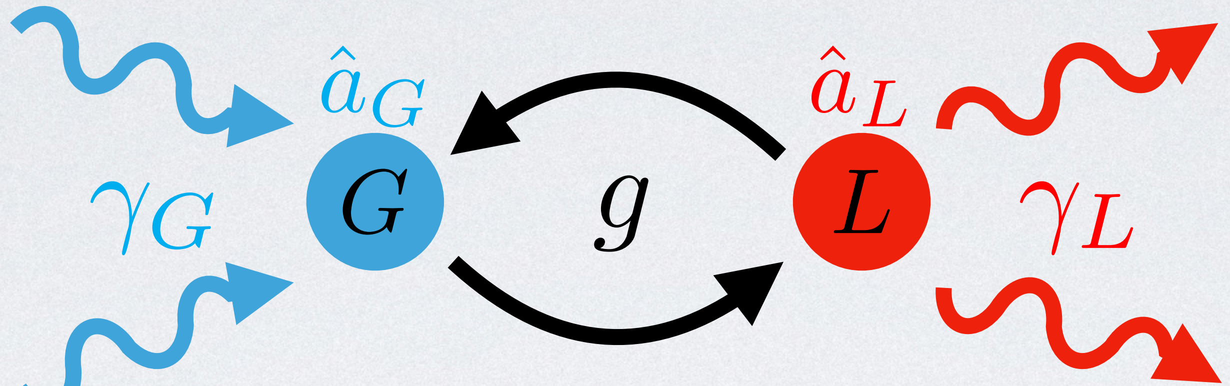
$g < \gamma$

broken phase

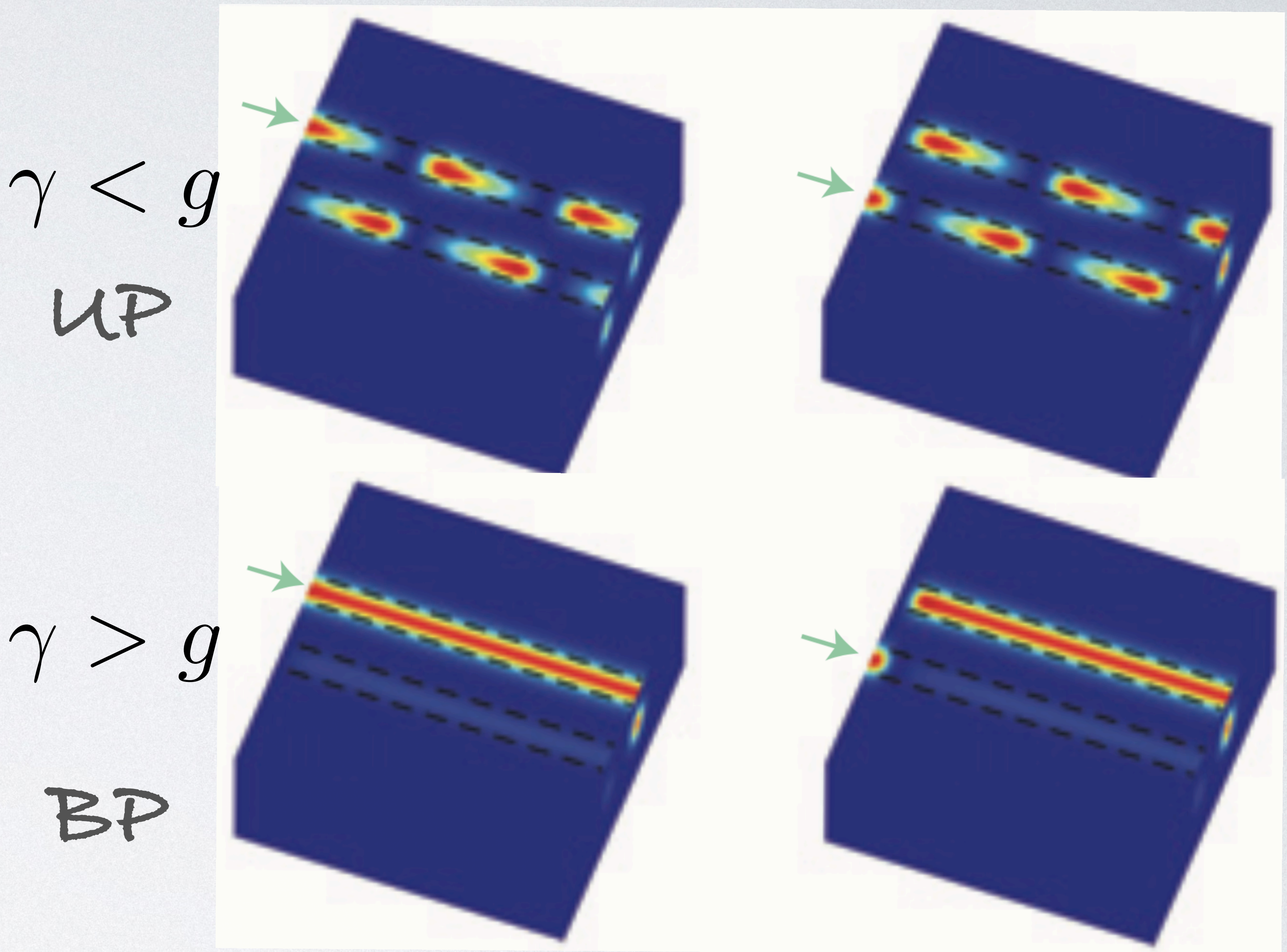
Imaginary eigenvalues

SIMULATION OF $\mathcal{PT}$ - SYMMETRIC HAMILTONIANS WITH MEAN FIELD QUANTUM OPTICS





$$\gamma_L = \gamma_G = \gamma$$




LETTERS

nature physics

PUBLISHED ONLINE: 24 JANUARY 2010 | DOI: 10.1038/NPHYS1515

Observation of parity–time symmetry in optics

Christian E. Rüter[1], Konstantinos G. Makris[2], Ramy El-Ganainy[2], Demetrios N. Christodoulides[2], Mordechai Segev[3] and Detlef Kip[1]*

the equation of motion for the average values of field operators

$$i\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\langle \hat{a}_L\rangle \\ \langle \hat{a}_G\rangle\end{pmatrix} = \begin{pmatrix}-i\gamma_L & g \\ g & i\gamma_G\end{pmatrix}\begin{pmatrix}\langle \hat{a}_L\rangle \\ \langle \hat{a}_G\rangle\end{pmatrix}$$

are PT symmetric when $\gamma_L = \gamma_G = \gamma$

WHAT ABOUT FLUCTUATIONS?

- Quantum fluctuations play a crucial role in open system dynamics e.g. in preserving commutation relations
- Quantum features of multipartite open system emerge at fluctuation level
- How are correlations in fluctuation related to the broken-unbroken phase?

SPOILER (TAKE HOME MESSAGE)

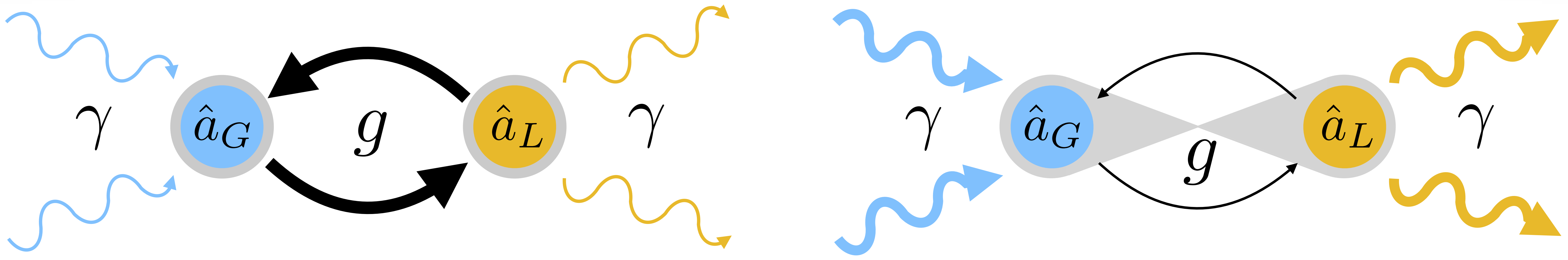
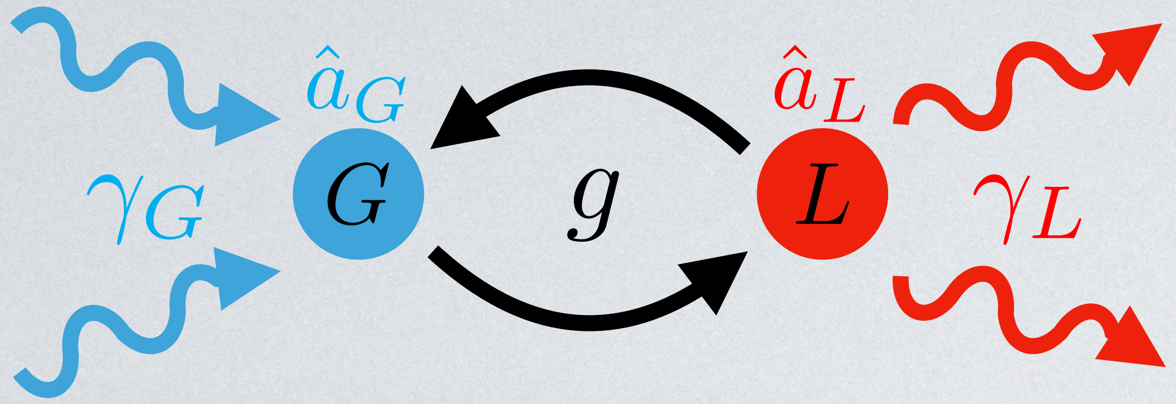


Figure 1. A pair of quantum oscillators G and L undergo a coherent exchange energy with rate g. Additionally, mode G (L) is subject to a local gain (loss) with rate γ. The mean-field dynamics is described by a $\mathcal{PT}$-symmetric Hamiltonian. (Left): when $\mathcal{PT}$ symmetry is preserved ($g > \gamma$), if each mode starts in a coherent state (zero correlations), after some time they will share only classical correlations. (Right): $\mathcal{PT}$ symmetry breaking ($g < \gamma$) is instead accompanied by stationary quantum correlations.



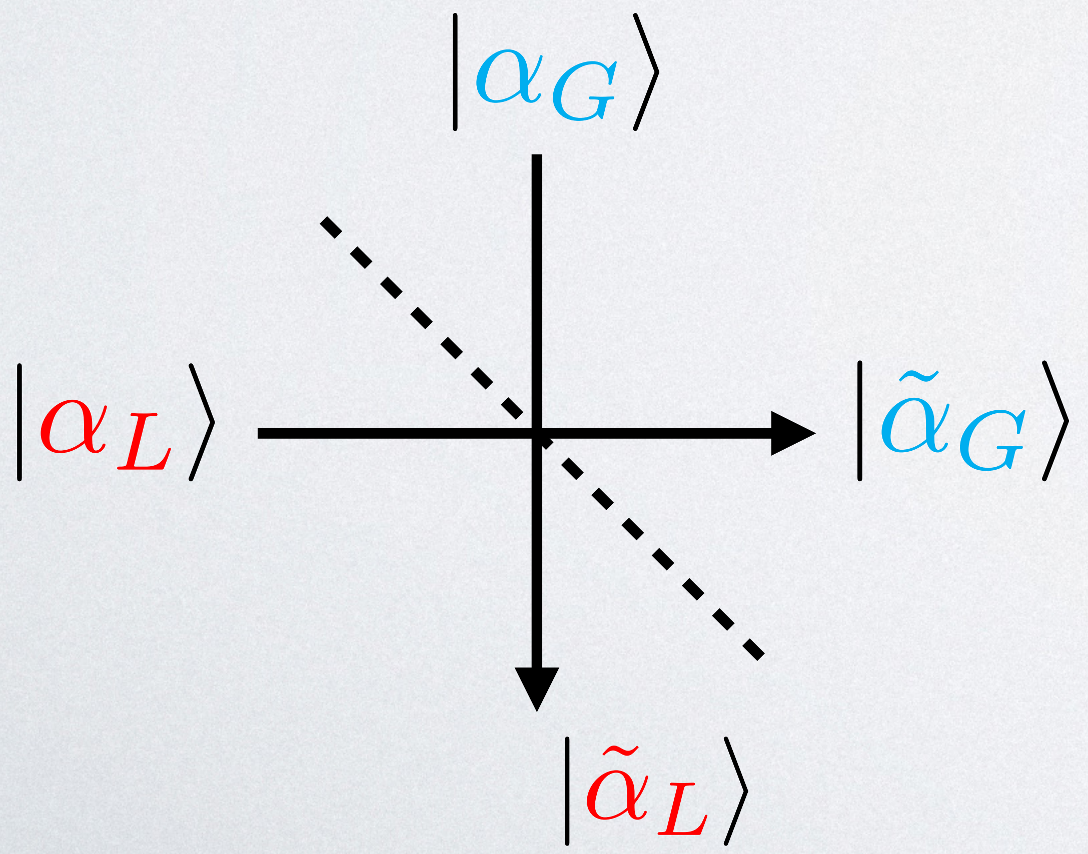
BEYOND MEAN FIELD: THE MASTER EQUATION

$$\dot{\rho} = -i[H,\rho] + 2\,\gamma_L \left(\hat{a}_L\,\rho\,\hat{a}_L^\dagger - \tfrac{1}{2}\{\hat{a}_L^\dagger\hat{a}_L,\rho\}\right) + 2\,\gamma_G \left(\hat{a}_G^\dagger\,\rho\,\hat{a}_G - \tfrac{1}{2}\{\hat{a}_G\hat{a}_G^\dagger,\rho\}\right)$$

$$H = g\,(\hat{a}_L^\dagger\hat{a}_G + \hat{a}_L\hat{a}_G^\dagger)\,.$$

$\gamma_L = \gamma_G \equiv \gamma$ PT line

$\rho(0) = |\alpha_G\rangle\langle\alpha_G| \otimes |\alpha_L\rangle\langle\alpha_L|$



- the m.eq. preserves gaussian states (the covariance matrix fully characterises the fluctuations dynamics)
- the beam splitter - like hamiltonian term does not create entanglement for coherent input states
- the gain transforms pure coherent input states into mixtures
- the average energy is not bounded

CORRELATION MEASURES

mutual information

$$\mathcal{I} = S_G + S_L - S,$$

quantum discord

$$\mathcal{D}_{LG} = S_G - S + \min_{\hat{G}_k} \sum_k p_k S(\rho_{L|k}),$$

$$\rho_{L|k} = \mathrm{Tr}_G(\hat{G}_k \rho)/p_k$$

In general (and in our case in particular)

$$\mathcal{D}_{LG} \neq \mathcal{D}_{GL},$$

We replace the VonNeuman Entropy with the Rény -2 Entropy

$$S(\varrho) = -\log \mathrm{Tr}(\varrho^2)$$

For Gaussian states, and Rény 2- Entropy (optimal measurement being phase insensitive)

$$\mathcal{D}_{LG} = \log\left(1 + \frac{e^{\mathcal{I}} - 1}{e^{S_G} + 1}\right), \qquad \mathcal{D}_{GL} = \log\left(1 + \frac{e^{\mathcal{I}} - 1}{e^{S_L} + 1}\right)$$

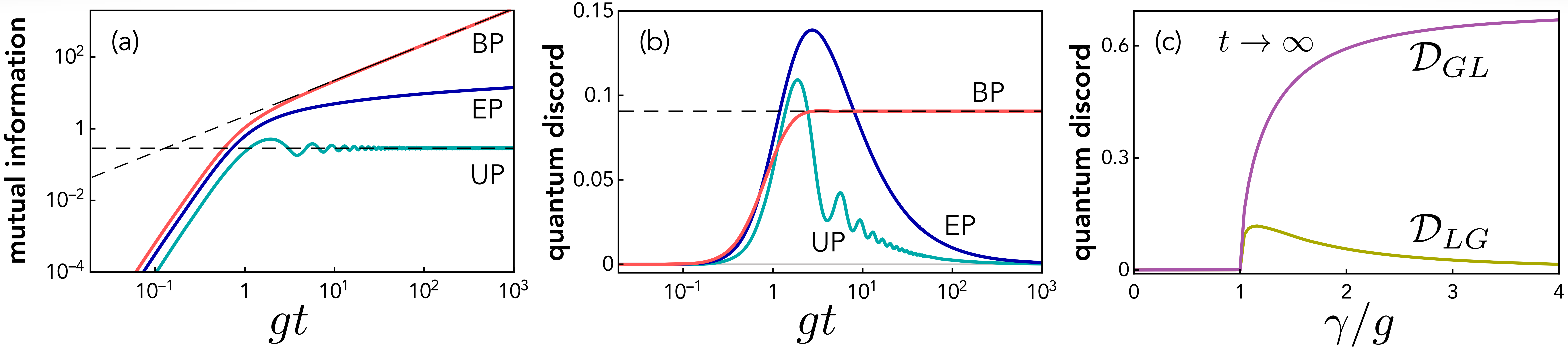


Figure 2. Evolution of total and quantum correlations on the $\mathcal{PT}$ line ($\gamma_L = \gamma_G = \gamma$). This comprises the UP $\gamma < g$, the EP $\gamma = g$ and the BP for $\gamma > g$. (a) and (b): mutual information $\mathcal{I}$ (a) and discord $\mathcal{D}_{LG}$ (b) for $\gamma = g/2$ (UP, green), $\gamma = 3g/2$ (BP, red) and $\gamma = g$ (EP, blue). A qualitatively analogous behavior is exhibited by $\mathcal{D}_{GL}$. (c): asymptotic value of discord, $\mathcal{D}_{LG}(\infty)$ (yellow) and $\mathcal{D}_{GL}(\infty)$ (purple). See appendix.

UP $\mathcal{I} \approx \log(\frac{g^2}{g^2-\gamma^2}),$

$$\mathcal{D}_{LG}, \mathcal{D}_{GL} \approx \frac{\gamma}{2g^2 t}$$

Purely classical Asymptotic correlations

BP $\mathcal{I} \approx 2\Omega\, t$, with $\Omega = \sqrt{\gamma^2 - g^2}$

$$\mathcal{D}_{GL} \approx \log\left(\frac{\gamma(3\gamma+\Omega)-g^2}{2\gamma^2}\right)$$

$$\mathcal{D}_{LG} \approx \log\left(\frac{\gamma(\gamma+\Omega)+g^2}{2\gamma^2}\right)$$

Asymptotic Q correlations in the presence of noise

EP $\mathcal{I} \approx \log(\frac{4g^2}{3}t^2)$

$$\mathcal{D}_{LG}, \mathcal{D}_{GL} \approx \frac{1}{gt}$$

Diverging asymptotic Classical correlations

Table 1. Asymptotic behavior of S and $S_{L(G)}$ on the $\mathcal{PT}$ line.

$\mathcal{PT}$ line	UP	EP	BP
S	$\log\left(\frac{4\gamma^2 g^2}{g^2-\gamma^2}t^2\right)$	$\log\left(\frac{4g^4}{3}t^4\right)$	$2\Omega t + \log\left(\frac{\gamma^3(\gamma+\Omega)}{\Omega^4}\right)$
S_L	$\log\left(\frac{2\gamma g^2}{g^2-\gamma^2}t\right)$	$\log\left(\frac{4g^3}{3}t^3\right)$	$2\Omega t + \log\left(\frac{\gamma g^2}{2\Omega^3}\right)$
S_G			$2\Omega t + \log\left(\frac{\gamma(\gamma+\Omega)^2}{2\Omega^3}\right)$

In the long time limit

$$\mathcal{D}_{LG} \approx \log\left(1 + \mathrm{e}^{-(S-S_L)} - \mathrm{e}^{-S_G}\right),$$

The asymptotic survival of Quantum Correlations depends only on $S - S_L$

CONCLUSIONS

- We have considered the fluctuations dynamics of two harmonic oscillators whose mean field dynamics simulates a gain-loss $\mathcal{PT}$ - symmetric hamiltonian
- In the "Unbroken phase" the long time correlations are purely classical
- In the "Broken phase" long term quantum correlation survive in the presence of noise