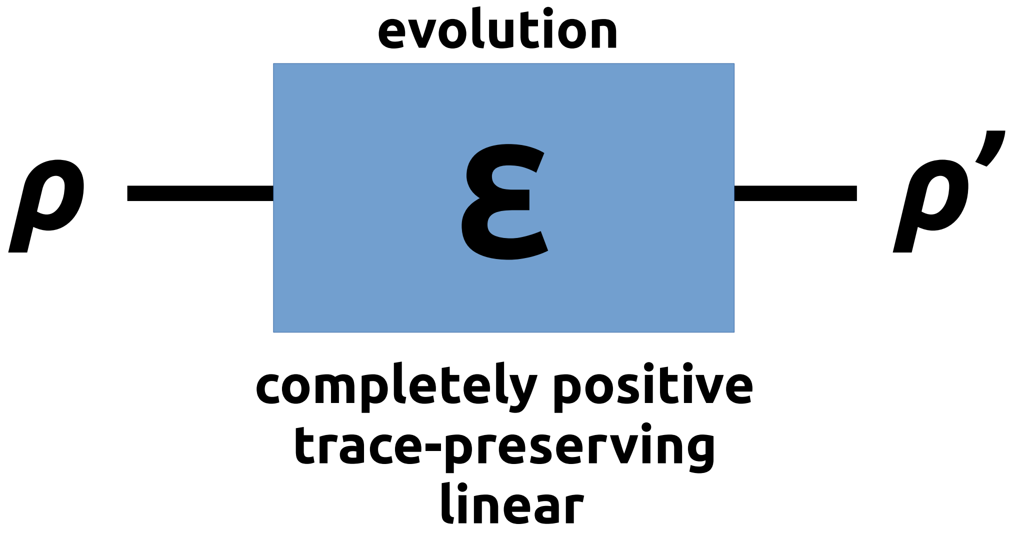
Probabilistic storing
of quantum dynamics

Mario Ziman

MESSAGE FROM STINESPRING



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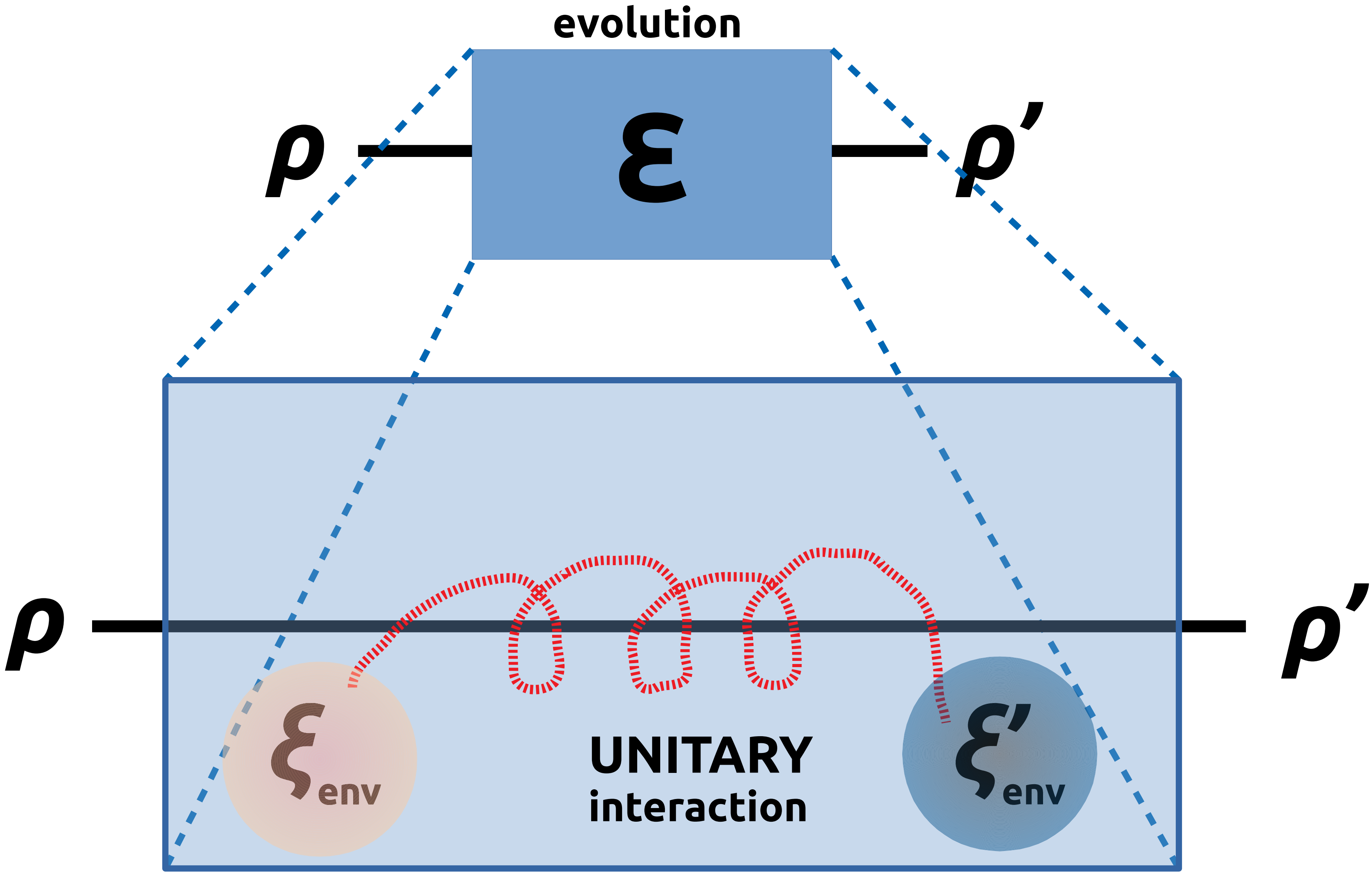

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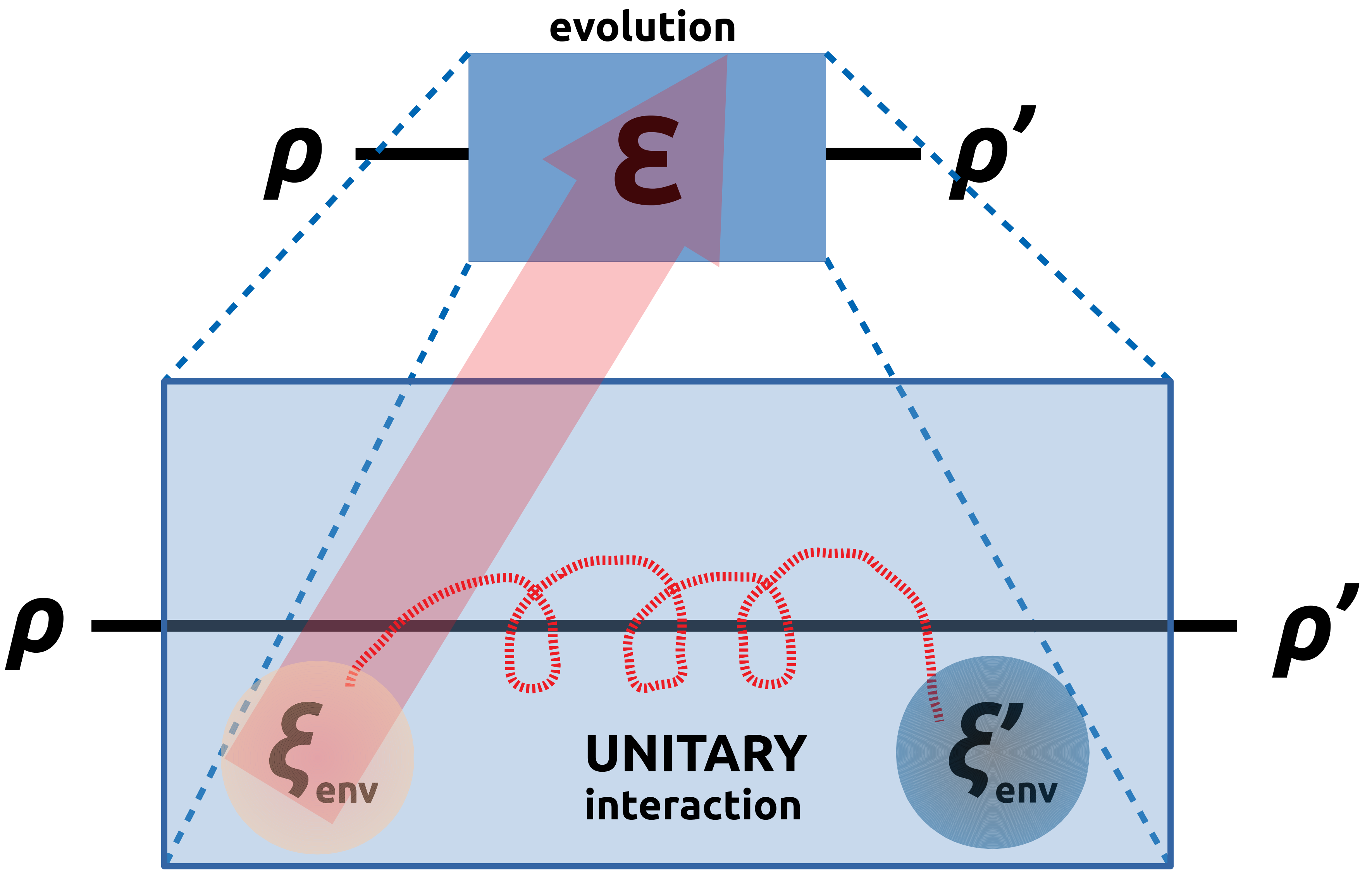

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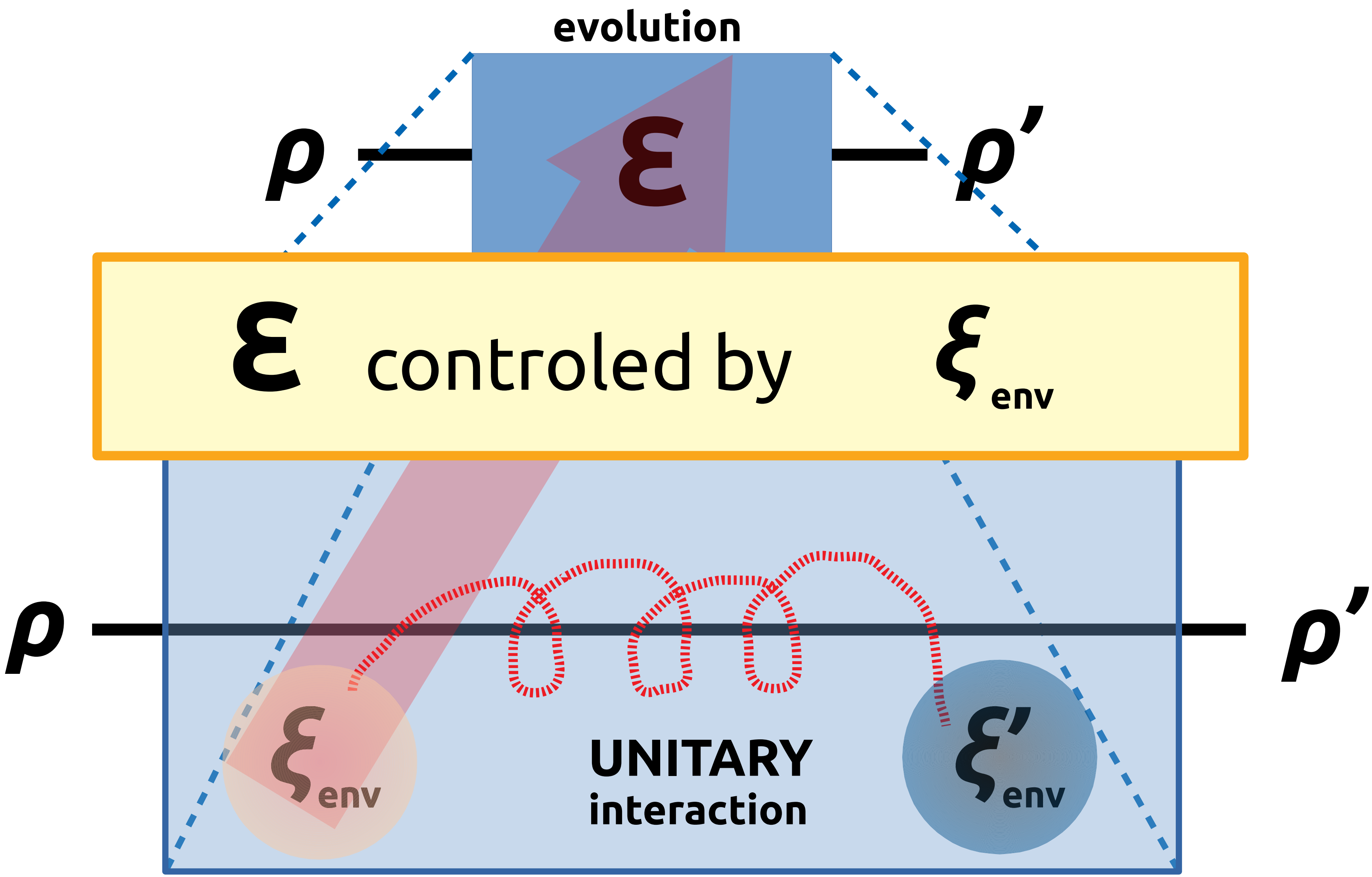

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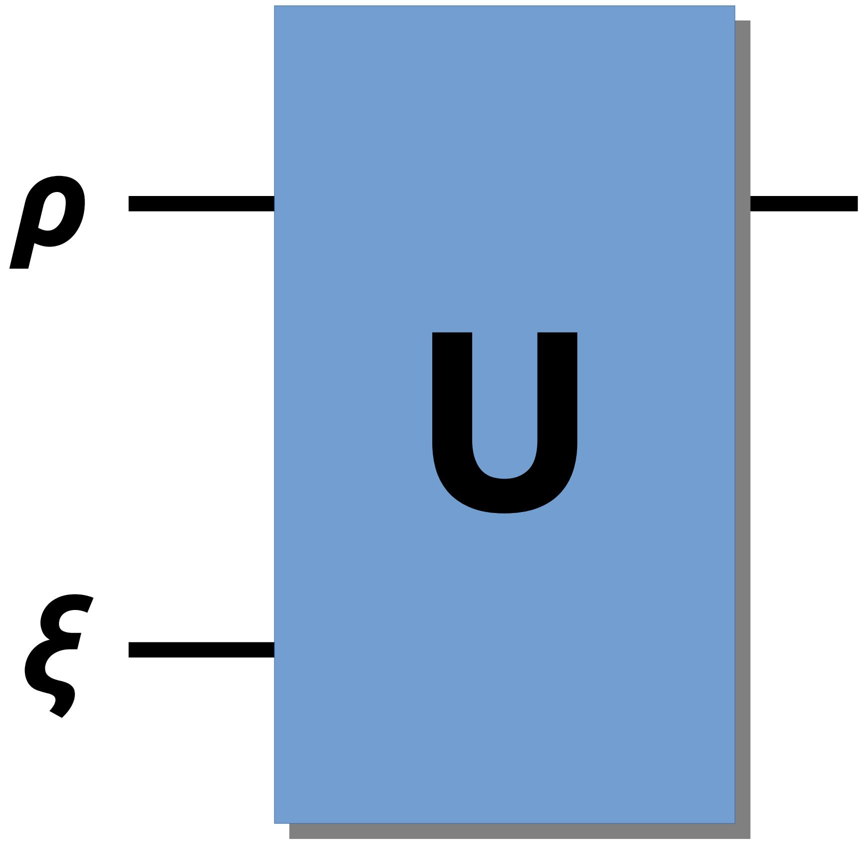
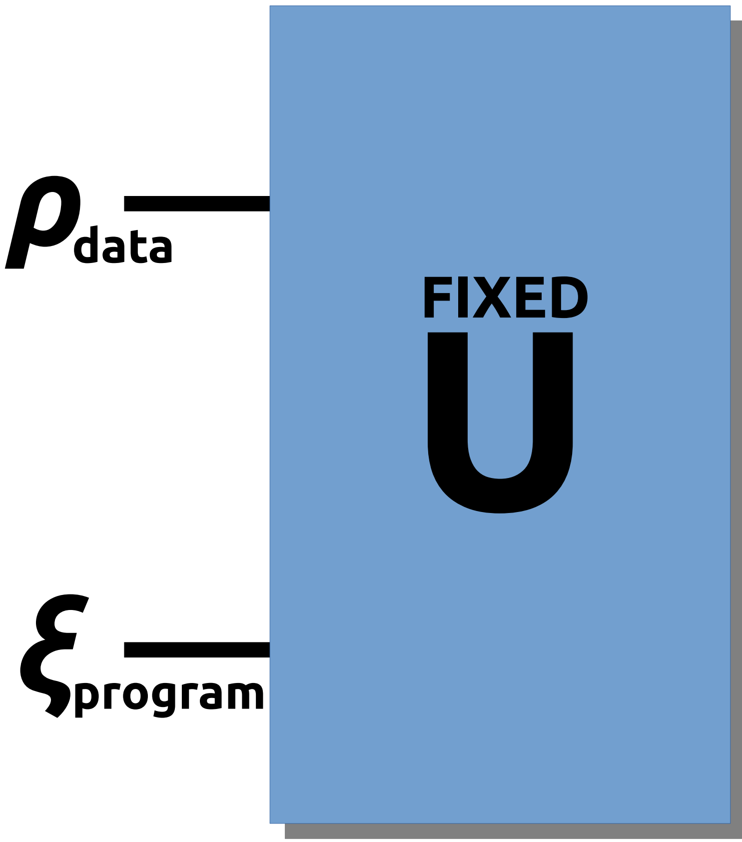


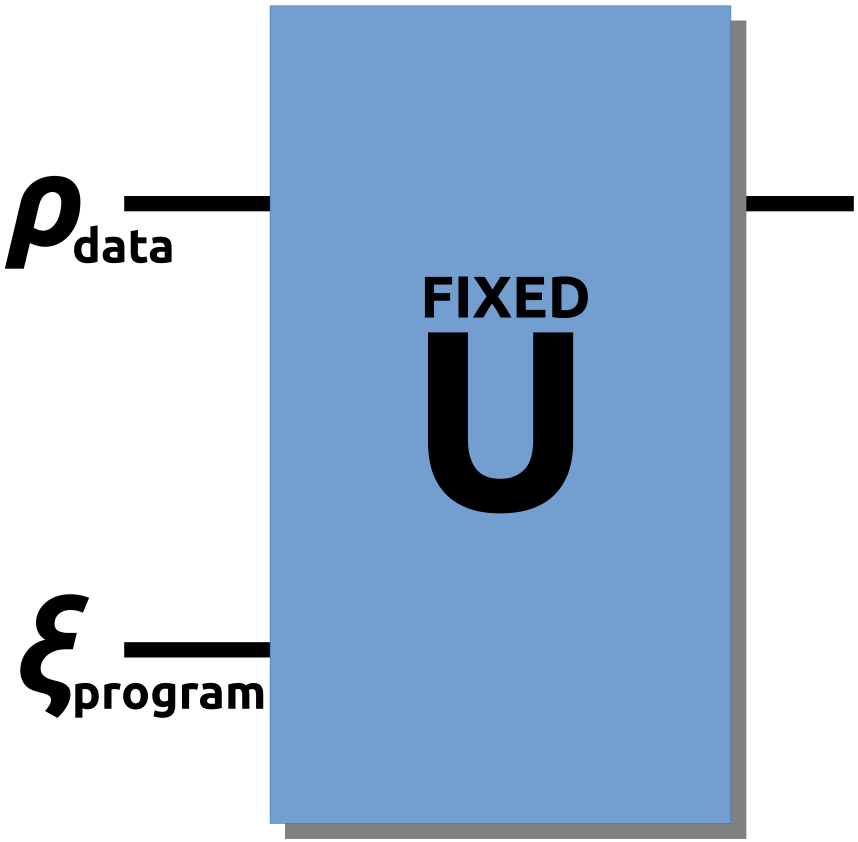
$$\rho' = \mathcal{E}_{\xi}(\rho)$$

PARADIGM OF QUANTUM PROGRAMMING



$$\rho' = \mathcal{E}_{\xi}(\rho)$$

PARADIGM OF QUANTUM PROGRAMMING



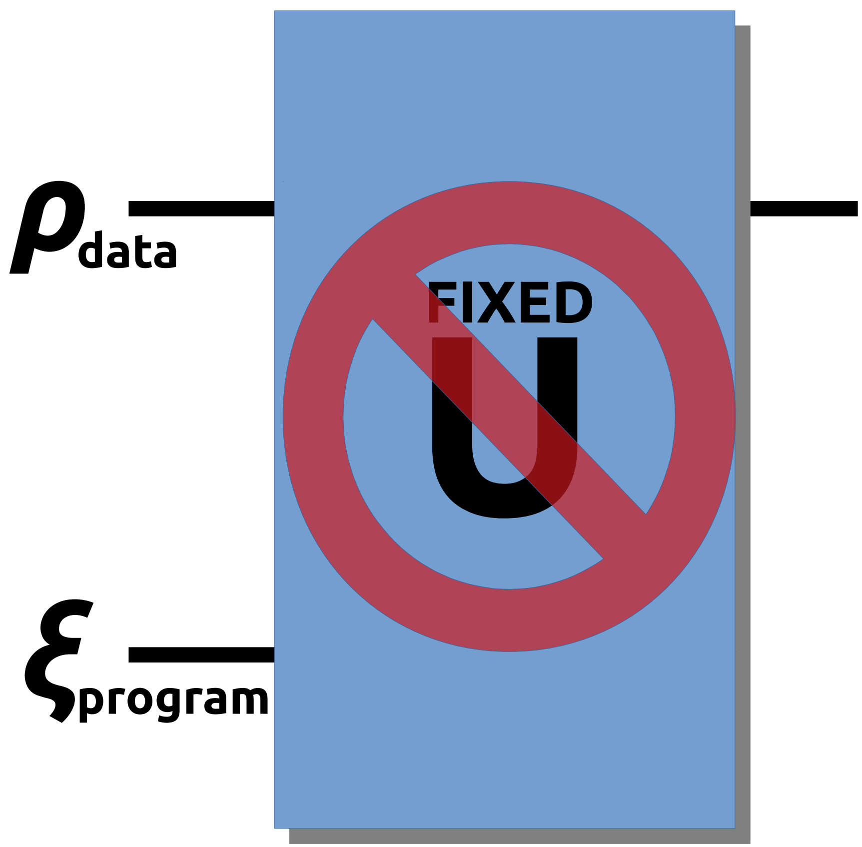
$$\rho' = \mathcal{E}_{\xi}(\rho)$$

QUESTION

quantum program

compatibility

PARADIGM OF QUANTUM PROGRAMMING



$$\rho' = \mathcal{E}_{\xi}(\rho)$$

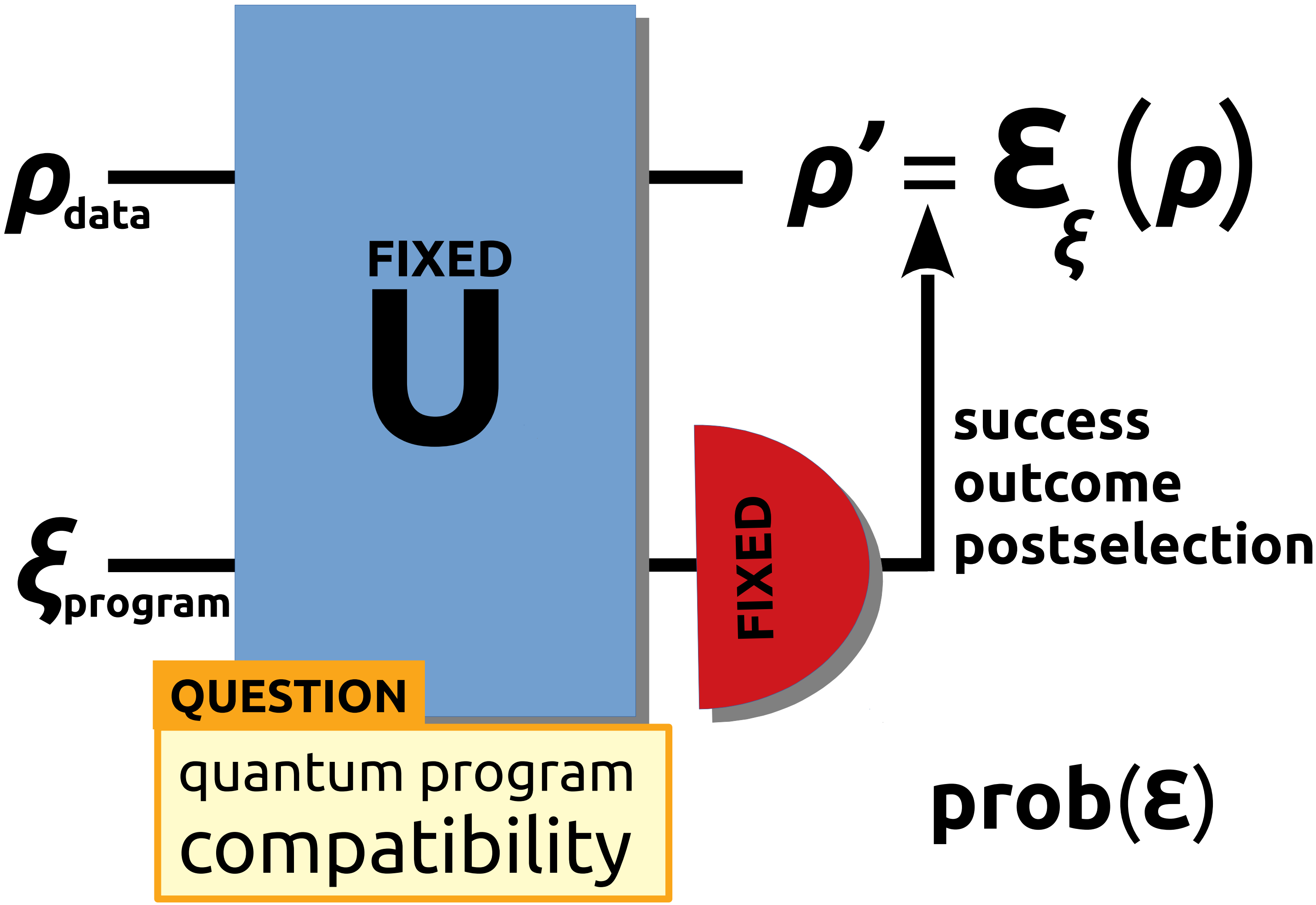
QUESTION

quantum program
compatibility

NO-PROGRAMMING THEOREM

M. Nielsen, I. Chuang, Phys. Rev. Lett. 79, 321–324 (1997)

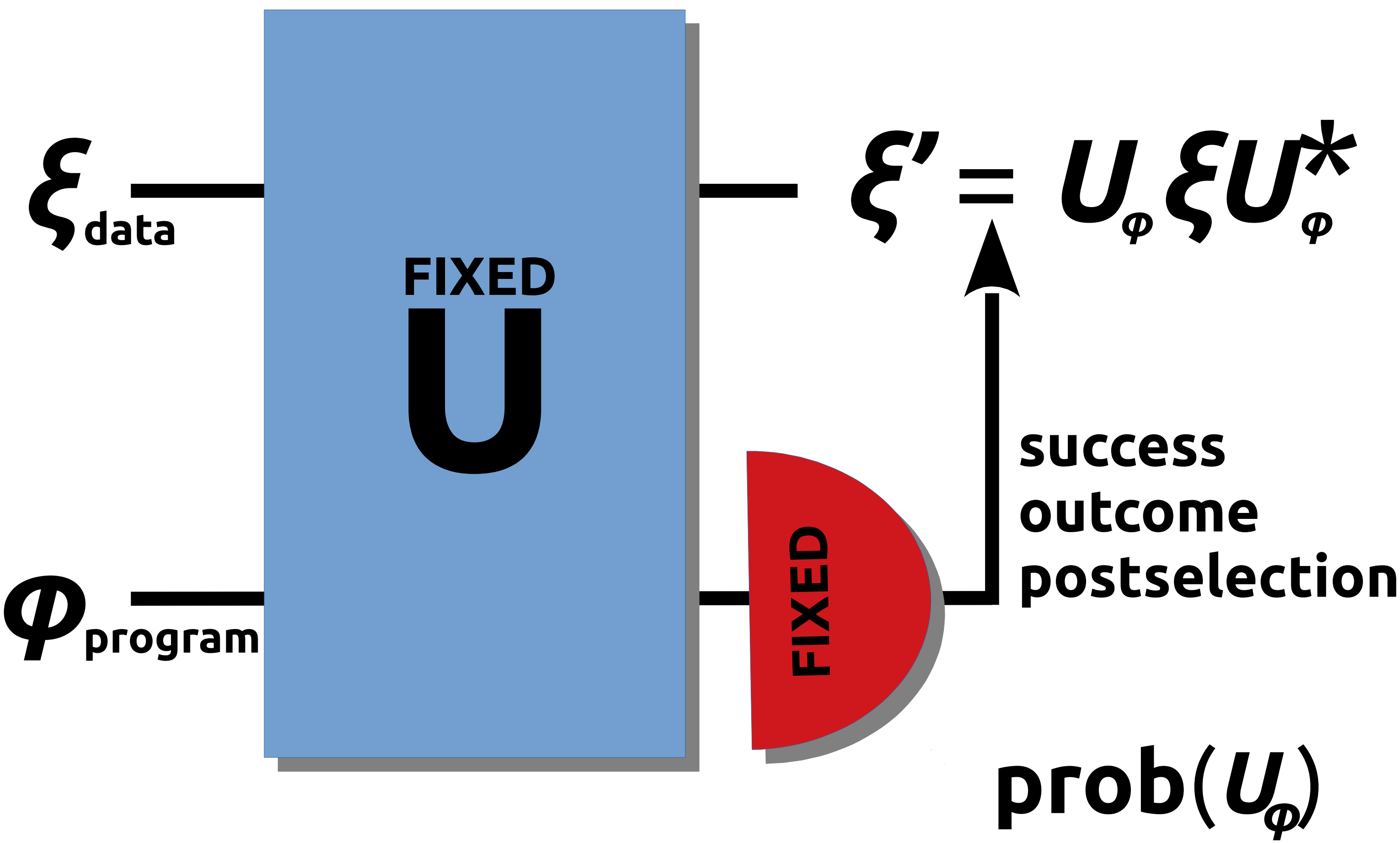
HERALDED UNIVERSALITY



M. Nielsen, I. Chuang, Phys. Rev. Lett. 79, 321–324 (1997)

PROGRAMMING THE PHASE

$$U_\varphi = |0\rangle\langle 0| + e^{i\varphi}|1\rangle\langle 1|$$

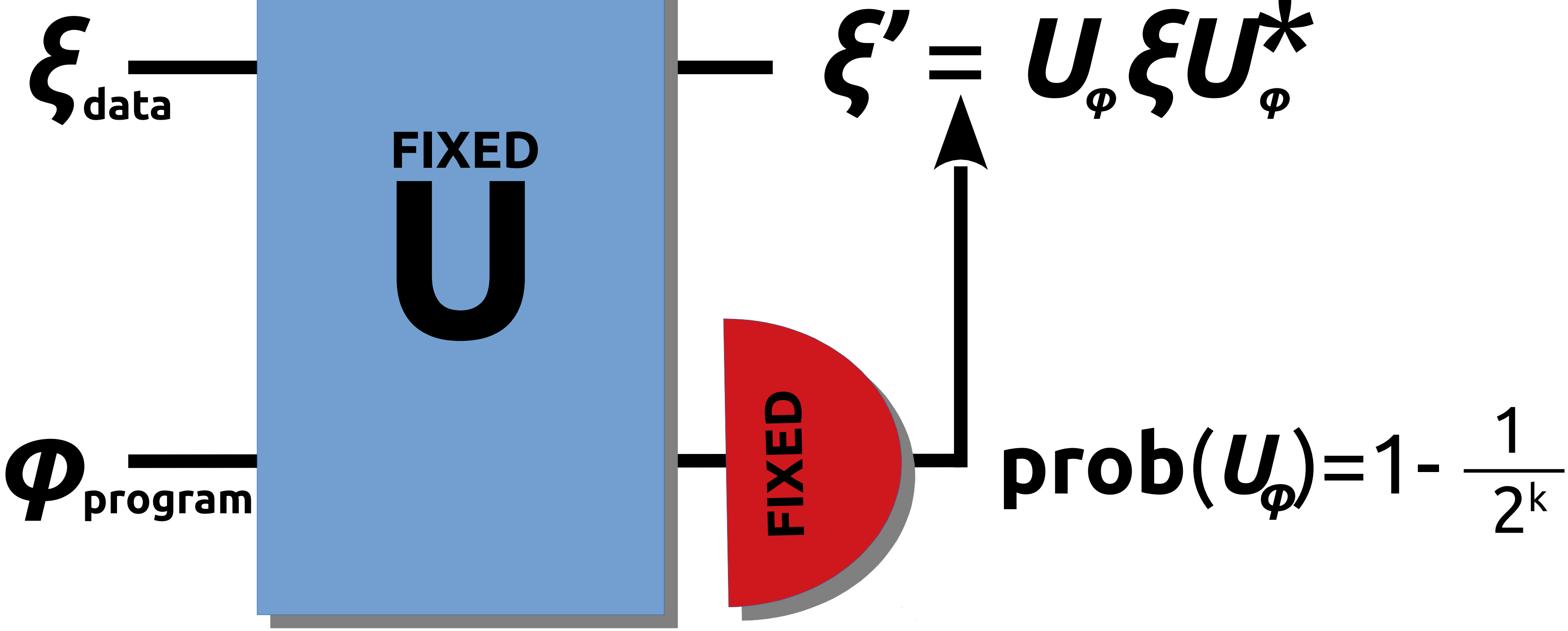


optimize $dim(H_{program})$ and success probability

G. Vidal, L. Masanes, J.I. Cirac, Phys. Rev. Lett. 88, 047905 (2002)

PROGRAMMING THE PHASE

$$U_\varphi = |0\rangle\langle 0| + e^{i\varphi}|1\rangle\langle 1|$$

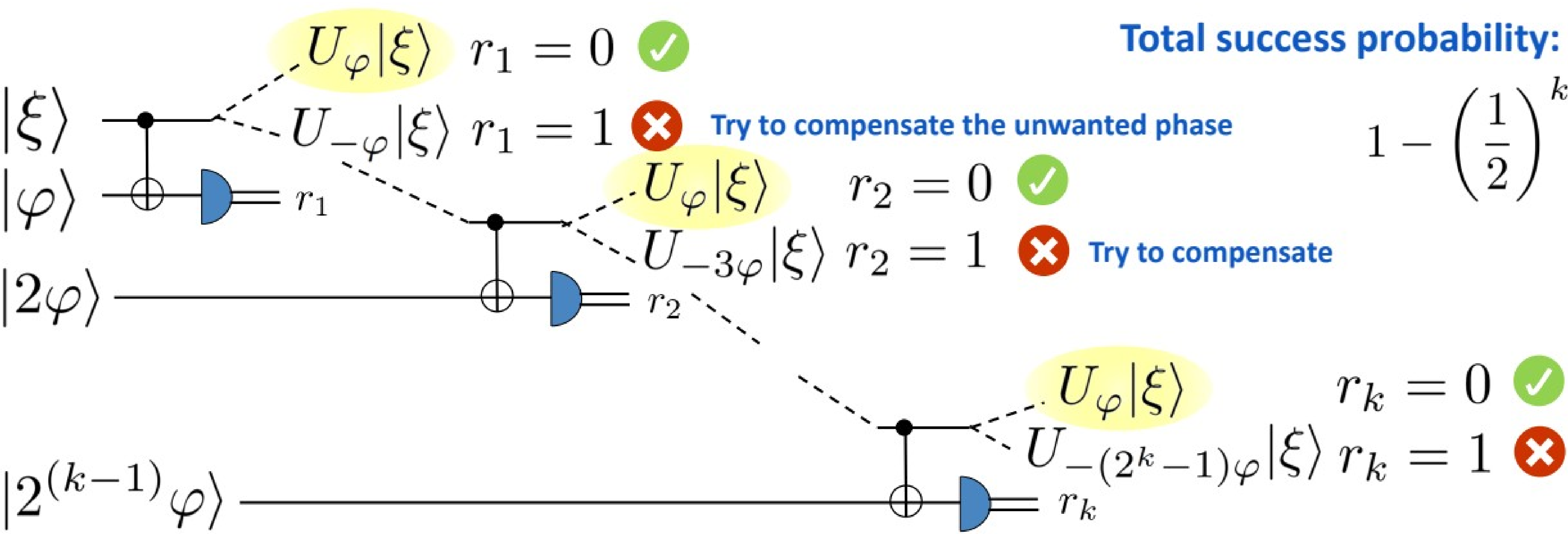


$$\Phi_{program} = |\varphi\rangle|2\varphi\rangle\ldots|2^{(k-1)}\varphi\rangle$$

$$|\varphi\rangle = |0\rangle + e^{i\varphi}|1\rangle$$

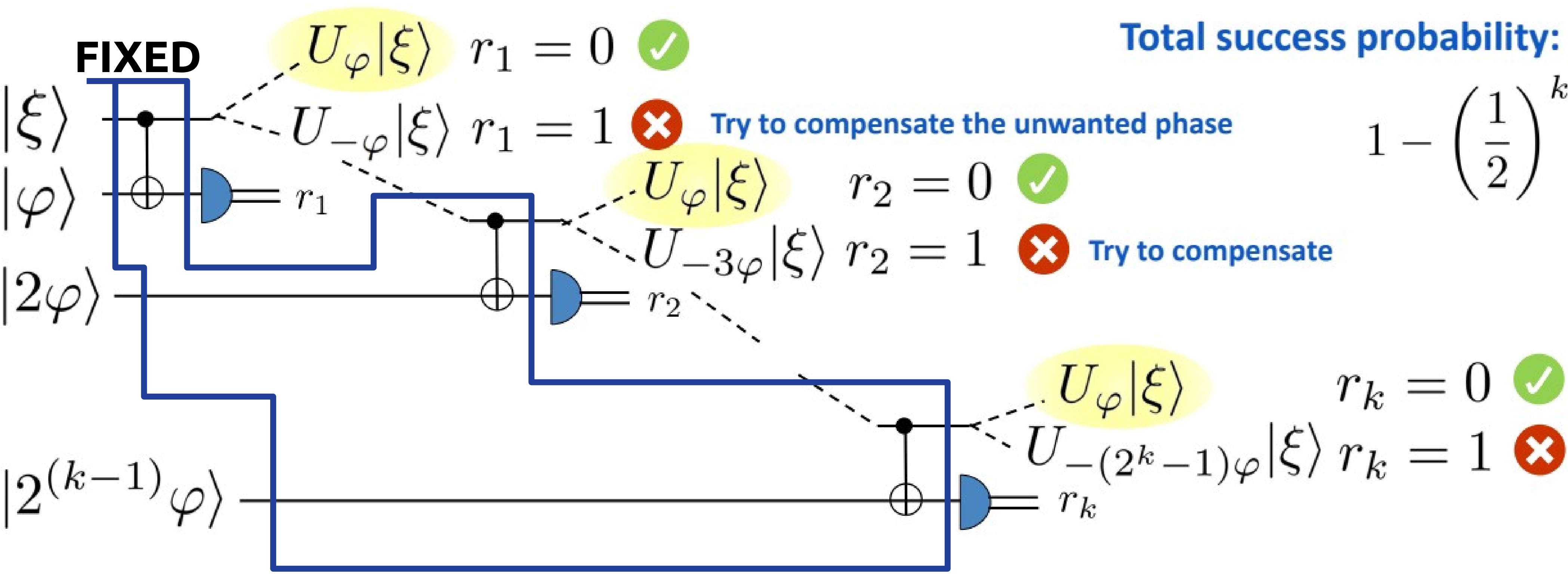
G. Vidal, L. Masanes, J.I. Cirac, Phys. Rev. Lett. 88, 047905 (2002)

PROGRAMMING THE PHASE



G. Vidal, L. Masanes, J.I. Cirac, Phys. Rev. Lett. 88, 047905 (2002)

PROGRAMMING THE PHASE



phase stored in

$$\Phi_{program} = |\varphi\rangle|2\varphi\rangle \ldots |2^{(k-1)}\varphi\rangle$$

G. Vidal, L. Masanes, J.I. Cirac, Phys. Rev. Lett. 88, 047905 (2002)

INVERSE QUESTION

dynamics stored in $\Phi_{program}$

HOW TO

STORE / LEARN

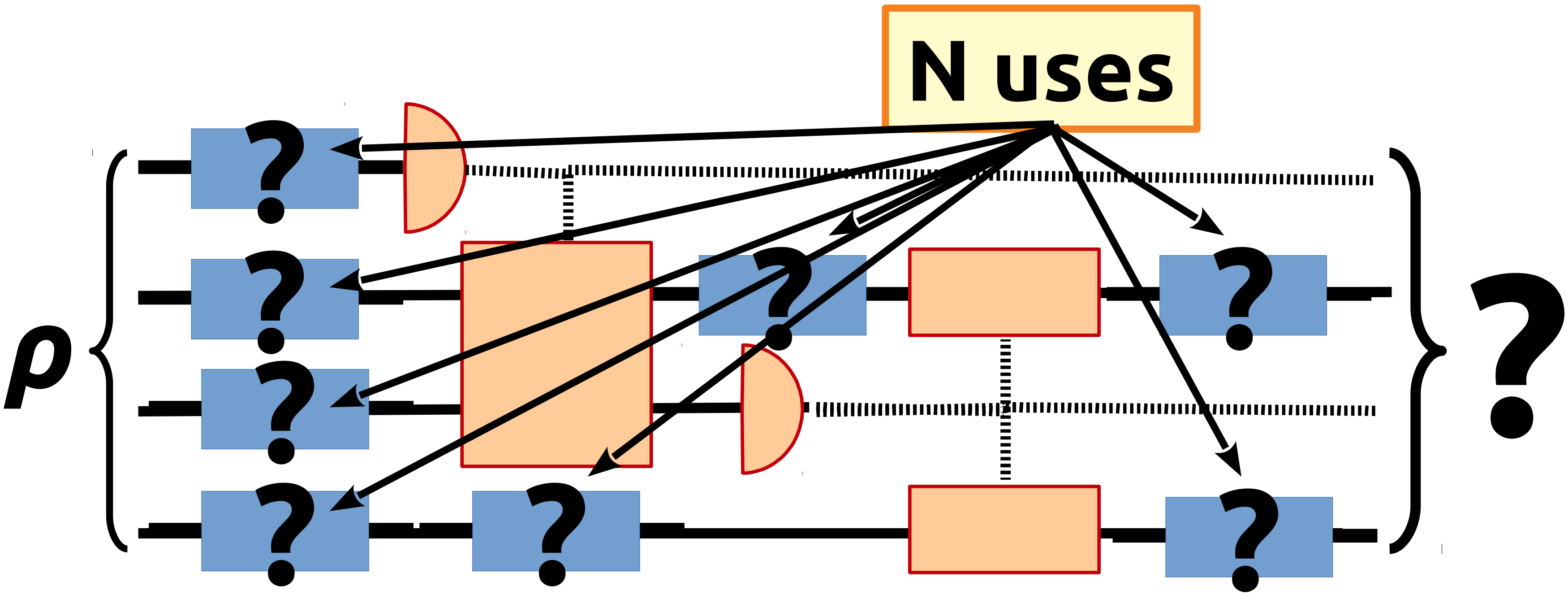
DYNAMICS?

INVERSE QUESTION

dynamics stored in $\Phi_{program}$

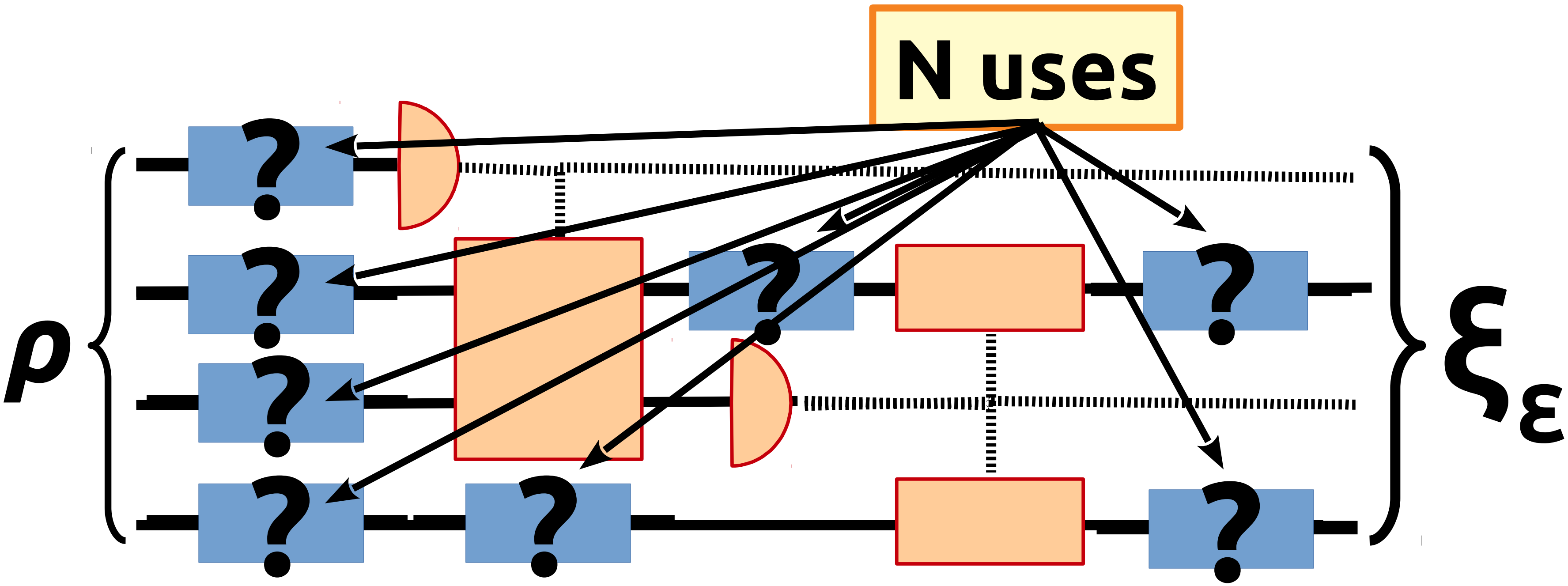
HOW TO
PROBABILISTICALLY
STORE / LEARN
UNKNOWN
DYNAMICS?

LEARNING THE PROCESS



result of learning
→ a program simulating the process
→ stored in ~~bits~~ ***qubits***

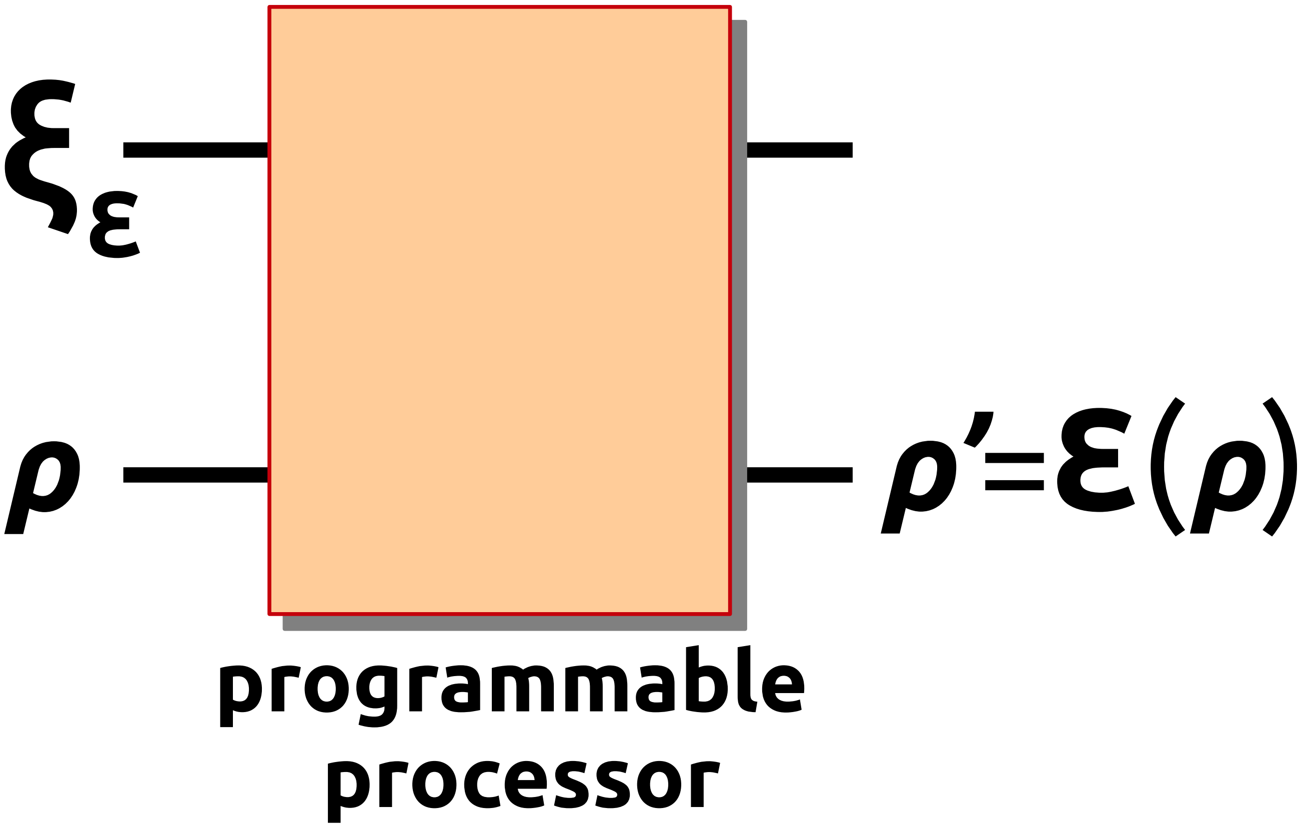
LEARNING THE PROCESS



Storing in quantum memory

$$\varepsilon \rightarrow \xi_\varepsilon$$

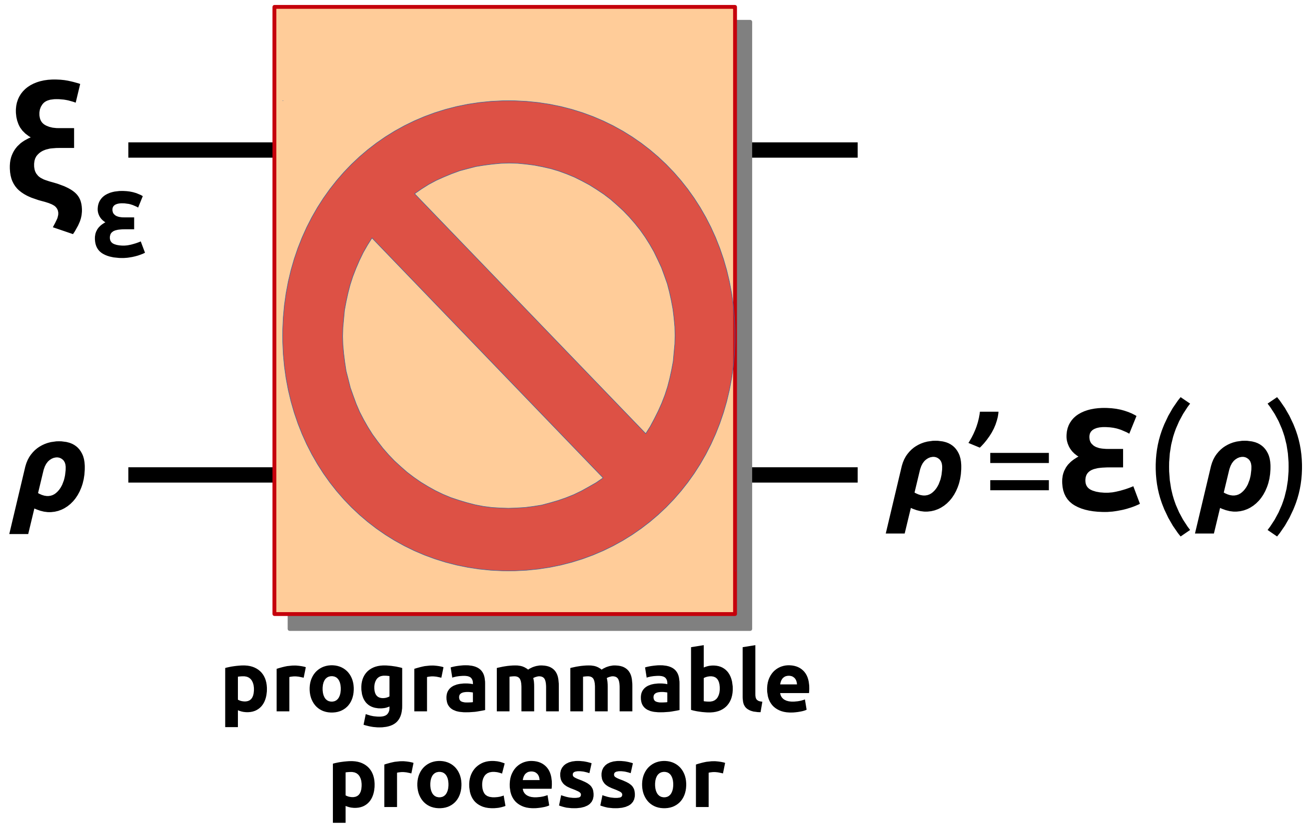
UNDONE LEARNING



N → 1 RETRIEVAL

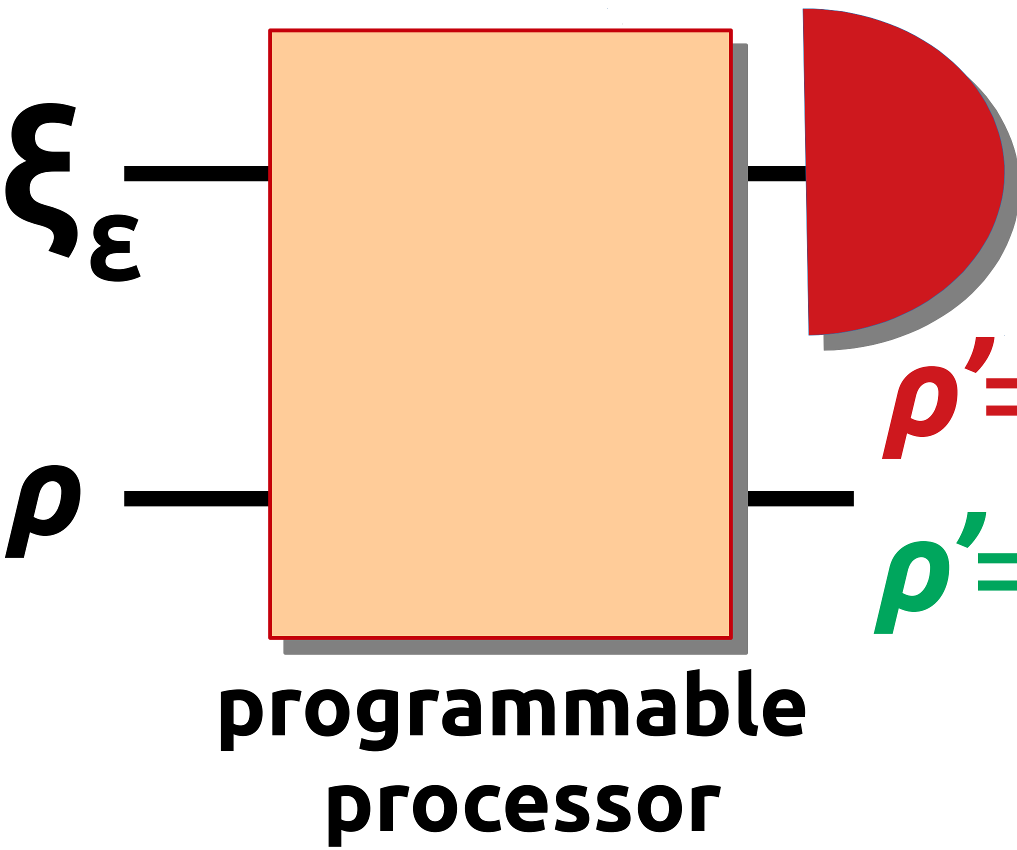
UNDONE LEARNING

NO PROGRAMMING



N → 1 RETRIEVAL

UNDONE LEARNING PROBABILISTICALLY
APPROXIMATIVELY



$\rho' = \mathcal{E}(\rho)$ prob($\mathcal{E}$)

$\rho' = \mathcal{E}'(\rho)$ dist($\mathcal{E}'$,$\mathcal{E}$)$\leq\varepsilon$

? N → 1 RETRIEVAL ?

QUANTUM LEARNING

Storing process in quantum memory.

N uses

$$\mathcal{E} \rightarrow \xi_{\mathcal{E}}$$

Retrieval through programming.

M applications

either approximate, or probabilistic

APPROXIMATE Q LEARNING

Optimal strategy for unitary channels

MEASURE-AND-ROTATE

A. Bisio, Gi. Chiribella, G. M. D'Ariano, S. Facchini, and P. Perinotti, Phys. Rev. A 81, 032324 (2010)

APPROXIMATE Q LEARNING

Optimal strategy for unitary channels

MEASURE-AND-ROTATE

→ optimal learning = optimal estimation
→ storing is classical

A. Bisio, Gi. Chiribella, G. M. D'Ariano, S. Facchini, and P. Perinotti, Phys. Rev. A 81, 032324 (2010)

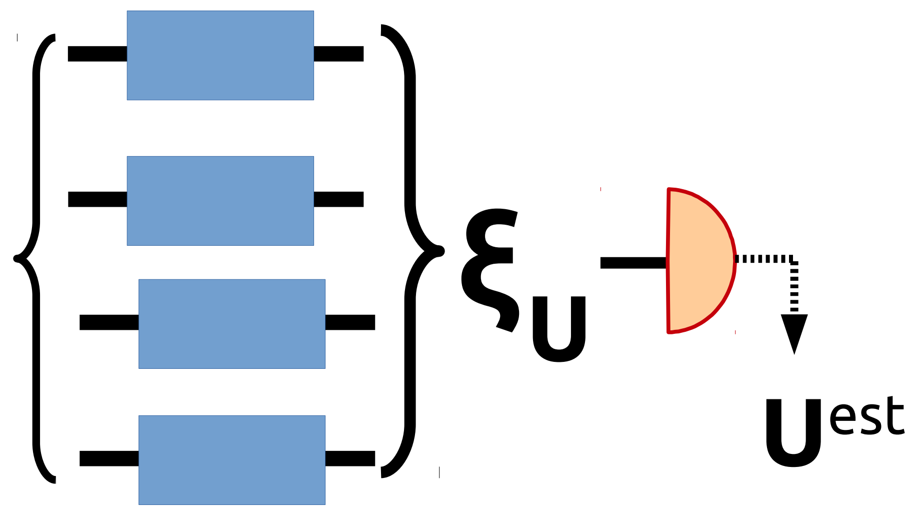
APPROXIMATE Q LEARNING

Optimal strategy for unitary channels

→ optimal state

$$|\psi\rangle = \bigotimes_{j \in IRR} \sqrt{p_j / d_j}\, |I_j\rangle\rangle$$

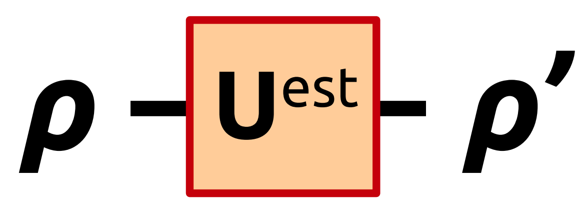
$$|I_j\rangle\rangle \in H_j \otimes H_j$$



→ optimal POVM (continuous)

$$E_{U^{est}} = |\eta_{U^{est}}\rangle\langle\eta_{U^{est}}|$$

$$|\eta_{U^{est}}\rangle = \bigotimes_{j \in IRR} \sqrt{d_j}\, |U_j\rangle\rangle$$



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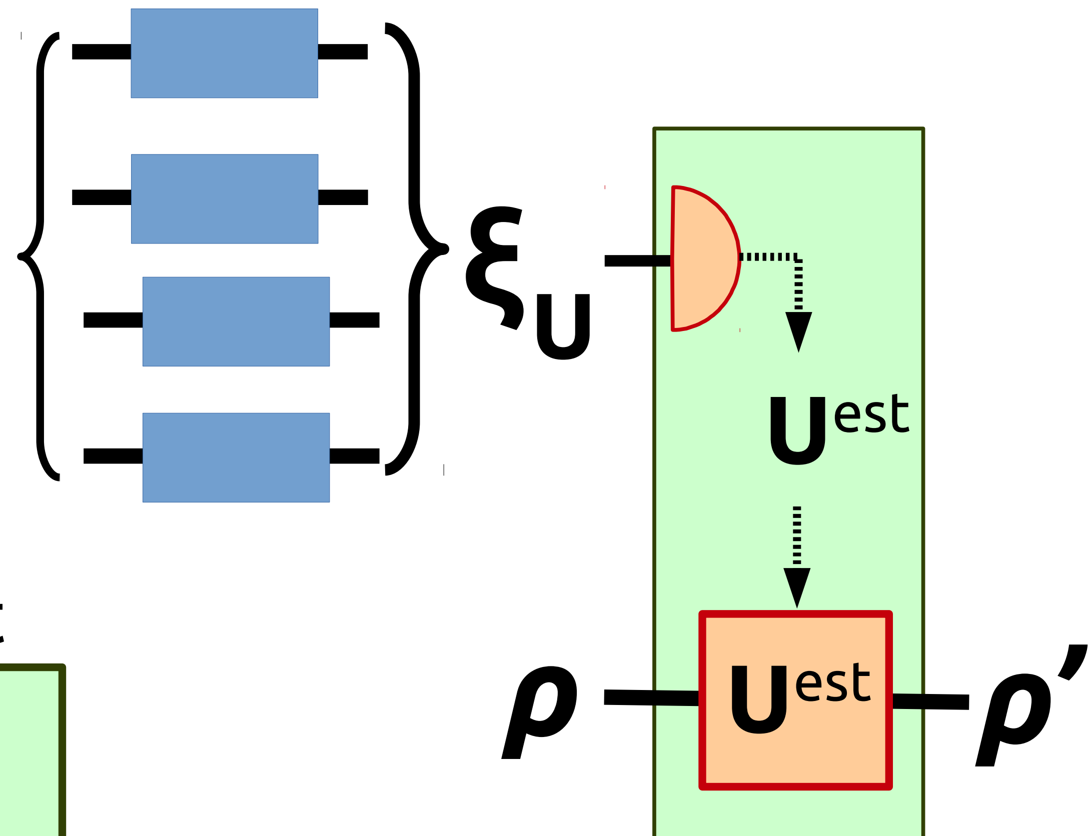
APPROXIMATE Q LEARNING

Optimal strategy for unitary channels



large N→1 for qubit

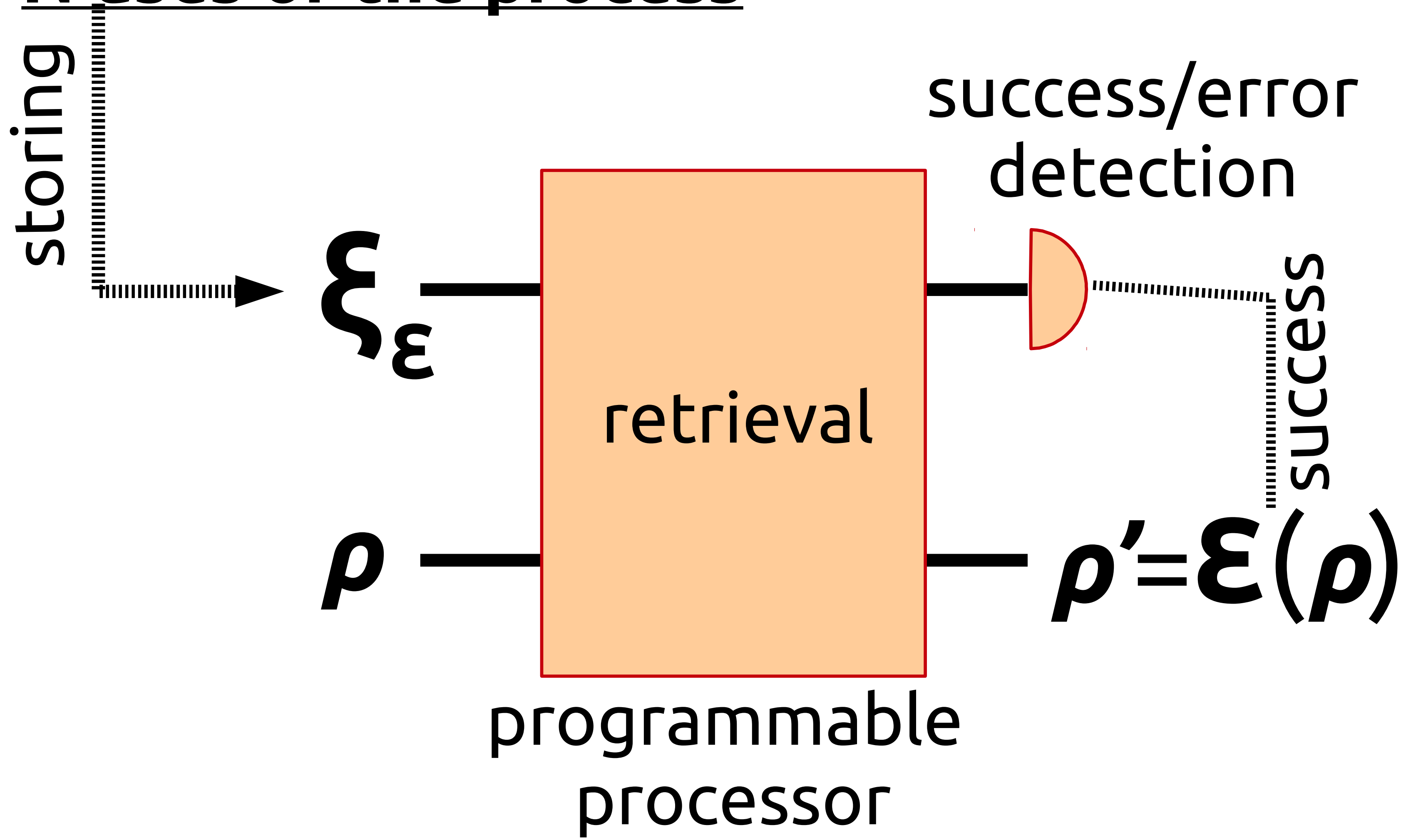
$$F \approx 1 - \pi^2/4N^2$$

A. Bisio, Gi. Chiribella, G. M. D’Ariano, S. Facchini, and P. Perinotti, Phys. Rev. A 81, 032324 (2010)

PROBABILISTIC Q LEARNING

→ **PROBABILISTIC PERFORMANCE**

→ **N uses of the process**

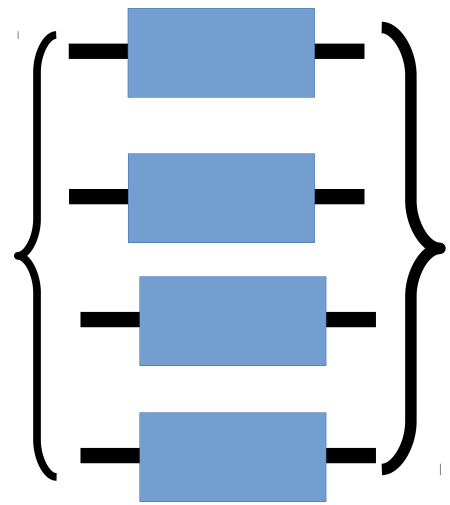


PROBABILISTIC Q LEARNING

Optimal strategy for unitary channels

→ optimal storing

$$|\psi\rangle = \bigotimes_{j \in IRR} \sqrt{p_j / d_j}\, |I_j\rangle\rangle$$



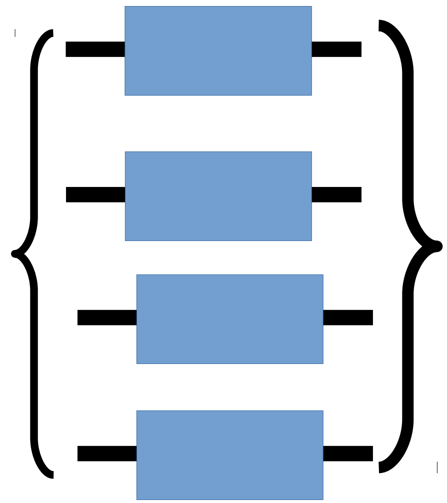
ξ_U

PROBABILISTIC Q LEARNING

Optimal strategy for unitary channels

→ optimal storing

$$|\Psi\rangle = \bigotimes_{j \in IRR} \sqrt{p_j / d_j}\,|I_j\rangle\rangle$$



ξ_U

→ optimal retrieval

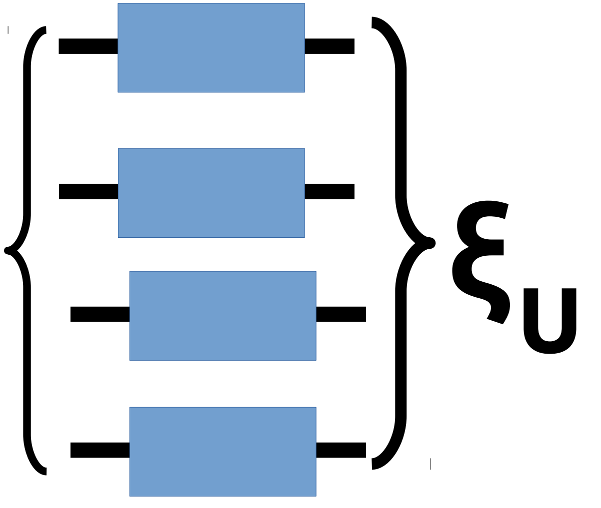
MEASURE-AND-ROTATE

PROBABILISTIC Q LEARNING

Optimal strategy for unitary channels

→ optimal storing

$$|\Psi\rangle = \bigotimes_{j \in IRR} \sqrt{p_j / d_j}\, |I_j\rangle\rangle$$

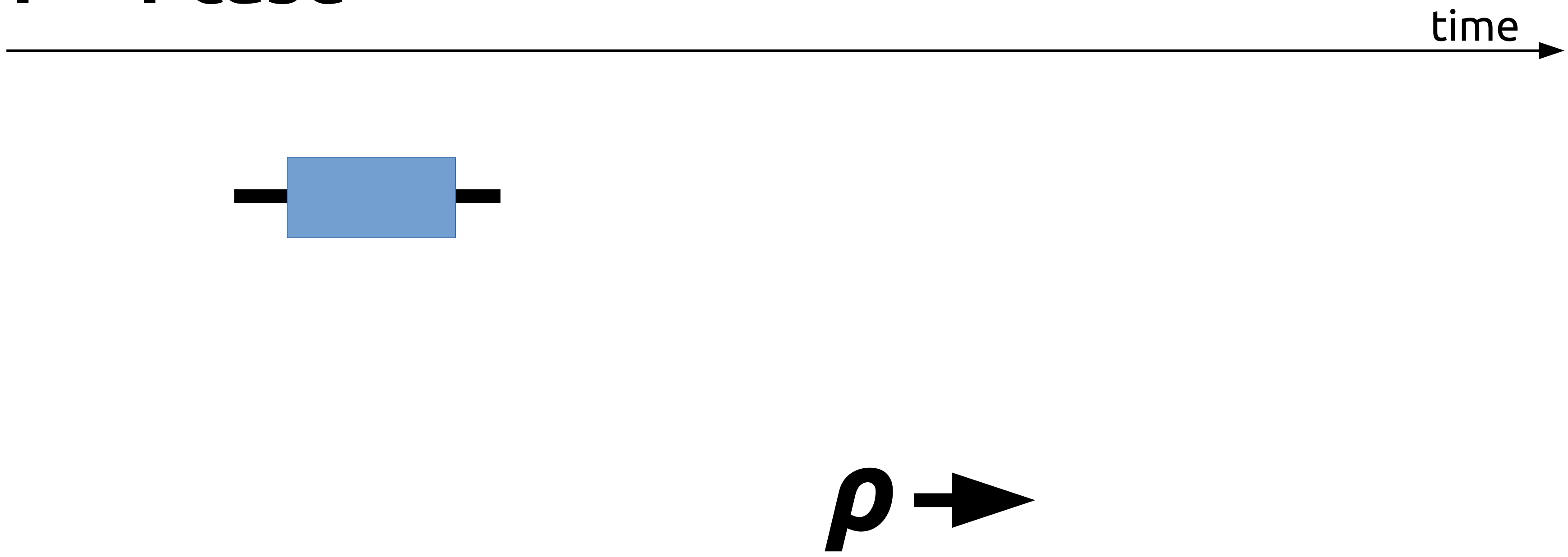


→ optimal retrieval

QUANTUM

PROBABILISTIC Q LEARNING

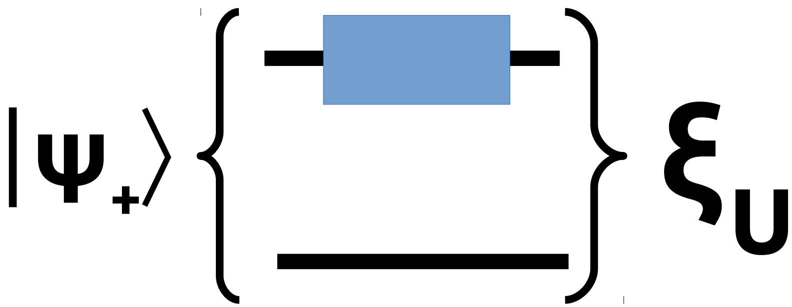
1→ 1 case



PROBABILISTIC Q LEARNING

1→ 1 case

time →

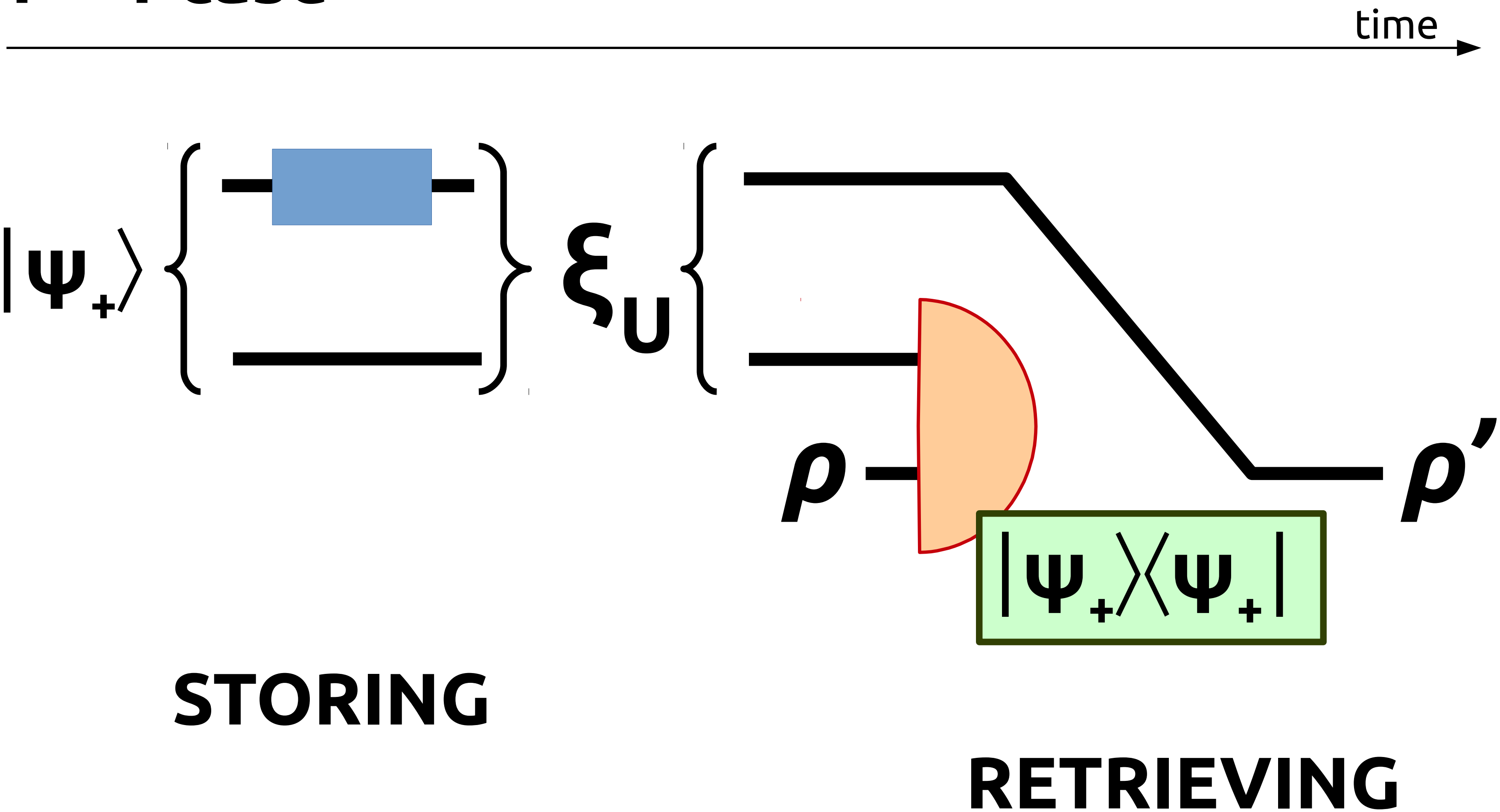


$\rho \rightarrow$

STORING

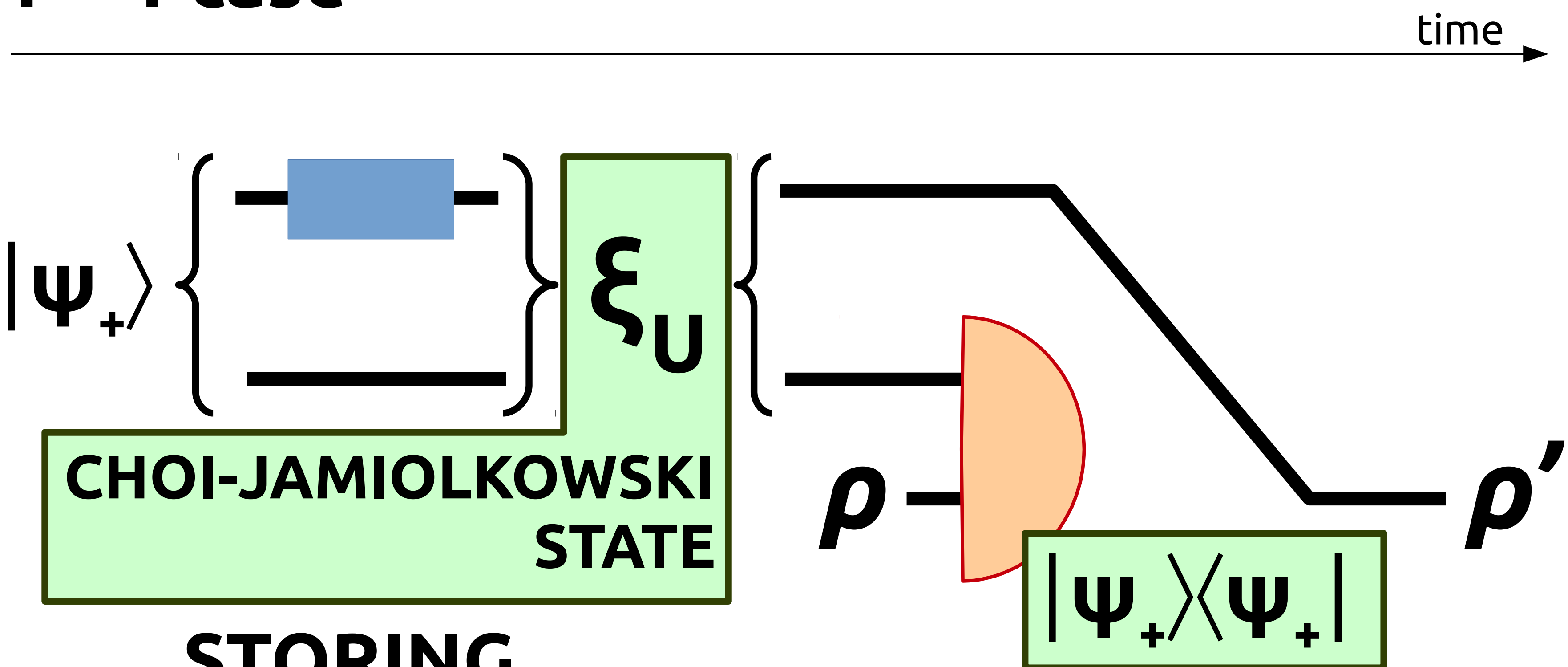
PROBABILISTIC Q LEARNING

1→ 1 case



PROBABILISTIC Q LEARNING

1→ 1 case



STORING

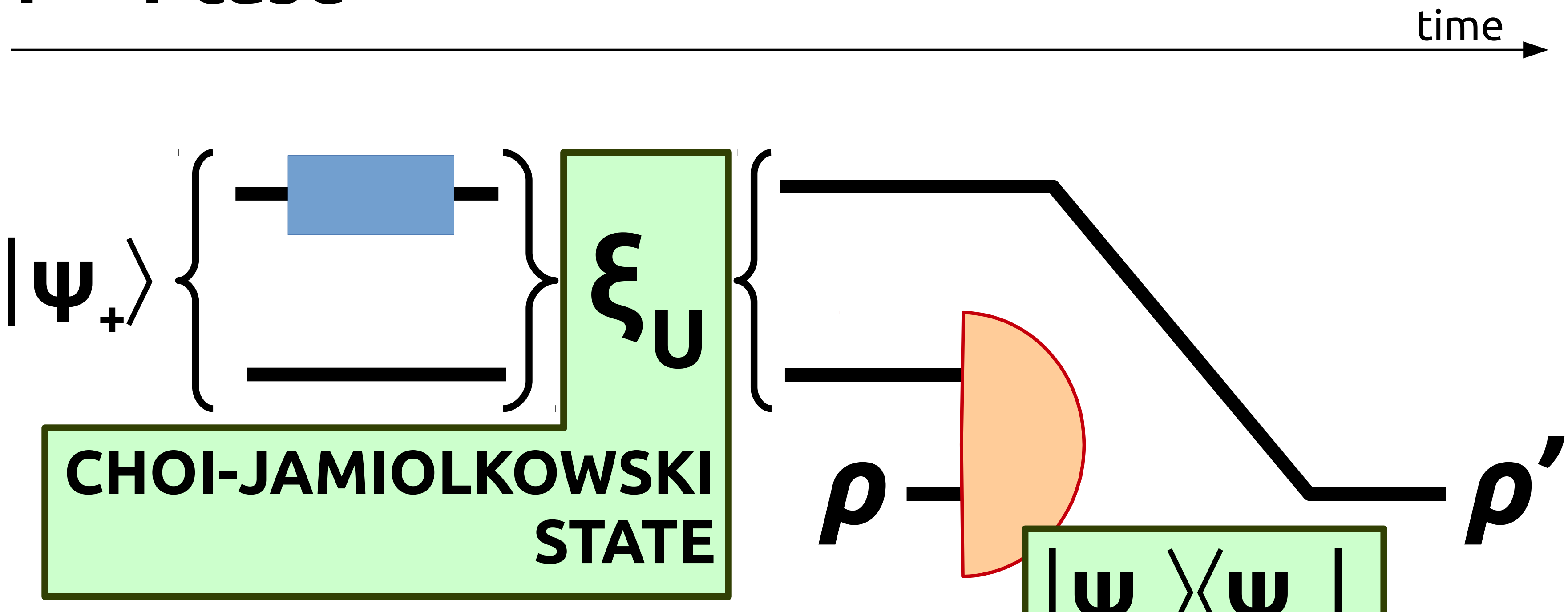
RETRIEVING

INCOMPLETE

TELEPORTATION

PROBABILISTIC Q LEARNING

1→ 1 case



STO

$$P_{success} = 1/d^2$$

INCO

VING

TELEPORTATION

PROBABILISTIC Q LEARNING

Optimal $N \to 1$ for unitary channels

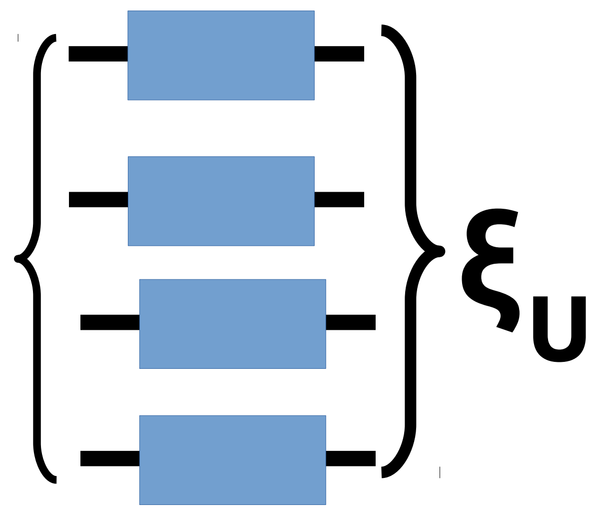
$$P_{success} = \frac{N}{N-1+d^2}$$

PROBABILISTIC Q LEARNING

Optimal N→ 1 for unitary channels

→ <u>optimal storing</u>

$$|\psi\rangle = \bigotimes_{j\in IRR} \sqrt{p_j/d_j}\,|I_j\rangle\rangle$$



PROBABILISTIC Q LEARNING*

Optimal N→ 1 for unitary channels

→ optimal retrieval (quantum comb formalism)

→ reduction to linear programming problem

$$\begin{aligned} \underset{\mu_J, p_j}{\text{maximize}} \quad & \lambda = \sum_J d_J^3 \mu_J, \\ \text{subject to} \quad & 0 \le d_J \mu_J \le \frac{p_j}{d_j^2} \quad \forall j \in \mathrm{j}_{JJ} \quad \forall J \\ & p_j \ge 0 \quad \sum_j p_j = 1, \end{aligned}$$

→ combinatorial identity for permutation group

$$\sum_j (c_j - r_j)^2 \frac{h_J}{h_j} = N - 1$$

→ result is analytical

M. Sedlák, A. Bisio, M.Ziman: Optimal probabilistic storage and retrieval of unitary channels, Phys. Rev. Lett. 122, 170502 (2019)
S. Ramgoolam, M. Sedlák: Quantum Information Processing and Composite Quantum Fields, J. High Energ. Phys. 2019, 170 (2019)
M. Sedlák, A. Bisio: On some new hook-content identities, Journal of Algebraic Combinatorics, https://doi.org/10.1007/s10801-019-00931-5
M. Sedlák, M. Ziman: Probabilistic storage and retrieval of qubit phase gates, Physical Review A 102, 032618 (2020) [arXiv:2008.09555]

RELATED RESULTS

Retrieval of inverse of U (undo) for qubits

→ the same success probability

→ difference only in retrieval

→ probabilistic alignment of reference frame

RELATED RESULTS

Trade-off for probabilistic processors

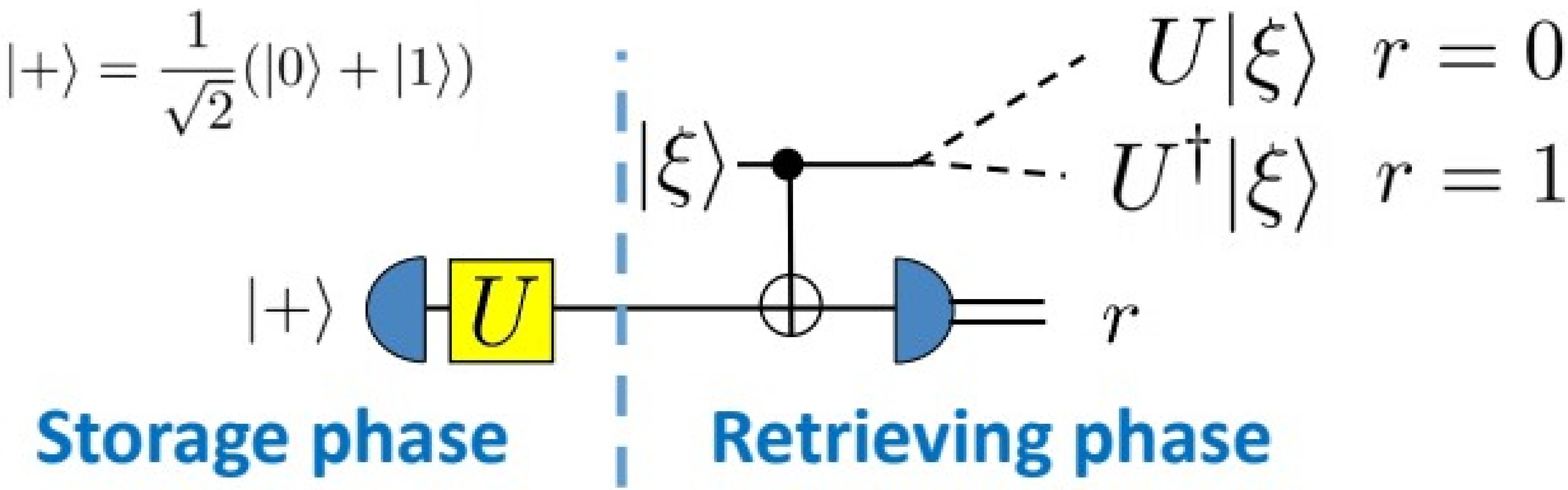
→ retrieval part provides best known bounds (exponentially better than before) on the memory size as a function of failure probability f

$$\dim H_{mem} = \sum_{j \in \mathbf{\mathit{IRR}}} d_j^2 = \binom{N-1+d^2}{N}$$

$$\propto (1/f)^{(d^2-1)}$$

PROBABILISTIC LEARNING OF PHASE

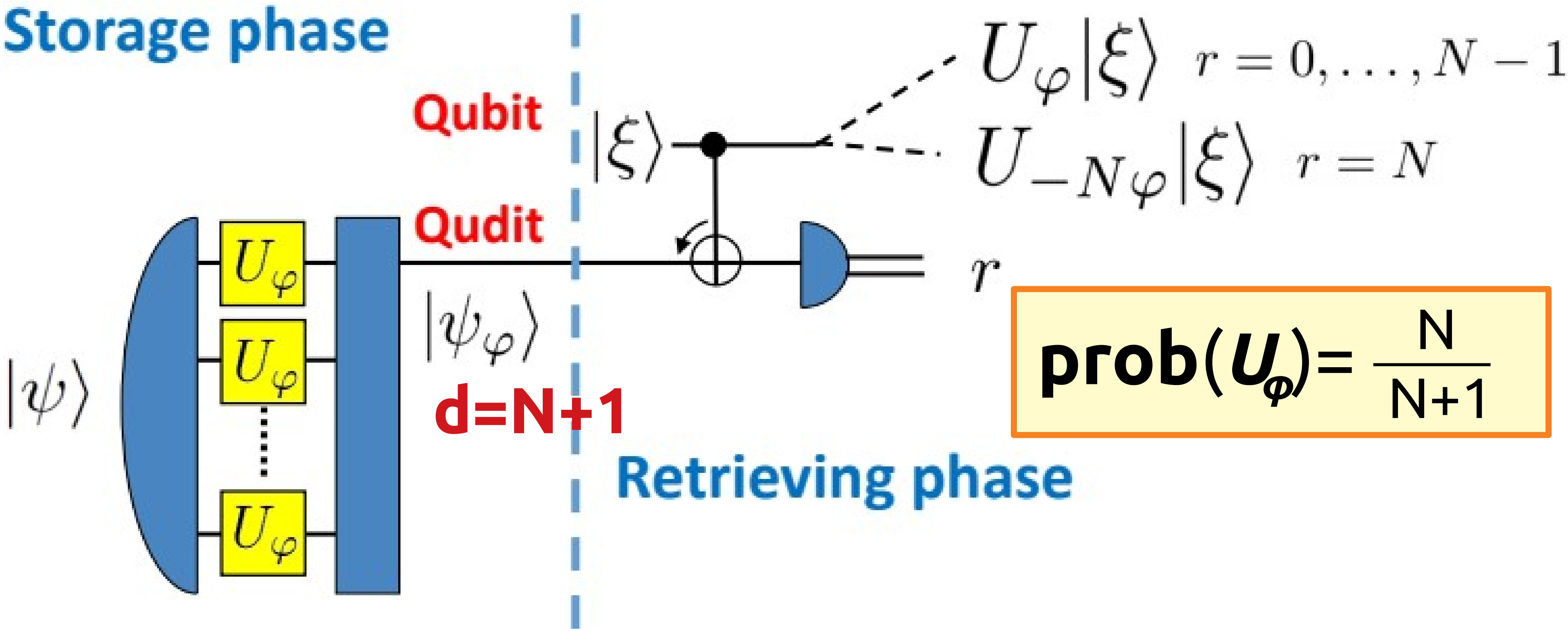
$$U_\varphi = |0\rangle\langle 0| + e^{i\varphi}|1\rangle\langle 1|$$



$P_{success} = 1/2$

1 → 1

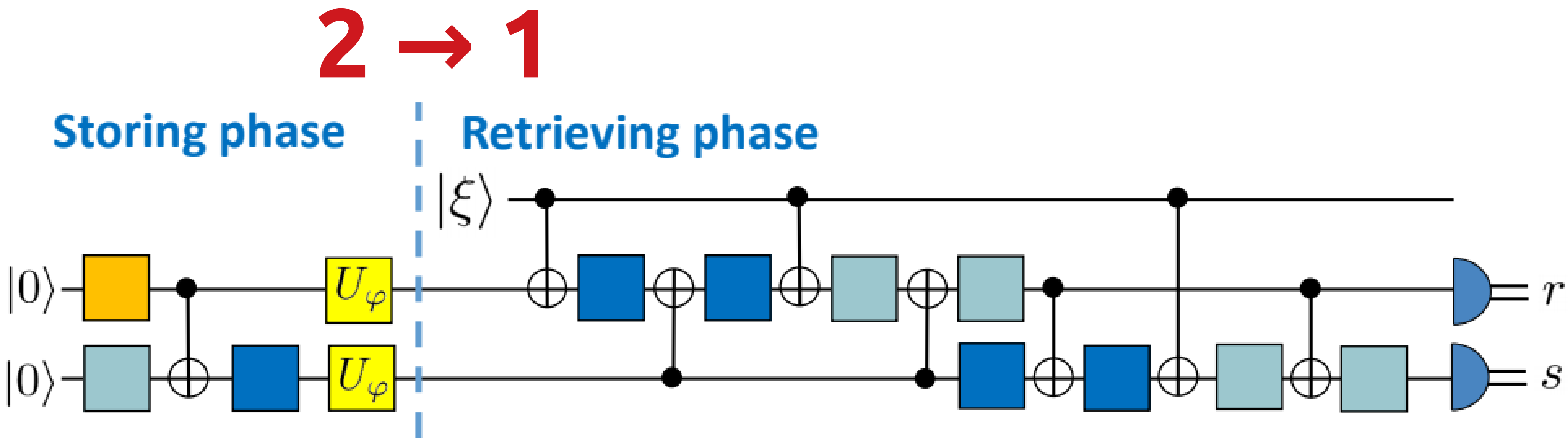
PROBABILISTIC LEARNING OF PHASE



$$\mathrm{prob}(U_\varphi) = \frac{N}{N+1}$$

N → 1

PROBABILISTIC LEARNING OF PHASE



PROBABILISTIC LEARNING OF PHASE

$$N=(2^k-1) \rightarrow 1$$

$$\text{prob}(U_\varphi)=1-\frac{1}{2^k}$$

PROBABILISTIC LEARNING OF PHASE

$$N=(2^k-1) \rightarrow 1$$

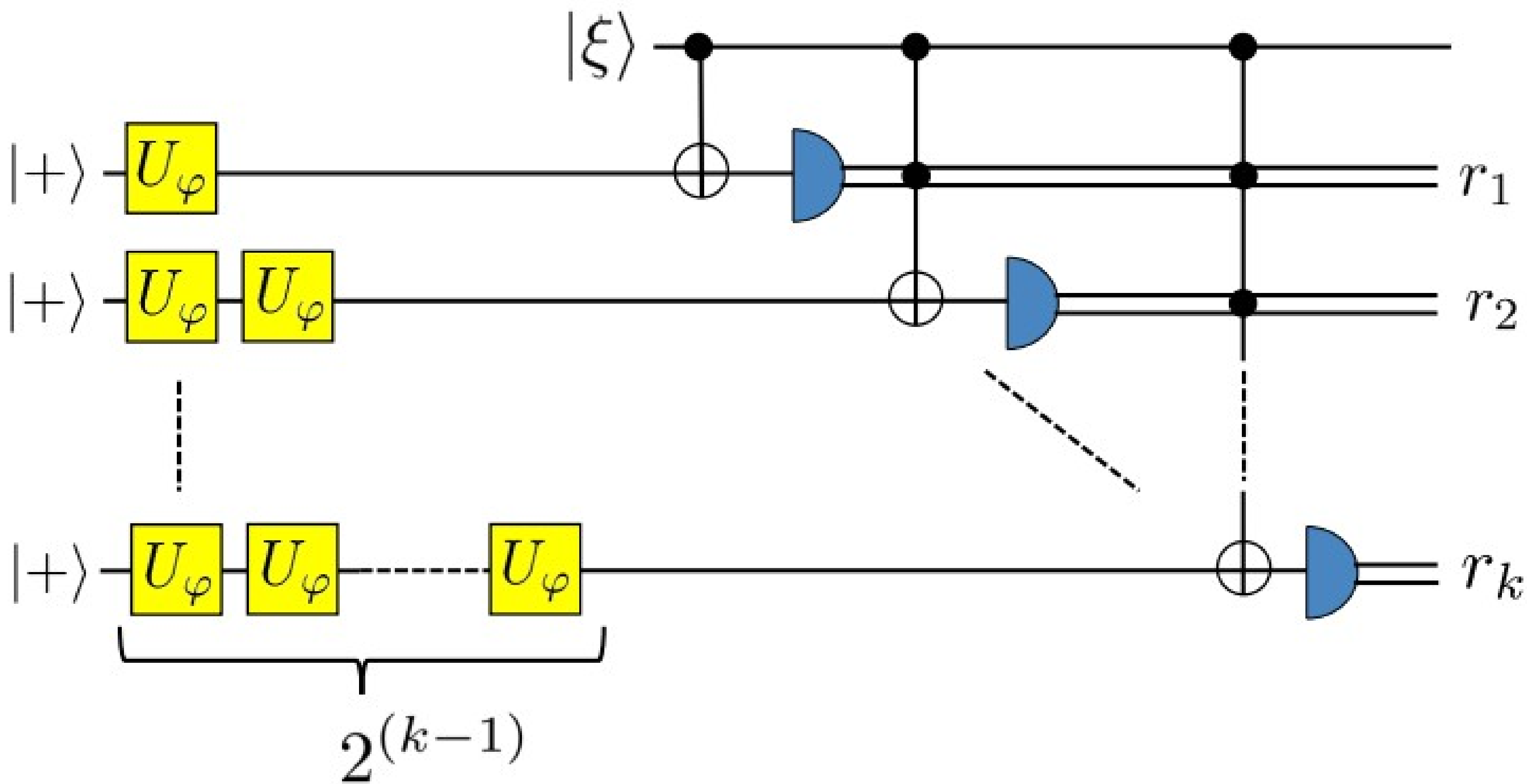
$$\mathrm{prob}(U_\varphi)=1-\frac{1}{2^k}$$

same as Vidal-Masanes-Cirac

PROBABILISTIC LEARNING OF PHASE

$N=(2^k-1) \rightarrow 1$

$\text{prob}(U_\varphi)=1-\frac{1}{2^k}$



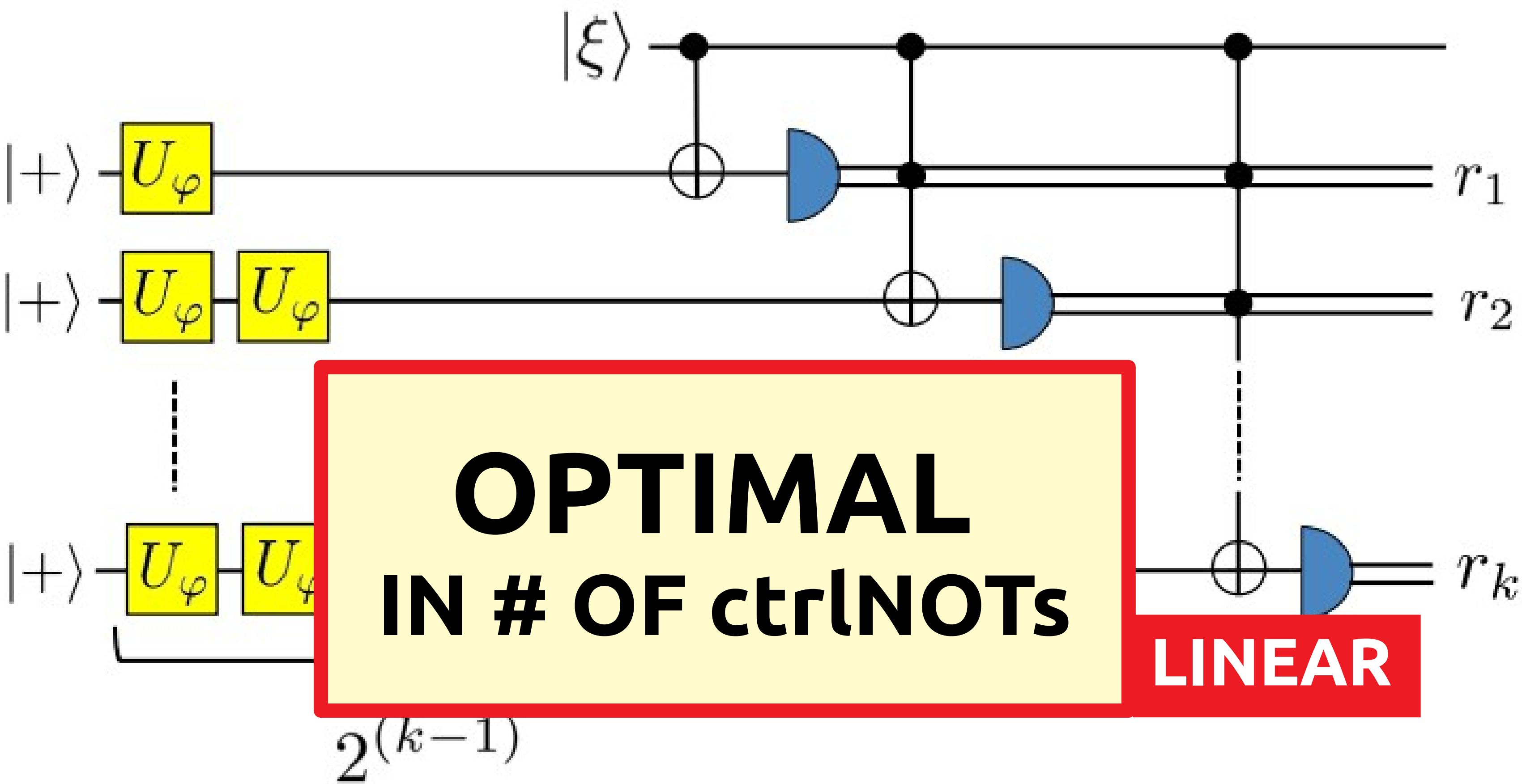
storing

Vidal-Masanes-Cirac

PROBABILISTIC LEARNING OF PHASE

$N=(2^k-1) \rightarrow 1$

$$\text{prob}(U_\varphi)=1-\frac{1}{2^k}$$



OPTIMAL
IN # OF ctrlNOTs

LINEAR

storing

Vidal-Masanes-Cirac

SUMMARY

store sampling **N** times
retrieve probabilistically **1** use

$$P^{universal}_{success} = \frac{N}{N-1+d^2}$$

$$P^{phase}_{success}(U_\varphi) = \frac{N}{N+1}$$

Probabilistic learning is teleportation, hence, quantum.

joint work with

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University of Pavia, Pavia, Italy

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S. Ramgoolam, M. Sedlák: Quantum Information Processing and Composite Quantum Fields, J. High Energ. Phys. 2019, 170 (2019)
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THANK YOU FOR YOUR ATTENTION