Markovianization

How quickly does nature forget?

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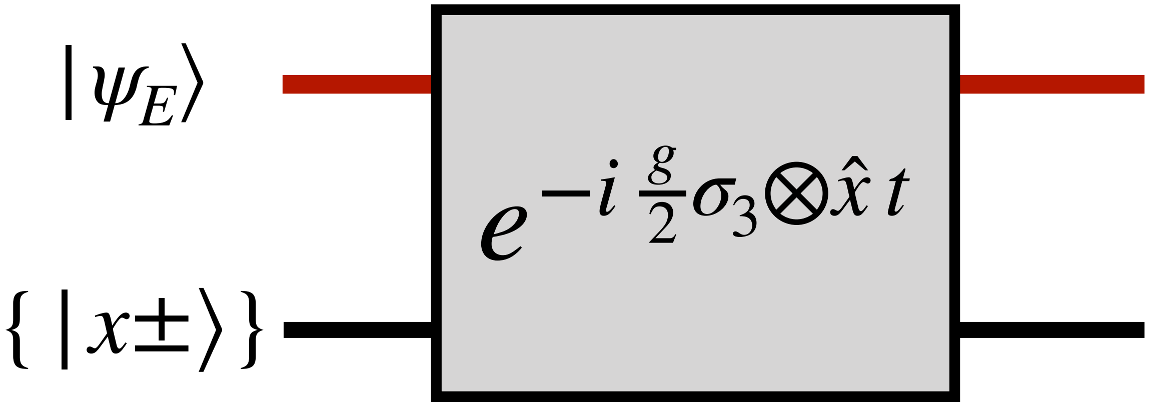
MonQIS

Pure dephasing dynamics
Time-independent GKLS

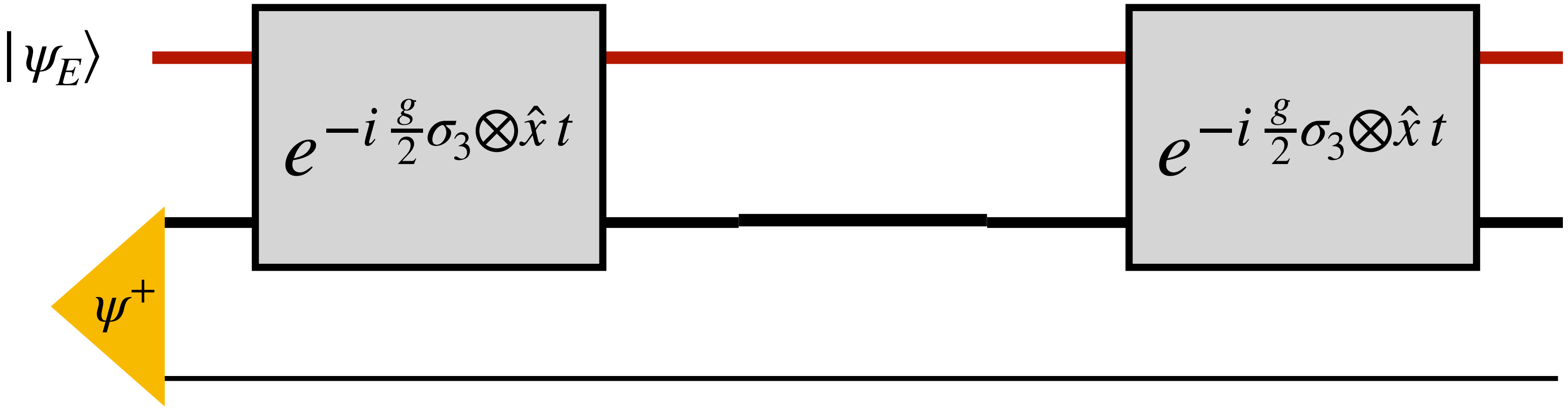
$$\langle x|\psi_E\rangle = \psi_E(x) = \sqrt{\frac{\mathscr{G}}{\pi}}\,\frac{1}{x+i\mathscr{G}}$$

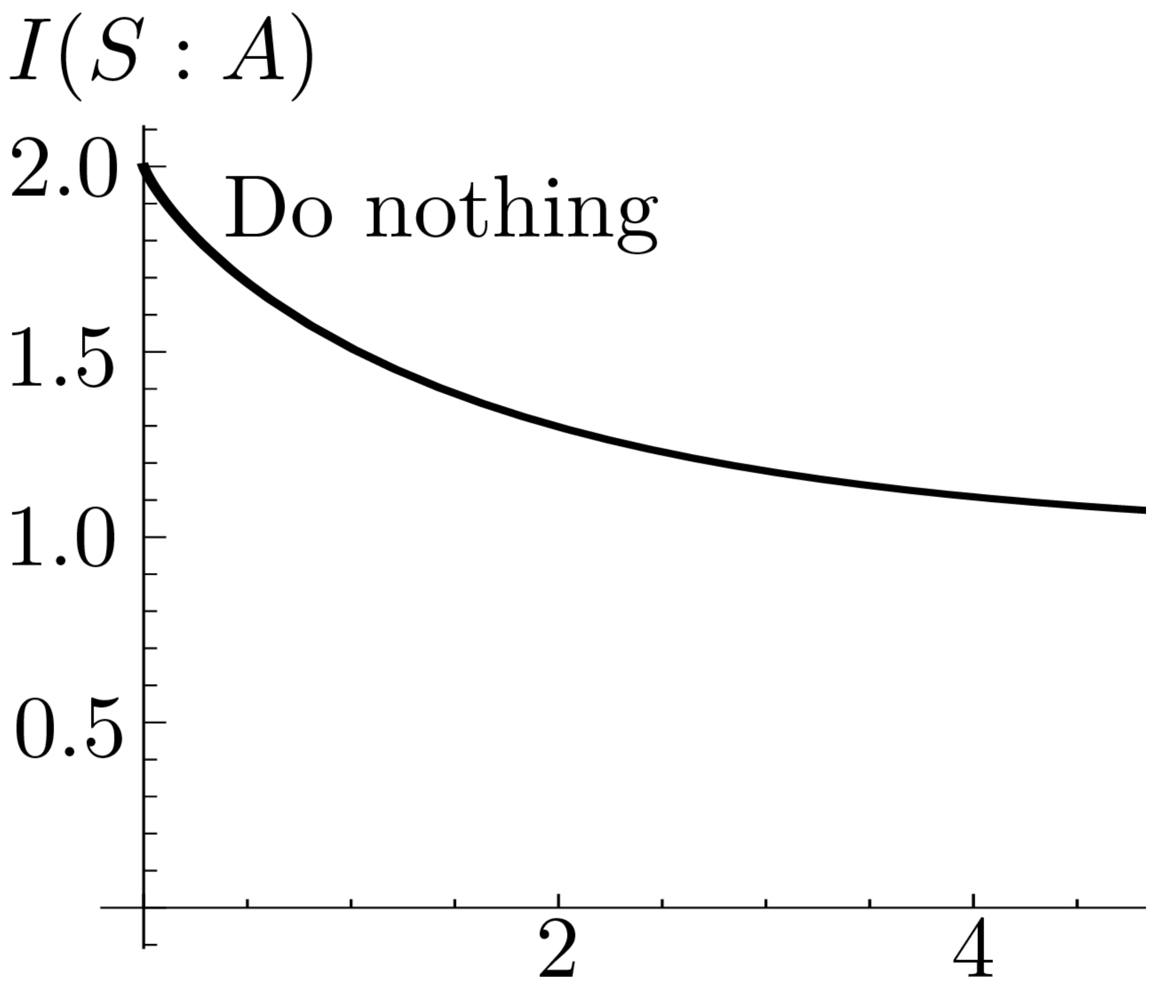


$$\rho_\pm(t) := \frac{1}{2}\begin{pmatrix} 1 & \pm e^{-\gamma t} \\ \pm e^{-\gamma t} & 1 \end{pmatrix} \quad \text{with } \gamma = g\,\mathscr{G}$$

Lindblad (1980)
Accardi-Fregerio-Lewis (1982)
Arenz-Hillier-Fraas-Burgarth (2015)

Pure dephasing dynamics but non-Markovian

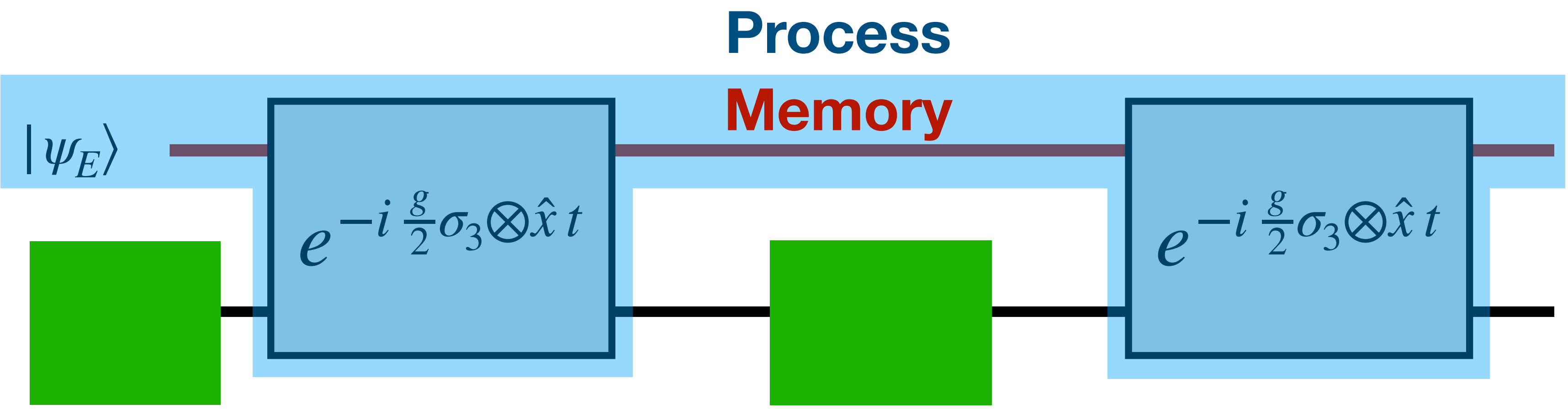


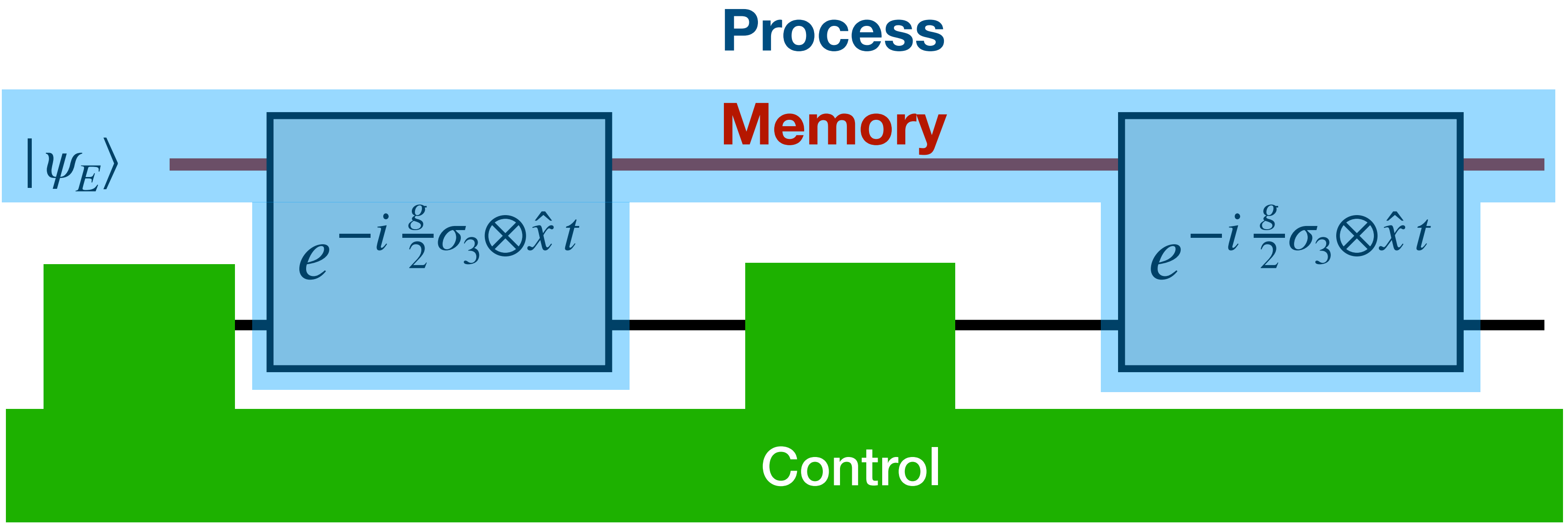


Taranto, Pollock, Modi arXiv (2019)

Time

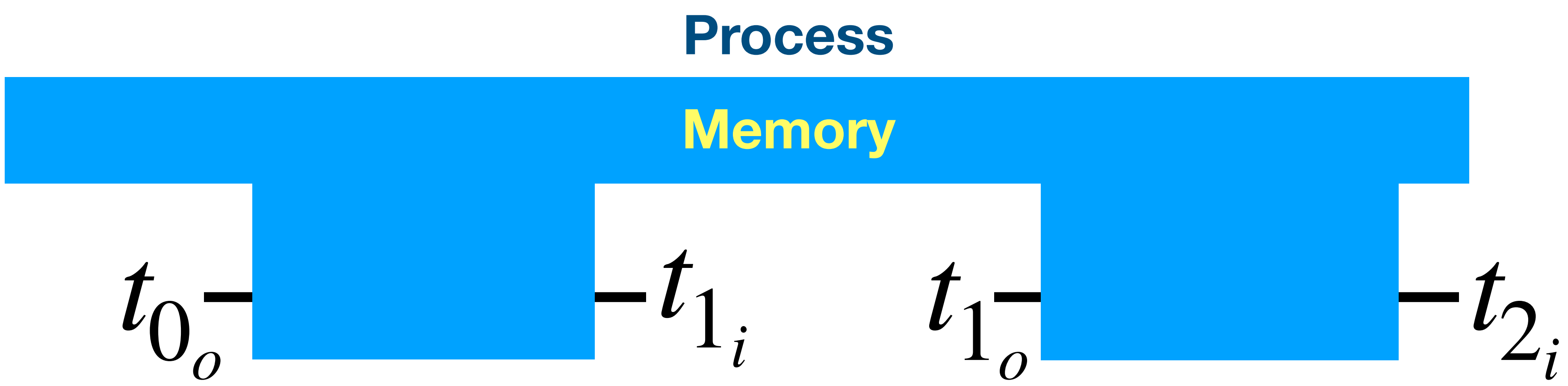
Describing the process independent of control



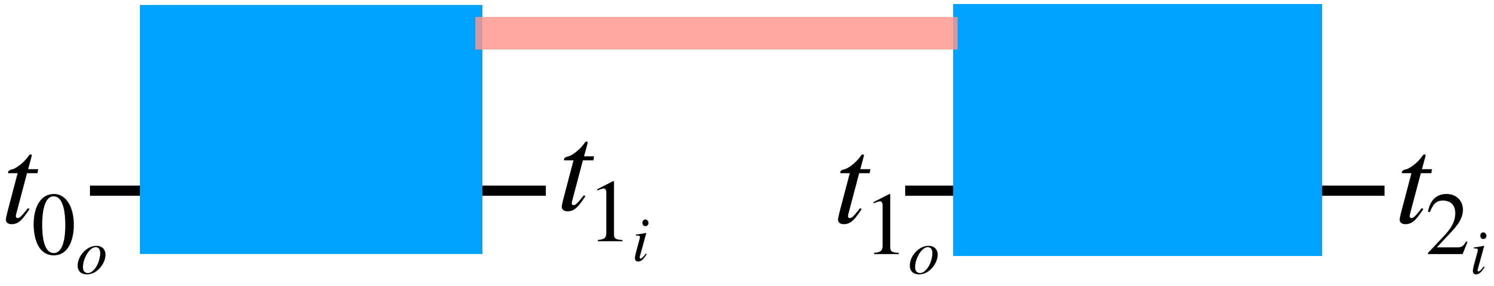


Milz, Modi PRX Quantum (2021)

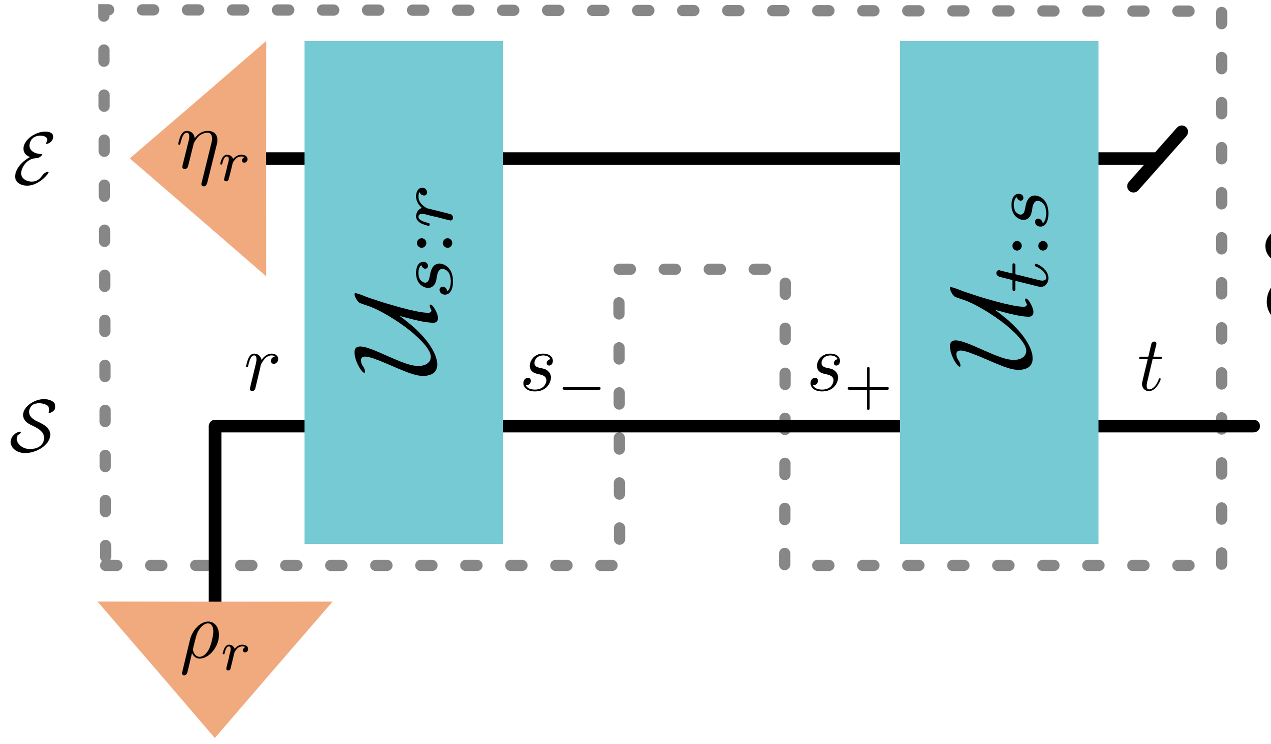
Divisible but non-Markovian



$$\Upsilon_{\{2t,t,0\}} = \begin{array}{c} \begin{array}{cccc} 0000 & 0011 & 1100 & 1111 \end{array} \\ \begin{pmatrix} 1 & e^{-\gamma t} & e^{-\gamma t} & e^{-2\gamma t} \\ e^{-\gamma t} & 1 & 1 & e^{-2\gamma t} \\ e^{-\gamma t} & 1 & 1 & e^{-2\gamma t} \\ e^{-2\gamma t} & e^{-\gamma t} & e^{-\gamma t} & 1 \end{pmatrix} \end{array} \neq \mathscr{E}_{t_{1_i}:t_{0_o}} \otimes \mathscr{E}_{t_{2_i}:t_{1_o}}$$



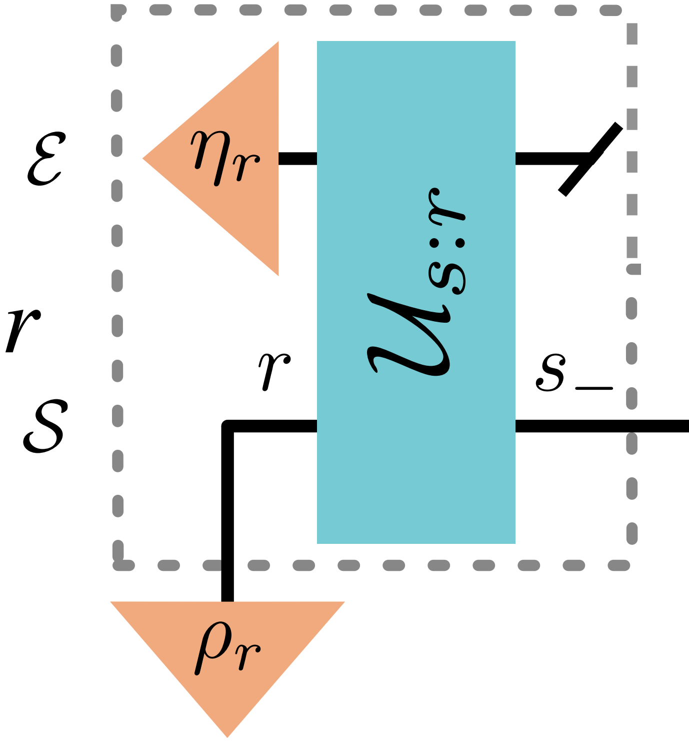
Milz, Modi PRX Quantum (2021)

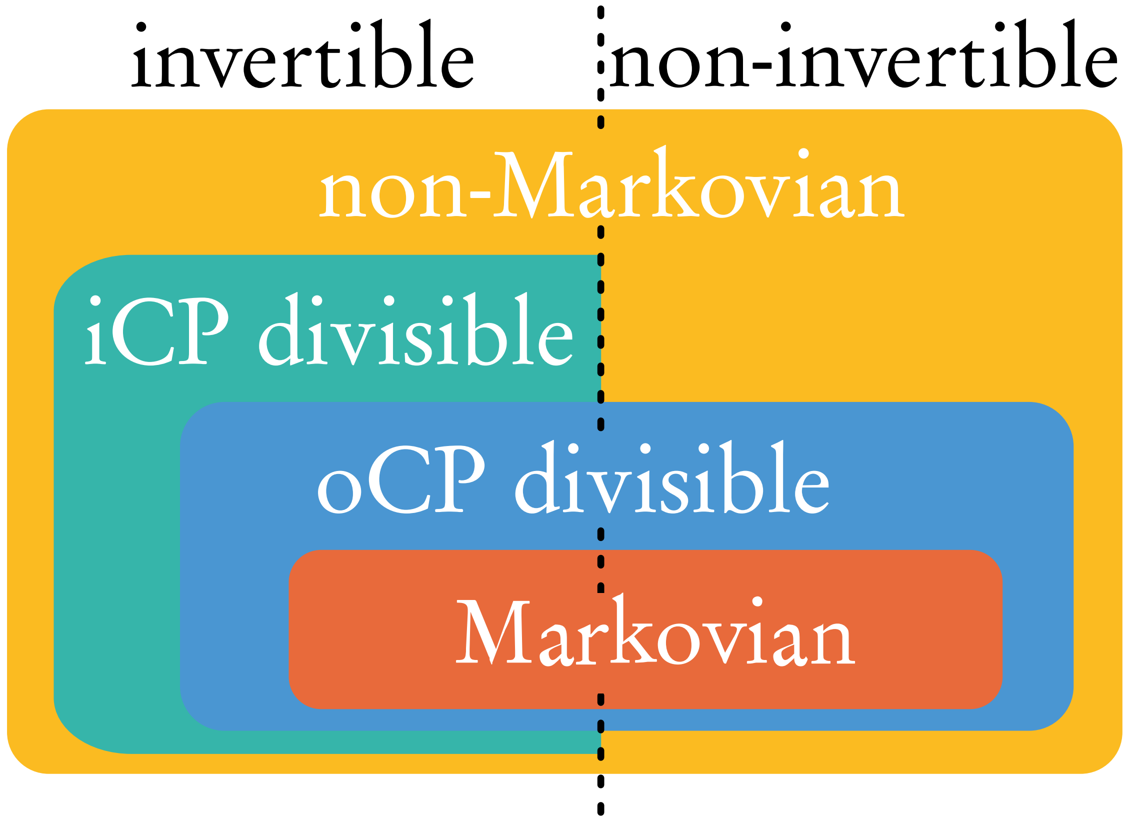


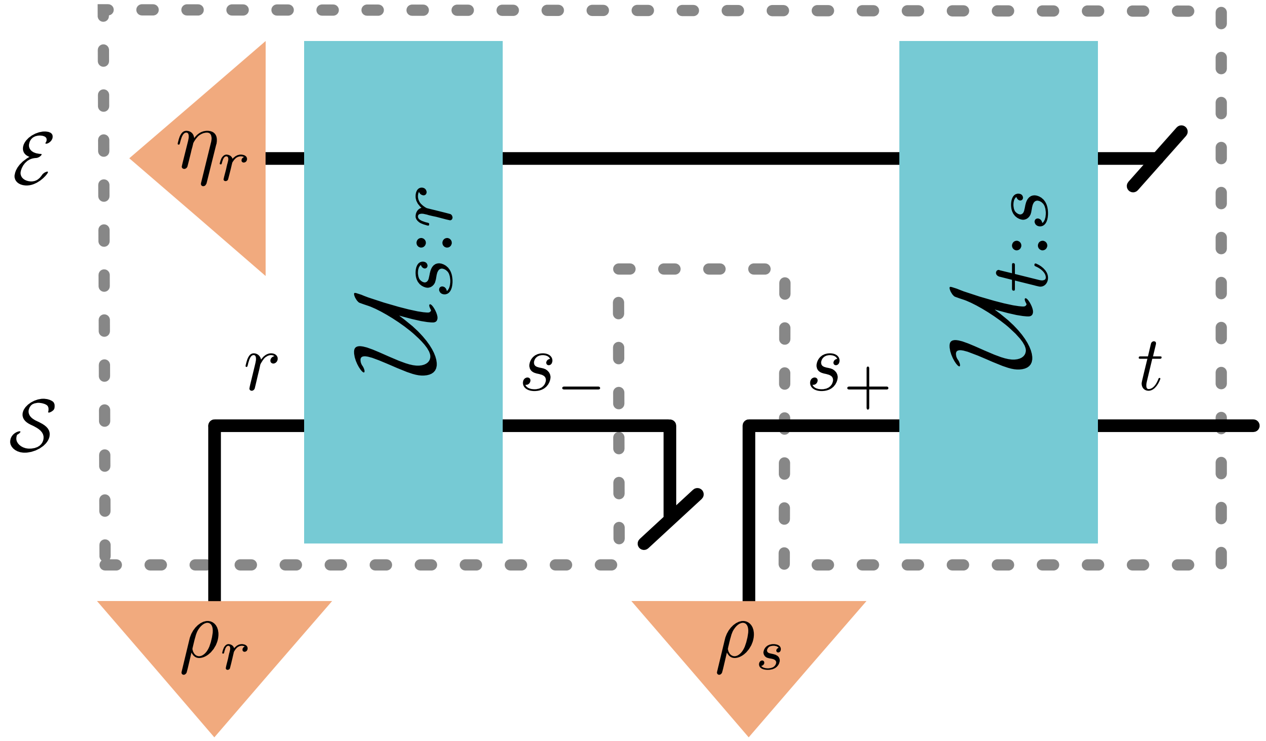
$$\mathscr{E}_{t:r} = \Phi_{t:s_+} \circ \mathscr{E}_{s_-:r}$$

$$\nparallel$$

$$\mathscr{E}_{t:s_+}$$

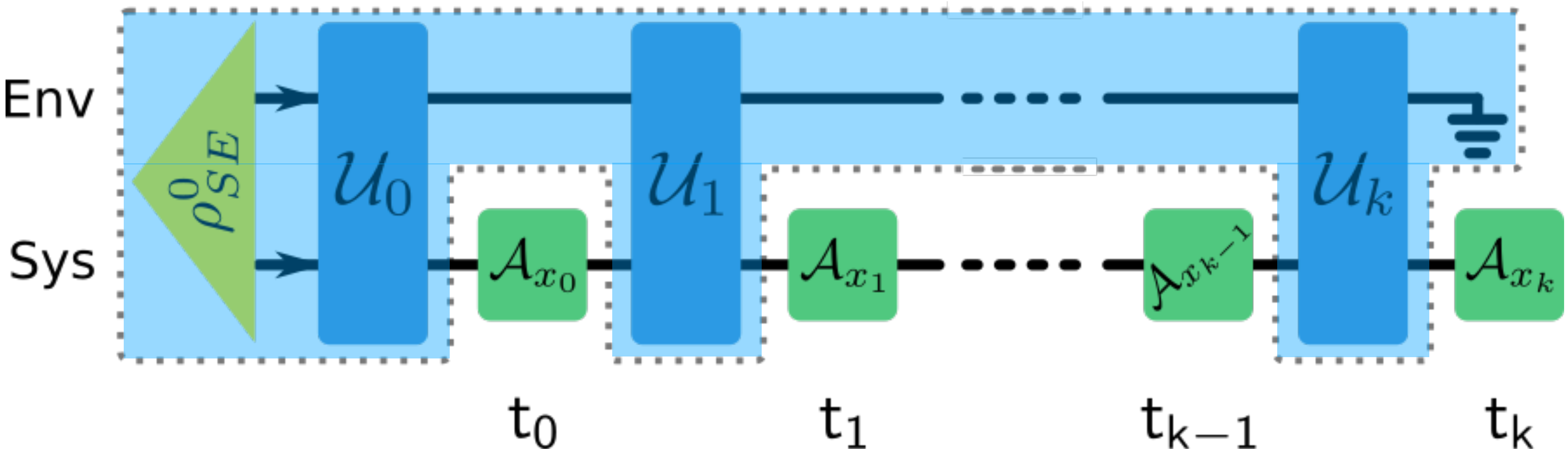






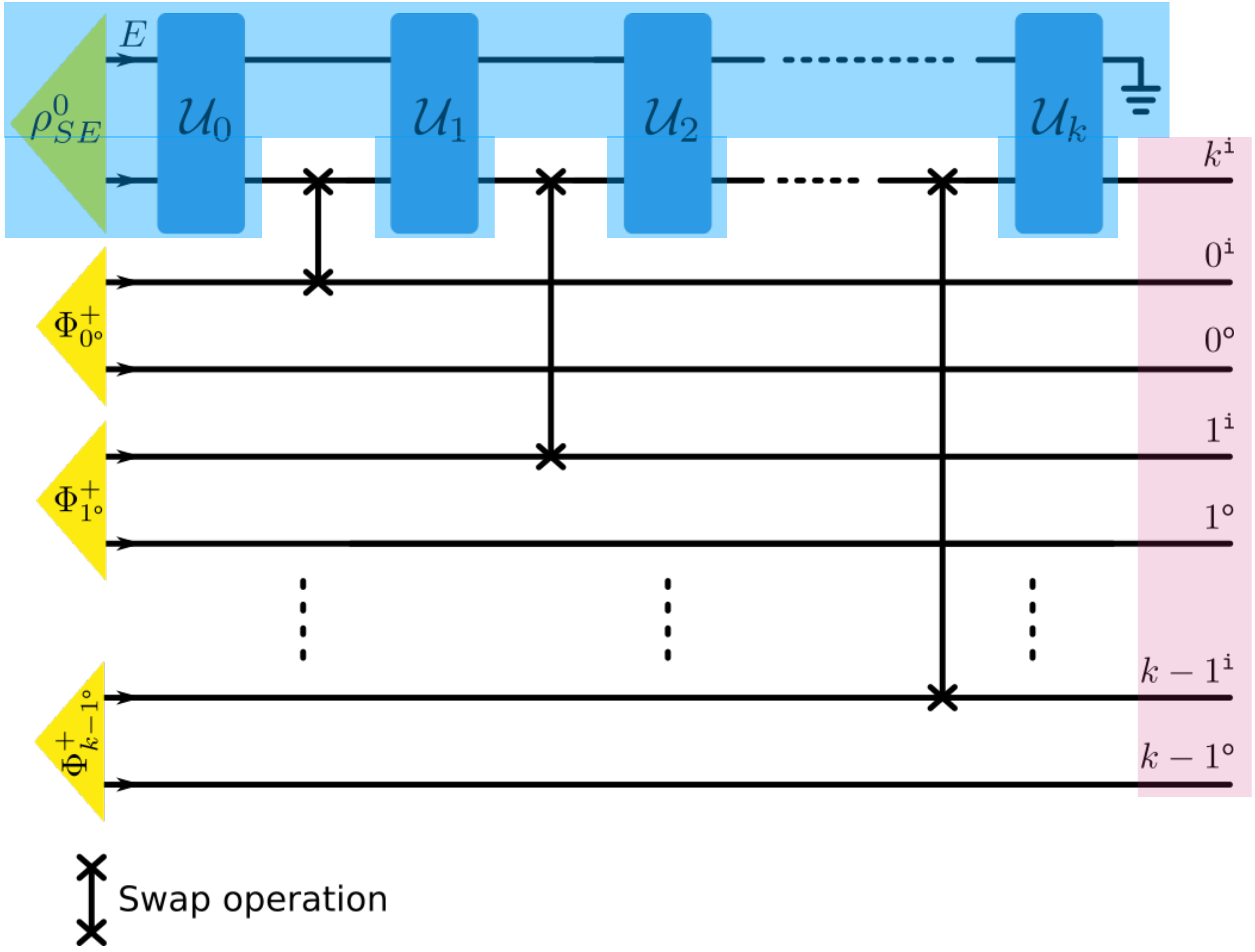
Milz, Kim, Pollock, Modi PRL (2019)

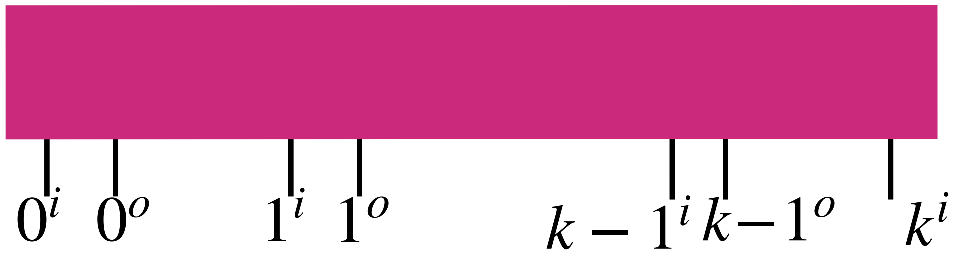
What is a quantum stochastic process?

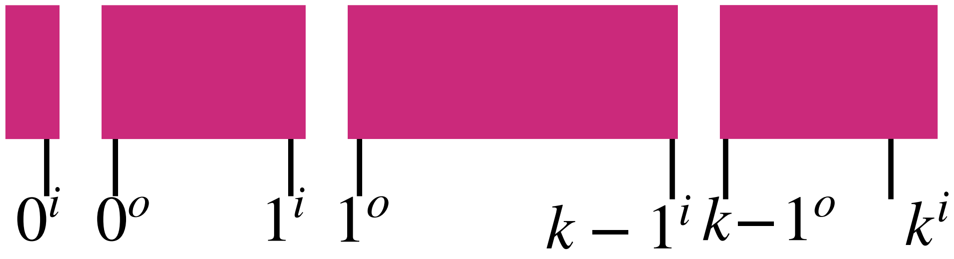


Pollock, Rodriguez-Rosario, Fraünheim, Paternostro, Modi PRA (2018)

Representation of quantum stochastic processes







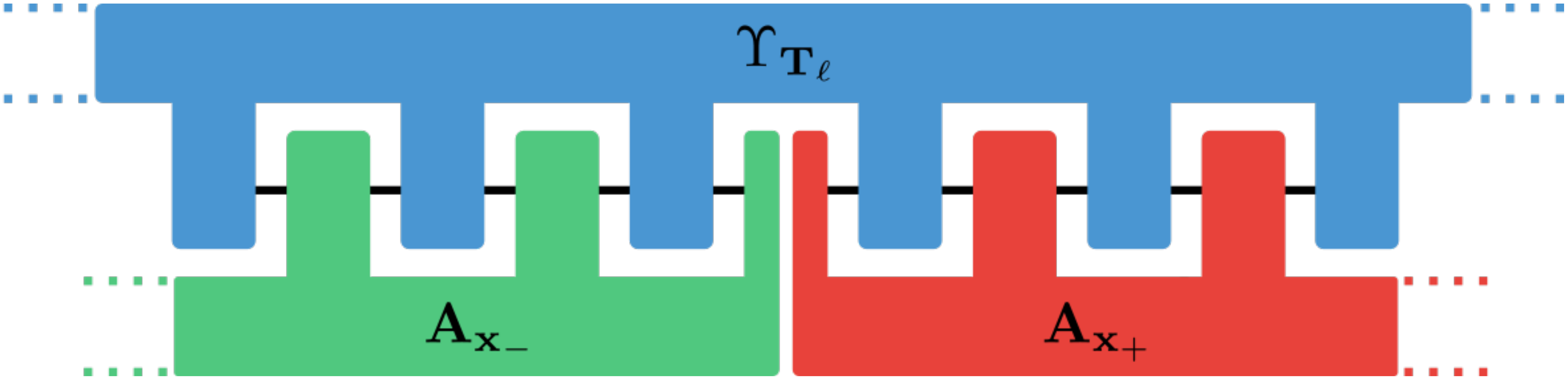
Markovian

$$\Upsilon^{(\mathrm{M})} = \mathcal{E}_{1:0} \otimes \cdots \otimes \mathcal{E}_{k:k-1}$$

Pollock, Rodriguez-Rosario, Fraünheim, Paternostro, Modi PRA (2018)

Quantum Markov condition

$$\mathbf{T}_- = \{t_0, \ldots, t_{k^-}\} \qquad \mathbf{T}_+ = \{t_{k^+}, \ldots, t_\ell\}$$



Markovian has the same meaning as in the classical theory

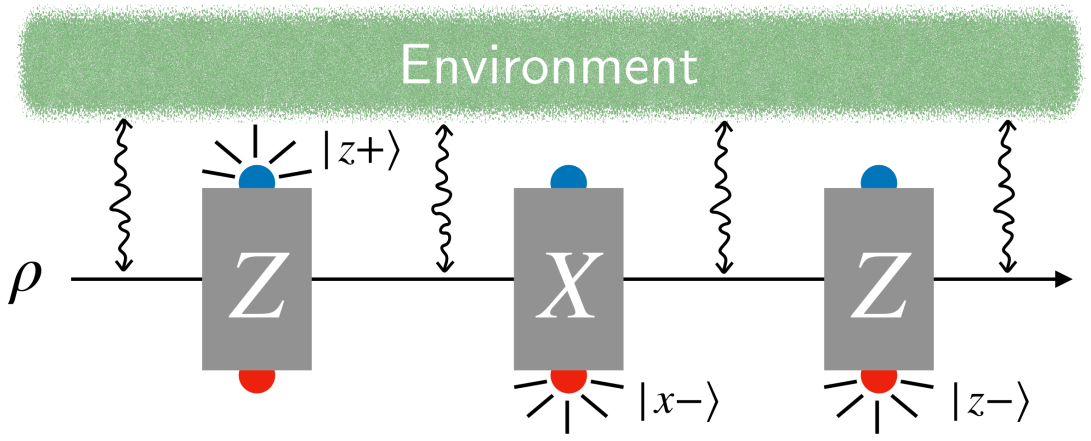
$$\mathbb{P}(\mathbf{x}_+|\mathcal{J}_+, \mathbf{x}_-) = \mathbb{P}(\mathbf{x}_+|\mathcal{J}_+),$$

As do the conditions for non-Markovianity

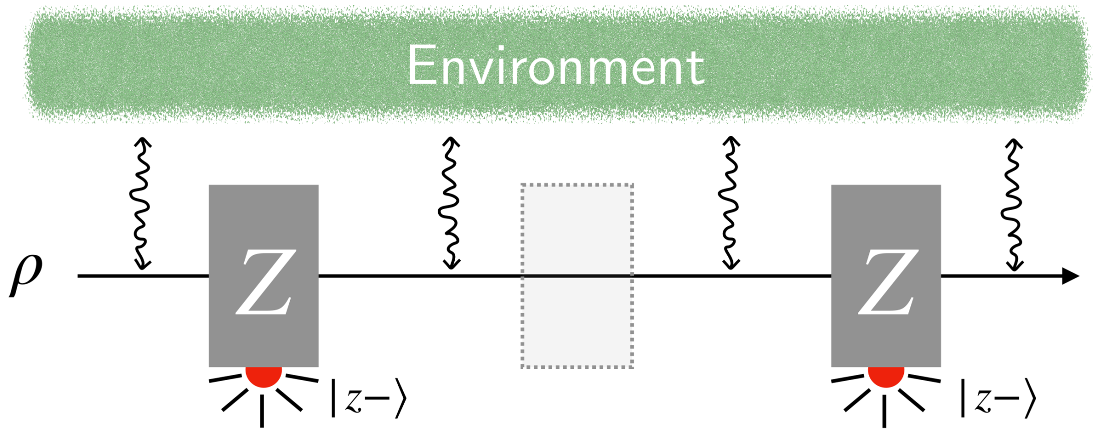
$$\mathbb{P}(\mathbf{x}_+|\mathcal{J}_+, \boxed{\mathbf{x}_-}) \neq \mathbb{P}(\mathbf{x}_+|\mathcal{J}_+, \boxed{\mathbf{x}'_-})$$

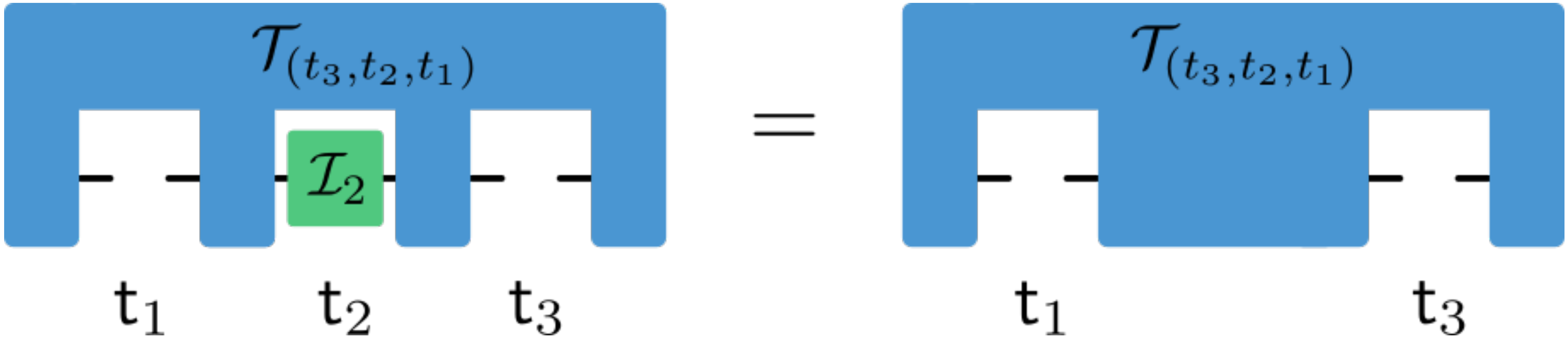
Pollock, Rodriguez-Rosario, Fraünheim, Paternostro, Modi PRL (2018)

Consistency conditions and Kolmogorov theorem



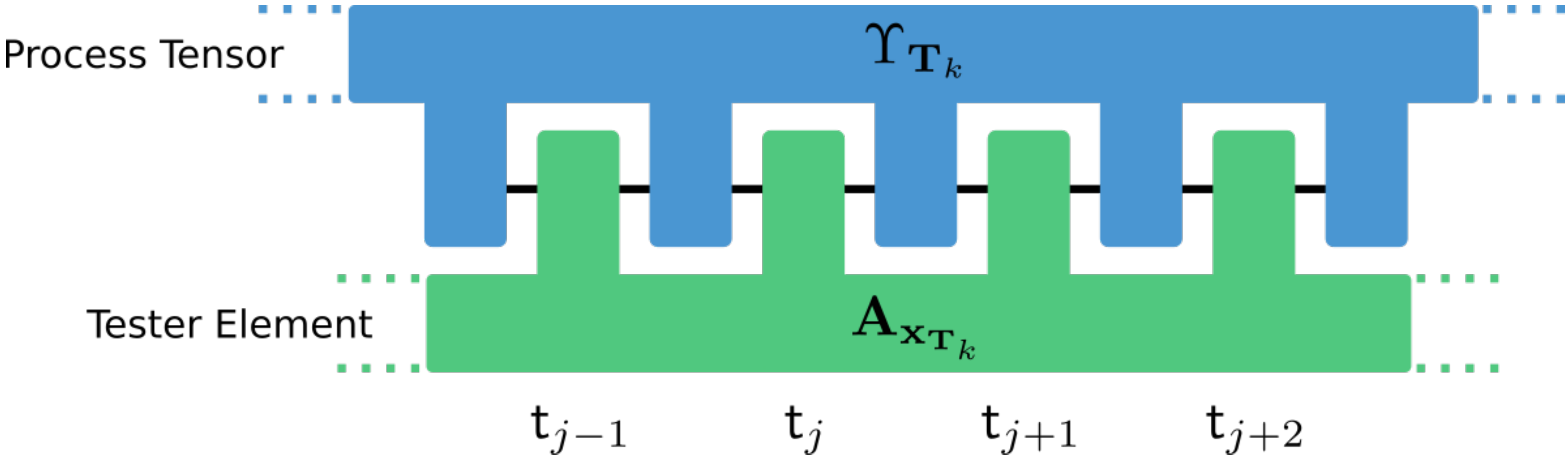
Intervention vs. No Intervention





Milz, Sakuldee, Pollock, Modi Quantum (2020)

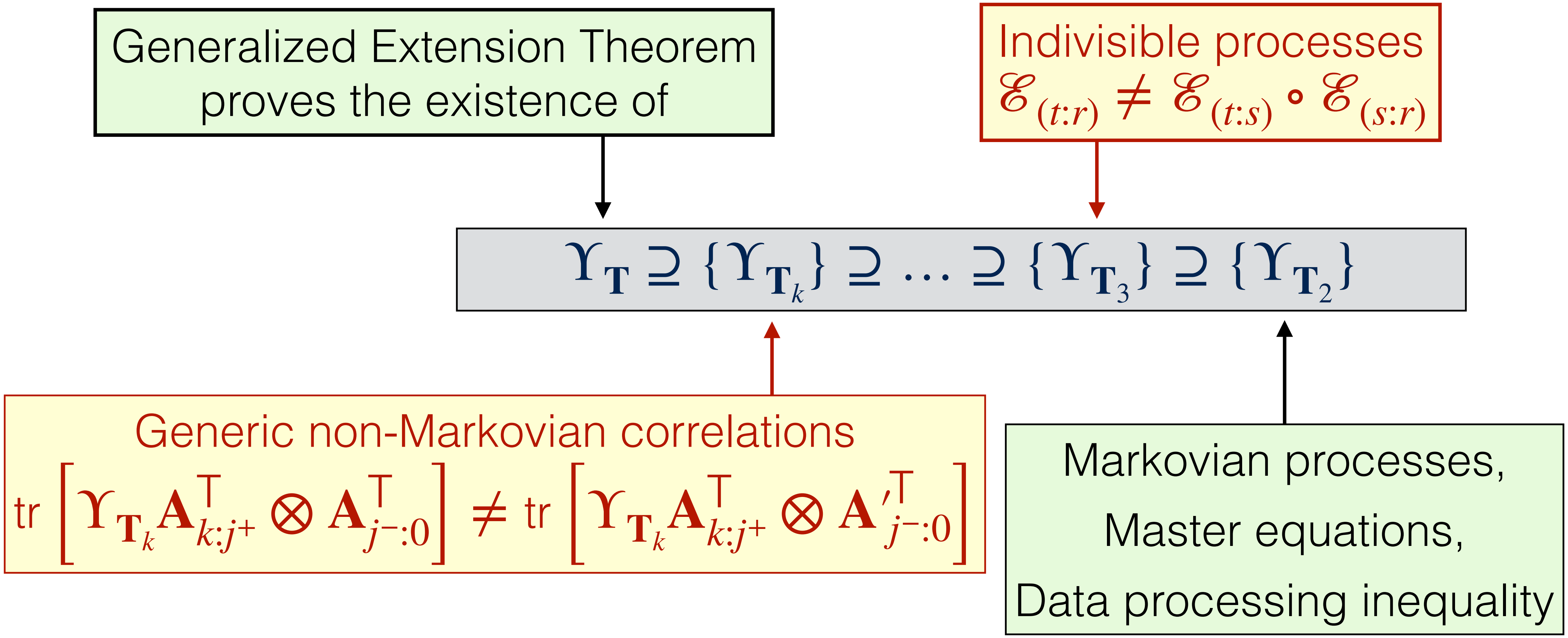
Spatiotemporal born rule



$$\mathbb{P}(\mathbf{x}_{\mathbf{T_k}} \,|\, \mathscr{J}) = \mathbf{tr}[\Upsilon_{T_k} \mathbf{A}^{T}_{\mathbf{x}_{T_k}}]$$

$$\mathbb{P}(0 \,|\, z) = \mathbf{tr}[\rho \Pi_0]$$

Hierarchy of temporal quantum correlations



Generalized Extension Theorem proves the existence of

Indivisible processes

$\mathcal{E}_{(t:r)} \neq \mathcal{E}_{(t:s)} \circ \mathcal{E}_{(s:r)}$

$$\Upsilon_{\mathbf{T}} \supseteq \{\Upsilon_{\mathbf{T}_k}\} \supseteq \dots \supseteq \{\Upsilon_{\mathbf{T}_3}\} \supseteq \{\Upsilon_{\mathbf{T}_2}\}$$

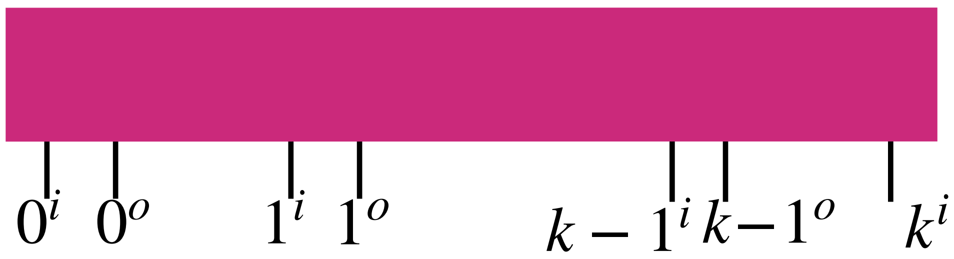
Generic non-Markovian correlations

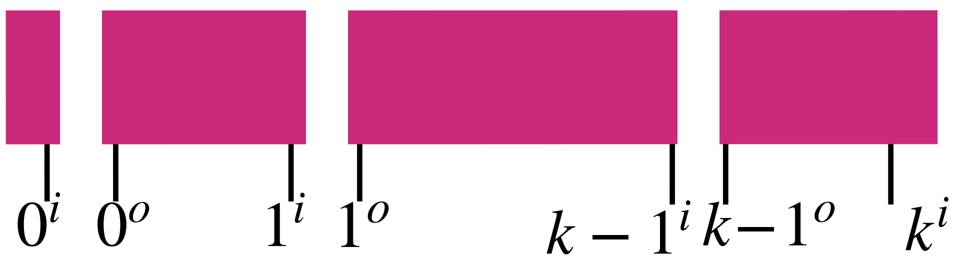
$$\mathrm{tr}\left[\Upsilon_{\mathbf{T}_k} \mathbf{A}_{k:j^+}^{\mathsf{T}} \otimes \mathbf{A}_{j^-:0}^{\mathsf{T}}\right] \neq \mathrm{tr}\left[\Upsilon_{\mathbf{T}_k} \mathbf{A}_{k:j^+}^{\mathsf{T}} \otimes \mathbf{A}'^{\mathsf{T}}_{j^-:0}\right]$$

Markovian processes,

Master equations,

Data processing inequality

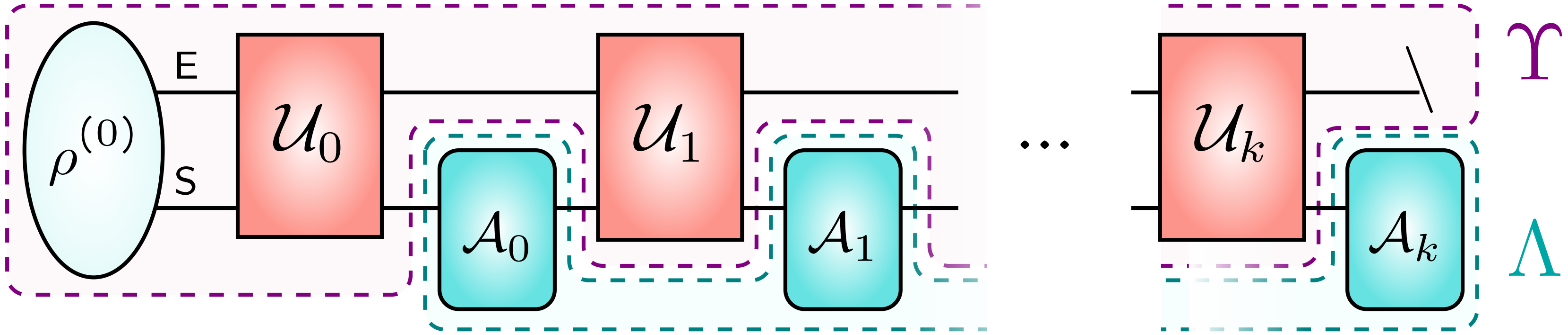




Markovian

Milz, Modi PRX Quantum (2021)

SE unitary processes → Markovian?



$$\mathcal{N}_{\blacklozenge} := \frac{1}{2} \min_{\Upsilon^{(\mathrm{M})}} \| \Upsilon - \Upsilon^{(\mathrm{M})} \|_{\blacklozenge} \approx 0$$

Haar random interactions

$$\boxed{\mathbb{P}[\mathcal{N} \geq \mathcal{B}_k(d_E, d_S) + \epsilon] \leq \mathrm{e}^{-\mathcal{C}(d_E, d_S)\epsilon^2}}$$

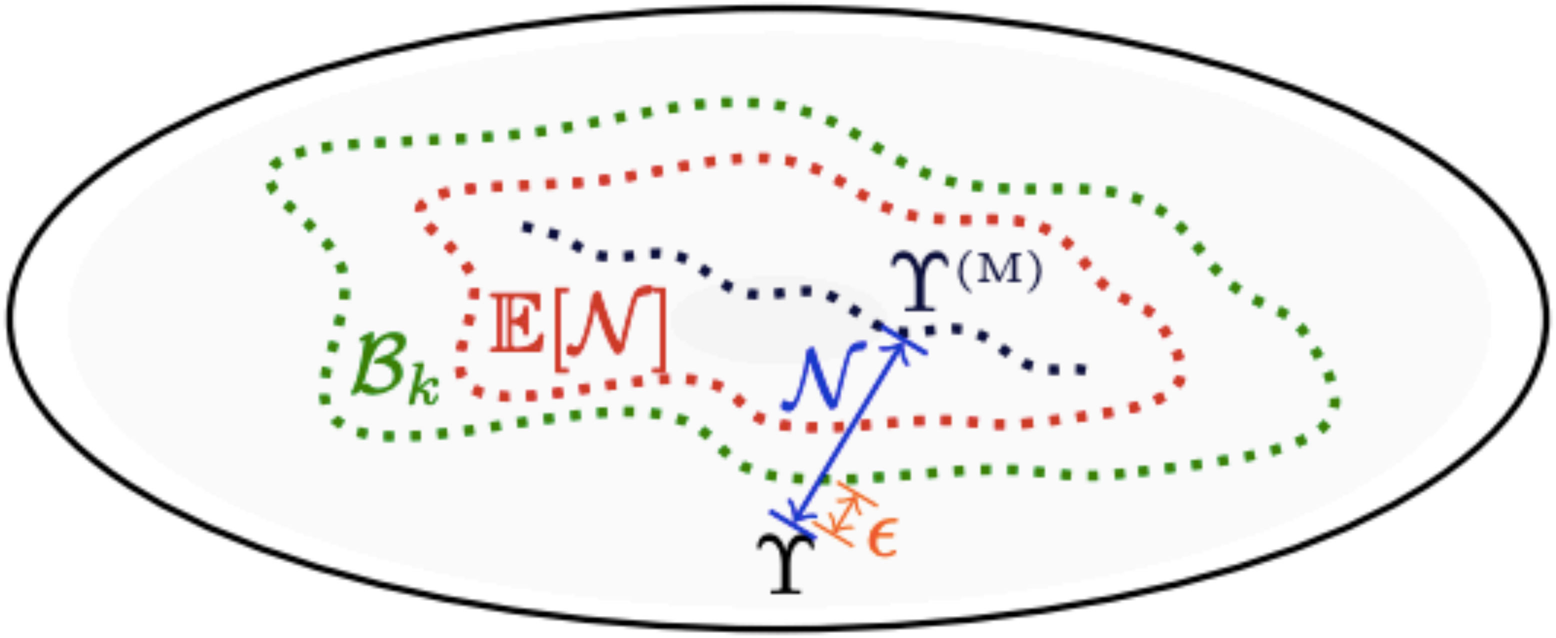
$$\mathcal{B}_k(d_E, d_S) \equiv \begin{cases} \frac{\sqrt{d_E\,\mathbb{E}[\mathrm{tr}(\Upsilon^2)] - x + y}}{2} & \text{if} \quad d_E < d_S^{2k+1} \\ \frac{\sqrt{d_S^{2k+1}\mathbb{E}[\mathrm{tr}(\Upsilon^2)] - 1}}{2} & \text{if} \quad d_E \geq d_S^{2k+1} \end{cases}$$

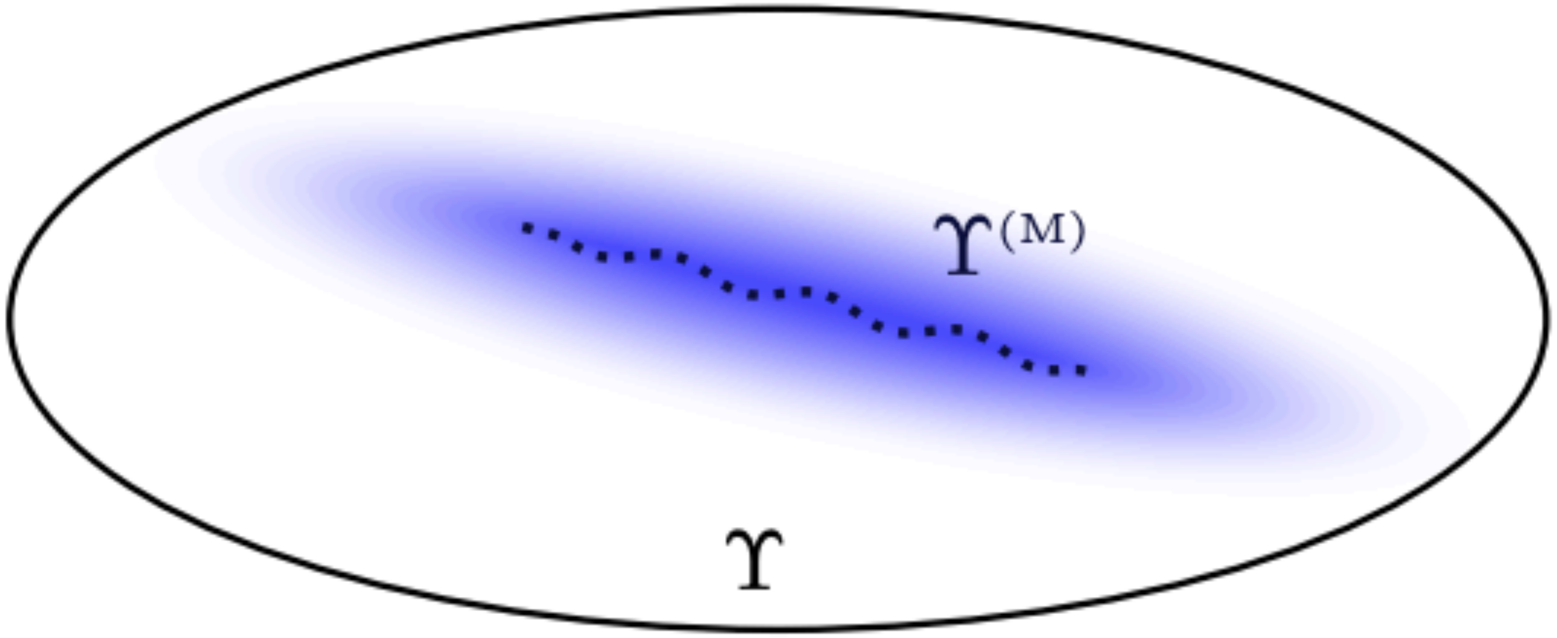
$$\mathcal{C}(d_E, d_S) = c\, d_E d_S \left(\frac{d_S - 1}{d_S^{k+1} - 1}\right)^2 \qquad x \equiv \frac{d_E}{d_S^{2k+1}}(1 + y),\ y \equiv 1 - \frac{d_E}{d_S^{2k+1}}$$

$$\mathbb{E}_{U_i}[\mathrm{tr}(\Upsilon^2)] = \frac{d_E^2 - 1}{d_E(d_E d_S + 1)} \left(\frac{d_E^2 - 1}{d_E^2 d_S^2 - 1}\right)^k + \frac{1}{d_E}$$

Figuroa-Romero, Modi, Pollock, Quantum (2019)

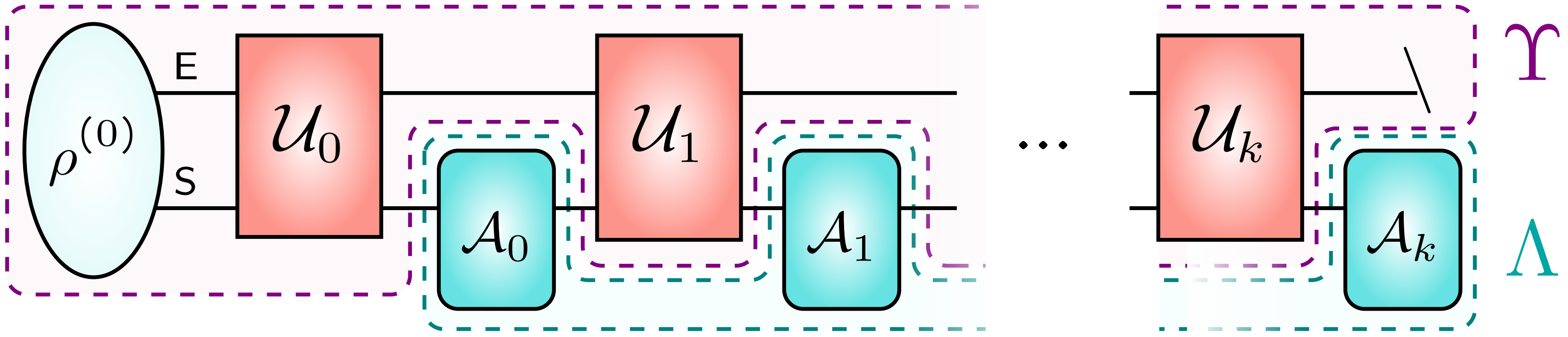
What does it really mean?





Figuroa-Romero, Modi, Pollock, Quantum (2019)

physical SE unitary processes → Markovian?



$$\mathcal{N}_\blacklozenge := \frac{1}{2} \min_{\Upsilon^{(\mathrm{M})}} \| \Upsilon - \Upsilon^{(\mathrm{M})} \|_\blacklozenge \approx 0$$

approximate t-design interactions

$$\left\| \mathbb{E}_{\mathsf{t}_\epsilon}\left[\mathcal{V}^{\otimes s}(X)\right] - \mathbb{E}_{\mathsf{h}}\left[\mathcal{U}^{\otimes s}(X)\right] \right\| \leq \epsilon, \ \ \forall s \leq \mathsf{t}$$

$$V \sim \mu_{\mathsf{t}_\epsilon} \qquad\qquad U \sim \mu_{\mathsf{h}}$$

Figueroa-Romero, Pollock, Modi Commun Phys (2021)

approximate t-design interactions

$$\boxed{\mathbb{P}_{\mathsf{t}_\epsilon}[\,\mathcal{N}_{\blacklozenge} \geq \delta] \leq \mathsf{B}}$$

$$m \in (0, t/4]$$

$$\mathsf{B} := \frac{d_{\mathsf{S}}^{3m(2k+1)}}{\delta^{2m}} \left[\left(\frac{m}{\mathcal{C}}\right)^m + (2\mathcal{B})^{2m} + \frac{\epsilon}{d_{\mathsf{SE}}^{\mathsf{t}}} \eta^{2m} \right]$$

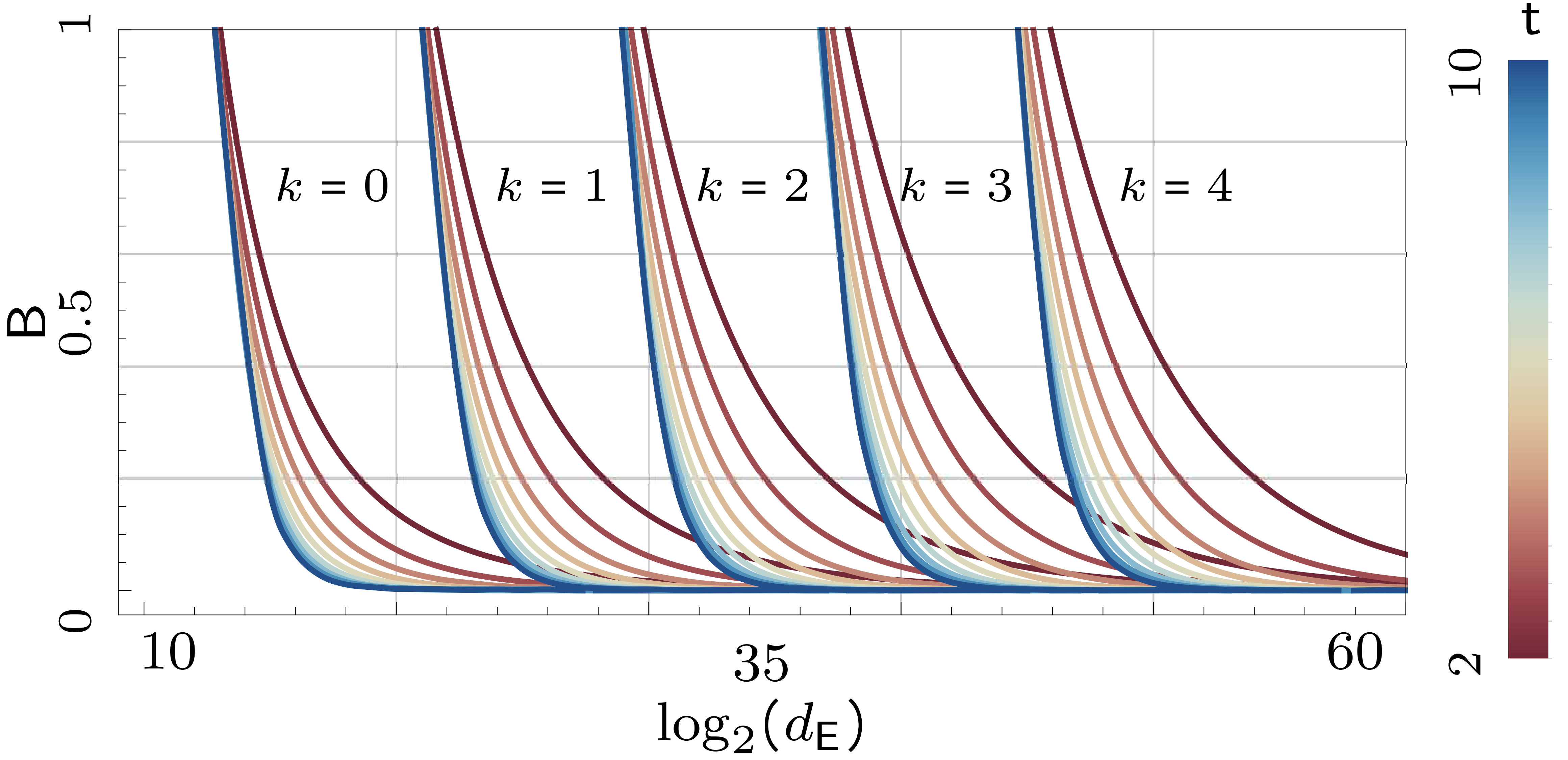
$$\mathcal{C} = \frac{d_{\mathsf{SE}}(k+1)}{16} \left(\frac{d_{\mathsf{S}} - 1}{d_{\mathsf{S}}^{k+1} - 1} \right)^2 \qquad \eta := \left(d_{\mathsf{SE}}^4 d_{\mathsf{S}}^{2k} + d_{\mathsf{S}}^{-(2k+1)} \right) / 4$$

Figueroa-Romero, Pollock, Modi Commun Phys (2021)

upper bounding non-Markovianity

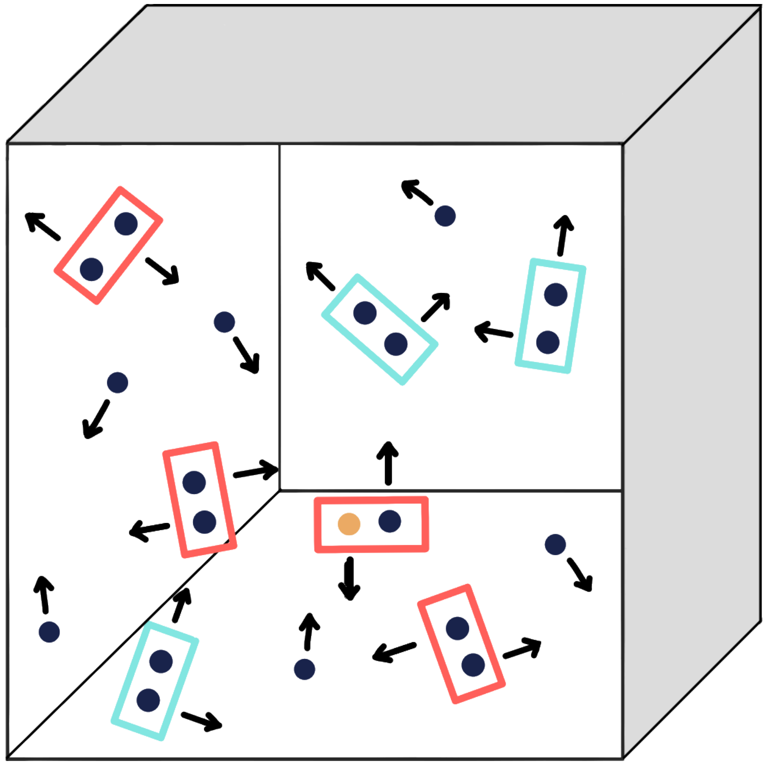
$\mathbb{P}_{t_\epsilon}[\mathcal{N} \geq 0.1]$

$$\epsilon \ll \delta^{2m}\left(2d_{\mathsf{E}}^{-2}d_{\mathsf{S}}^{-(10k+11)/4}\right)^{4m} d_{\mathsf{SE}}^{\mathsf{t}}$$

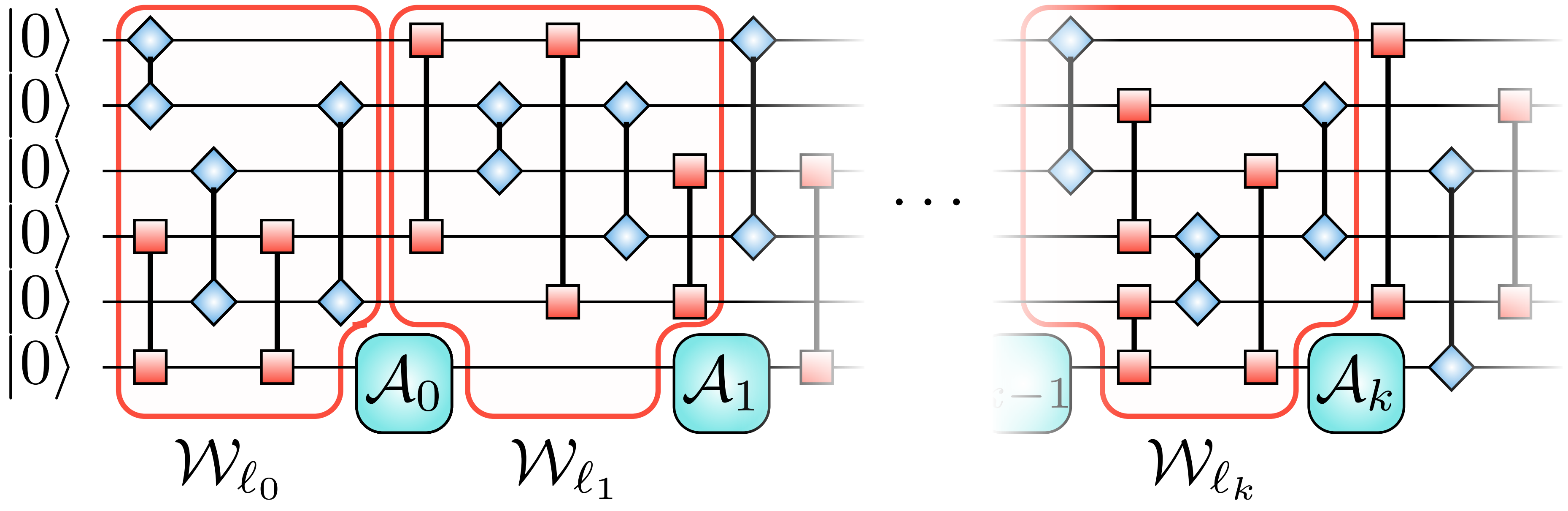


Figueroa-Romero, Modi, Pollock, Quantum (2021)

How quickly does nature forget?



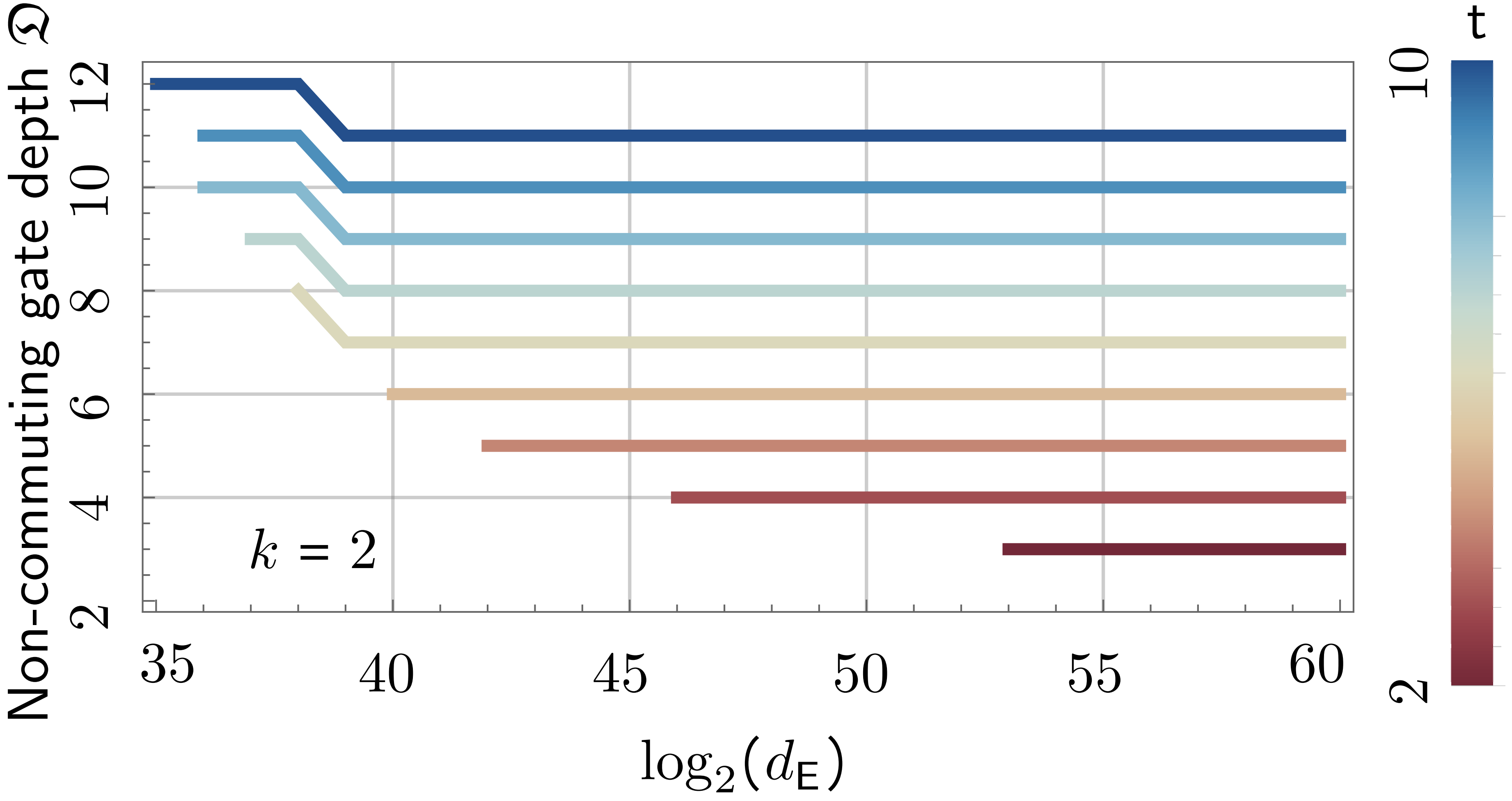
Repeating won't help!



Figueroa-Romero, Pollock, Modi Commun Phys (2021)
Nakata et al. PRX (2017)

Pretty damn fast!

$$\epsilon = 10^{-12}$$



At least, if we don’t look at higher order correlations.

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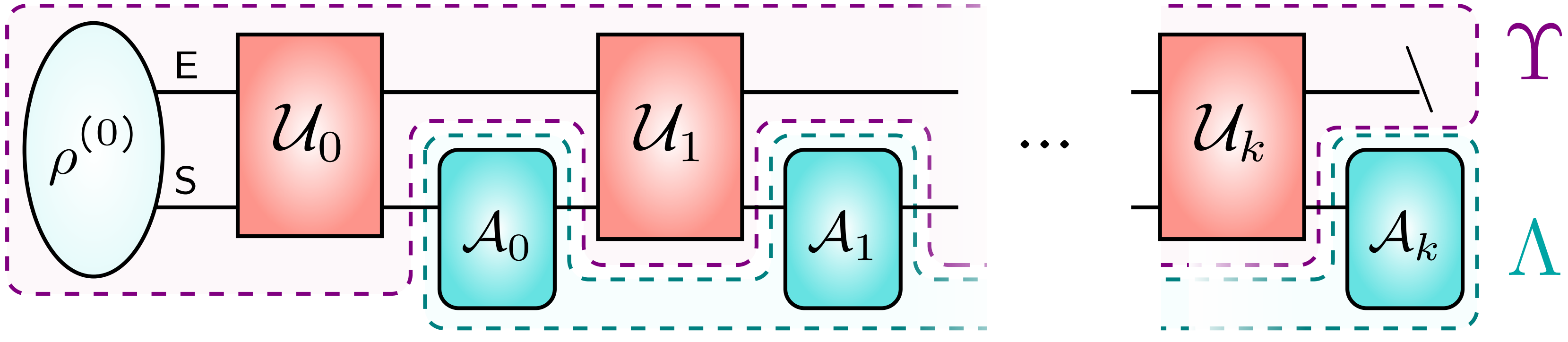
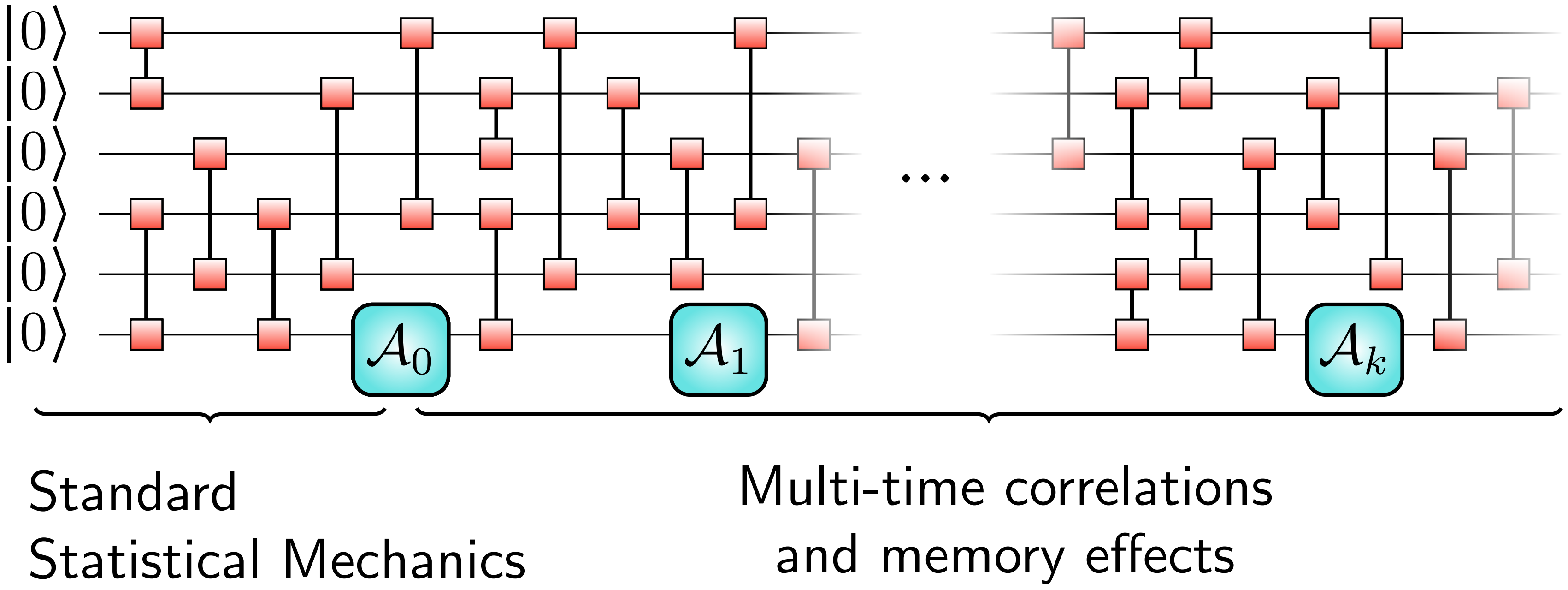
What about thermalization?



(a)



(b)

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Open questions

Does Markovianization imply thermalization?

Are these processes related to ETH?

What about equilibration?

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