Diffusive limit of non-Markovian quantum jumps

Phys. Rev. Lett. 125, 150403 (2020)

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52 Symposium on Mathematical Physics, Toruń
15.06.2021

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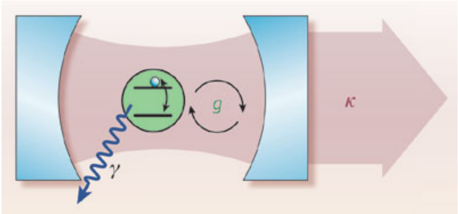
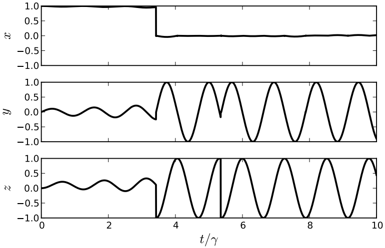
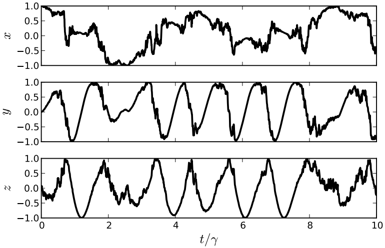
Motivation





direct photodetection

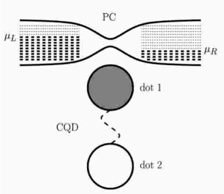


*dyne measurement

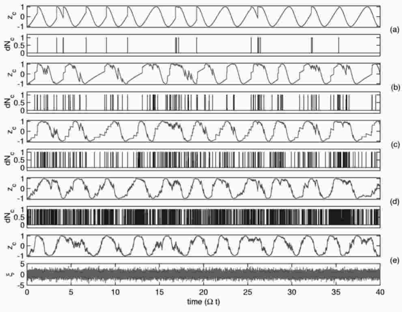
Motivation

- Double quantum dot (DQD) coupled via point-contact to a dissipative reservoir Goan,Milburn, PRB (2001) [1].
- GKSL master equation:

$$\dot{\rho}_t = -i[H_{DQD}, \rho_t] + \mathcal{D}[\mathcal{T}\mathbb{1} + \mathcal{X} n_1]\rho_t, \quad \mathcal{D}[X]\rho = X\rho X^\dagger - \frac{1}{2}\{X^\dagger X, \rho\}$$



Schematic of the system



Limit $\frac{|\mathcal{X}|}{|\mathcal{T}|} \to 0.$

- $\frac{|\mathcal{X}|}{|\mathcal{T}|} \to 0$ corresponds to a weakly responding detector [2].

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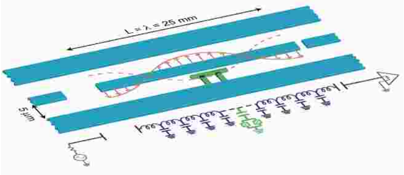
Motivation

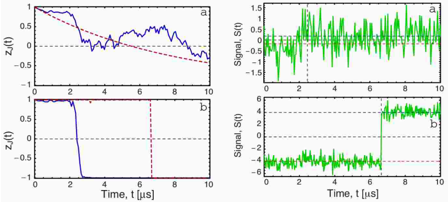
- Super conducting qubit dispersively coupled to a resonator Gambetta et.al, PRA (2008) [3]
- GKSL master equation:

$$\dot{\rho}_t = -i[H_{\text{eff}}, \rho_t] + \kappa \mathcal{D}[a]\rho_t + \gamma_1 \mathcal{D}[\sigma_-]\rho_t + \gamma_\phi \mathcal{D}[\sigma_z]\rho_t.$$

- Signal to noise ratio $S \propto \frac{\kappa\chi^2}{\gamma_1(\kappa^2/4+\chi^2)}$, χ is the dispersive coupling const.



Schematic of the system



Increasing S leads to quantum jumps

- Increasing S corresponds to more strongly responding detector.

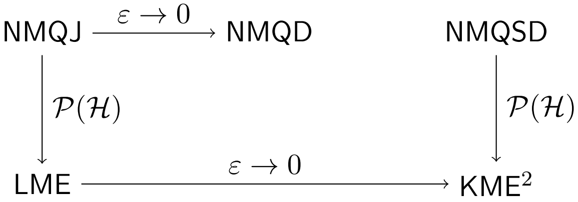
Motivation

- In Markov regime the diffusive limit is well known.
- Carmichael [4], Breuer and Petruccione [5],$\cdots$
- Piecewise deterministic dynamics is given by quantum jumps (QJ) and diffusive dynamics by quantum state diffusion (QSD)

$$\begin{array}{ccc} \text{QJ} & \xrightarrow{\varepsilon \to 0} & \text{QSD} \\ \Big\downarrow \mathcal{P}(\mathcal{H}) & & \Big\downarrow \mathcal{P}(\mathcal{H}) \\ \text{LME} & \xrightarrow{\varepsilon \to 0} & \text{FPE} \end{array}$$

Markov regime

Main result



Beyond Markov regime

- NMQJ – Non-Markovian Quantum Jumps [6]
- NMQSD – Non-Markovian Quantum State Diffusion [7].

Main result

In this talk: Markov = positive decay rates at all times.

Main result

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2. Probability densities on $\mathcal{P}(\mathcal{H})$
3. Master equations beyond GKSL
4. Non-Markovian Quantum State Diffusion
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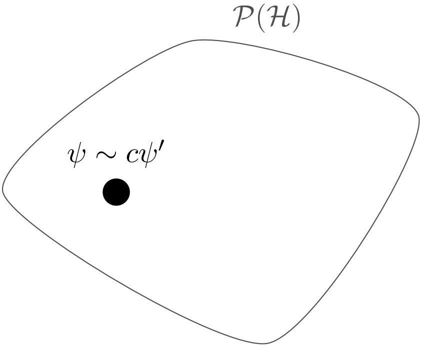
7. Example: Driven TLA
8. Summary and outlook

Probability densities on $\mathcal{P}(\mathcal{H})$



- Vectors $\psi \in \mathcal{H} = \mathbb{C}^d$, $||\psi|| < \infty$.
- We identify each $\psi, \psi' \in \mathcal{H}$ for which $\psi = c\psi'$ and $c \in \mathbb{C}$.
- Coordinates $\psi_k = \langle k|\psi\rangle$ on $\mathcal{P}(\mathcal{H})$ where $\{|k\rangle\}_k$ is ONB of $\mathcal{H}$.

- For any state $\rho \in \mathcal{S}(\mathcal{H})$

$$\rho_{ij} = \int \mathrm{d}\psi\, P[\psi]\, \langle i|\psi\rangle \langle\psi|j\rangle = \int \mathrm{d}\psi \underbrace{P[\psi]}_{\text{prob. density}} \psi_i^* \psi_j,$$

where $\mathrm{d}\psi \equiv \prod_i \mathrm{dRe}\psi_i \mathrm{dIm}\psi_i$.

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Probability densities on $\mathcal{P}(\mathcal{H})$

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Master equations beyond GKSL

Quantum dynamical semigroup — GKSL master equation [8, 9]

$$\dot{\rho} = -i[H,\rho] + \sum_k \gamma_k (L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\})$$

- ρ is the density matrix.
- typically requires weak system bath coupling.
- time scale separation $\tau_C \ll \tau_S, \tau_R$.
- $\gamma_k \geq 0$ are the decay rates.

Master equations beyond GKSL

Master equation with non-definite decay rates

$$\dot{\rho}_t = -i[H, \rho_t] + \underbrace{\sum_{k=1}^{N} \gamma_k(t) \left(L_k \rho_t L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho_t\} \right)}_{:=\text{dissipator}},$$

- $\gamma_k(t)$ can be negative for some values of t.
- N decay channels.

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Master equations beyond GKSL

Symmetry of the dissipator

$$M = 2N, \quad K_k = \mathbb{1} + \varepsilon \xi_k L_k, \quad K_{k+N} = \mathbb{1} + \varepsilon \xi_{k+N} L_k,$$

$$\xi_k + \xi_{k+N} = 0, \quad \tilde{\gamma}_k(t) = \tilde{\gamma}_{k+N}(t) = \frac{\gamma_k(t)}{2\varepsilon^2 |\xi_k|^2},$$

$$\xi_k \in \mathbb{C}, \quad \varepsilon > 0.$$

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Master equations beyond GKSL

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Non-Markovian Quantum State Diffusion

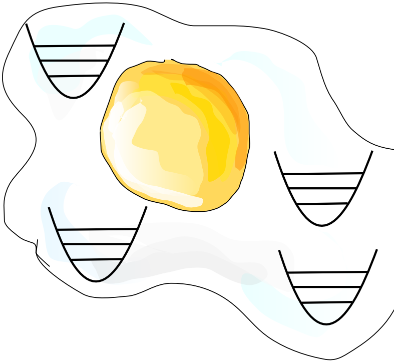
$$\partial_t|\psi(\mathbf{z}^*,t)\rangle = \left(-iH(t) + z_t^* L\right)|\psi(\mathbf{z}^*,t)\rangle - \int_0^t \mathrm{d}s\, \alpha(t,s) L^\dagger \frac{\delta|\psi(\mathbf{z}^*,t)\rangle}{\delta z_s^*}.$$

- Exact representation of the Schrödinger equation for the total system.
- Defines a stochastic propagator $|\psi(\mathbf{z}^*,t)\rangle = G_t(\mathbf{z}^*)|\psi_0\rangle$, $G_0(\mathbf{z}^*) = \mathbb{1}$.
- $G_t(\mathbf{z}^*)$ is not unitary and $|\det G_t(\mathbf{z}^*)| \neq 1$.
- $\alpha(t-s)$ is the bath correlation function.
- More details in [7, 10, 11].

Non-Markovian Quantum State Diffusion

$$H_T(t) = H(t) + H_I(t),$$
$$H_I(t) = \sum_\lambda g_\lambda (L \otimes b_\lambda^\dagger e^{i\omega_\lambda t} + L^\dagger \otimes b_\lambda e^{-i\omega_\lambda t}).$$

$$\partial_t |\Psi_t\rangle = -iH_T(t)|\Psi_t\rangle, \quad |\Psi_0\rangle = |\psi(0)\rangle \otimes |\mathbf{0}\rangle$$



An open system

Non-Markovian Quantum State Diffusion

$$|\Psi_t\rangle = \mathbb{1} \otimes \mathbb{1} |\Psi_t\rangle$$

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Non-Markovian Quantum State Diffusion

$$
\begin{aligned}
|\Psi_t\rangle &= \mathbb{1} \otimes \mathbb{1} |\Psi_t\rangle \\
&= \mathbb{1} \otimes \left(\int \mathrm{d}^2\mathbf{z}\, p(\mathbf{z}) ||\mathbf{z}\rangle\langle\mathbf{z}|| \right) |\Psi_t\rangle
\end{aligned}
$$

Non-Markovian Quantum State Diffusion

$$
\begin{aligned}
|\Psi_t\rangle &= \mathbb{1} \otimes \mathbb{1} |\Psi_t\rangle \\
&= \mathbb{1} \otimes \int \mathrm{d}^2\mathbf{z}\, p(\mathbf{z}) ||\mathbf{z}\rangle \color{red}{\langle \mathbf{z}|||\Psi_t\rangle}
\end{aligned}
$$

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Non-Markovian Quantum State Diffusion

$$
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\end{aligned}
$$

Non-Markovian Quantum State Diffusion

$$\begin{aligned}|\Psi_t\rangle &= \mathbb{1} \otimes \mathbb{1} |\Psi_t\rangle \\ &= \mathbb{1} \otimes \int \mathrm{d}^2\mathbf{z}\, p(\mathbf{z}) ||\mathbf{z}\rangle \langle \mathbf{z}|||\Psi_t\rangle \\ &= \int \mathrm{d}^2\mathbf{z}\, p(\mathbf{z}) |\psi(\mathbf{z}^*, t)\rangle ||\mathbf{z}\rangle.\end{aligned}$$

- $||\mathbf{z}\rangle$ are Bargmann coherent states [12].
- $\mathbf{z} = (z_1, \ldots, z_\lambda, \ldots)$.
- unnormalized, overcomplete and not orthogonal
- $||z_\lambda\rangle = e^{z_\lambda b_\lambda^\dagger} |0_\lambda\rangle$.

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Non-Markovian Quantum State Diffusion

$$
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- $p(\mathbf{z})$ is a zero mean multivariate Gaussian prob. density.
- NMQSD is the equation of motion for $|\psi(\mathbf{z}^*, t)\rangle$.

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Non-Markovian Quantum State Diffusion

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- $||\mathbf{z}\rangle$ are Bargmann coherent states [12].
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- $p(\mathbf{z})$ is a zero mean multivariate Gaussian prob. density.
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Key property

$\langle \mathbf{z}' || \mathbf{z}\rangle$ behaves like $\delta(\mathbf{z}^* - \mathbf{z}'^*)$ under $\int \mathrm{d}^2 z\, p(\mathbf{z})$ for any analytical function $f(\mathbf{z}^*)$.

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Non-Markovian Quantum State Diffusion

- Enforcing (the approximation)

$$\frac{\delta}{\delta z_s^*}|\psi(\mathbf{z}^*,t)\rangle \approx f(t,s)L|\psi(\mathbf{z}^*,t)\rangle,$$

on NMQSD leads to

$$\partial_t|\psi(\mathbf{z}^*,t)\rangle = \left[-iH - F(t)L^\dagger L + z_t^* L\right]|\psi(\mathbf{z}^*,t)\rangle,$$

- $F(t) = \gamma(t) + iS(t)$ and $F(t) = \int_0^t \mathrm{d}s\, f(t,s)\alpha(t-s)$.
- $K(t) = H + (S(t) - i\gamma(t))L^\dagger L$.
- Hierarchy methods [13] and perturbation theory can go beyond the crude approximation used here.

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Non-Markovian Quantum State Diffusion

- The reduced state evolves according to

$$\dot{\rho}_t = -i[H_S + S(t)L^\dagger L, \rho_t] + 2\gamma(t)\left(L\rho_t L^\dagger - \frac{1}{2}\{L^\dagger L, \rho_t\}\right),$$

  where the reduced state $\rho_t = \mathsf{E}\left[|\psi(\mathbf{z}^*, t)\rangle\langle\psi(\mathbf{z}^*, t)|\right]$.
- $F(t) = \gamma(t) + iS(t)$ and $F(t) = \int_0^t \mathrm{d}s\, f(t,s)\alpha(t-s)$.

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Non-Markovian Quantum State Diffusion

- Probability density in $\mathcal{P}(\mathcal{H})$ for NMQSD is

$$P_Q[\psi, t] = \mathsf{E}\left[\prod_i \delta(\psi_i - \psi_i(\mathbf{z}^*, t))\delta(\psi_i - \psi_i(\mathbf{z}^*, t))^*\right].$$

- We can show that P_Q satisfies a 2nd order partial differential equation[1] (KME[2])

$$\partial_t P_Q = \sum_k \partial_k c_k(\psi) P_Q + \partial_k^* c_k(\psi)^* P_Q + \sum_{kl} \partial_k \partial_l^* d_{kl}(\psi) P_Q,$$

which is a Fokker-Planck equation iff $\gamma(t) \geq 0$.

[1]We demand tr $\{L\} = 0$ and denote $\partial_k = \frac{\partial}{\partial \psi_k}$.

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Non-Markovian Quantum State Diffusion

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Non-Markovian Quantum Jumps

- Master equation

$$\dot{\rho} = -i[H_S + S(t)L^\dagger L, \rho] + \sum_k 2\gamma_k(t)\left(L_k \rho L_k^\dagger - \frac{1}{2}\{L_k L_k, \rho\}\right)$$

is unravelled by the following linear NMQJ process [6]

$$\mathrm{d}|\psi\rangle = -iK'(t)|\psi\rangle \mathrm{d}t + (L_k - \mathbb{1})|\psi\rangle \mathrm{d}N_k^+ + \int \mathrm{d}\psi' (|\psi'\rangle - |\psi\rangle)\mathrm{d}N_{k,\psi}^-.$$

- $K'(t) = H_S + S(t)L_k^\dagger L_k - i\sum_k \gamma_k(t)(L_k^\dagger L_k - \mathbb{1})$.

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Non-Markovian Quantum Jumps

- The Poisson increments satisfy

$$\mathrm{d}N_k^+(t)\mathrm{d}N_l^+(t) = \delta_{kl}\mathrm{d}N_k^+(t), \quad \mathrm{d}N_k^+(t)\mathrm{d}N_{l,\psi}^-(t) = 0,$$

$$\mathrm{d}N_{k,\psi}^-(t)\mathrm{d}N_{l,\psi'}^-(t) = \delta(\psi - \psi')\delta_{kl}\mathrm{d}N_{k,\psi}(t).$$

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Non-Markovian Quantum Jumps

- The Poisson increments have the statistics

$$\mathsf{E}\left[\mathrm{d}N_k^+(t)\right] = 2\gamma_+(t)\mathrm{d}t,$$
$$\mathsf{E}\left[\mathrm{d}N_{k,\psi}^-(t)\right] = 2\gamma_-(t)\frac{P_t[\psi']}{P_t[\psi]}\delta(\psi - L_k\psi')\mathrm{d}t.$$

- $\gamma_k(t) = \gamma_k^+(t) - \gamma_k^-(t)$, $\gamma_k^\pm(t) \geq 0$.

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Non-Markovian Quantum Jumps

- Negative jump probability depends on the total ensemble of states.
- At t: $\gamma_k(t) \leq 0$ and trajectory is in state ψ:
  - If $L_k\phi = \psi$ a reverse jump is possible with probability $2|\gamma_k(t)|\frac{P_t[\phi]}{P_t[\psi]}\mathrm{d}t$.
  - Reverse jumps can be computed (numerically) if:
    1. if L_k is invertible then $\phi = L_k^{-1}\psi$ and
    2. if $P_t[\psi]$ can be evaluated at any point on $\mathcal{P}(\mathcal{H})$.

Non-Markovian Quantum Jumps

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Diffusive limit of non-Markovian quantum jumps

- Master equation

$$\dot{\rho}_t = -i[H_S + S(t)L^\dagger L, \rho_t] + 2\gamma(t)\left(L\rho_t L^\dagger - \frac{1}{2}\{L^\dagger L, \rho_t\}\right),$$

is equivalent to

$$\dot{\rho} = -i[H_S + S(t)L^\dagger L, \rho] + \sum_k 2\gamma_k(t)\left(L_k\rho L_k^\dagger - \frac{1}{2}\{L_k L_k, \rho\}\right)$$

- Dissipator (blue) is equivalent to ME (red) when

$$\gamma_k(t) = \frac{\gamma(t)}{2m|\xi_k|\varepsilon^2}, \quad L_k = \mathbb{1} + \varepsilon\xi_k L,$$

where $\xi_k \in \mathbb{C}$, $\xi_k + \xi_{k+M} = 0$, $1 \le k \le 2M$ and $\varepsilon > 0$.

Diffusive limit of non-Markovian quantum jumps

$$\gamma_k(t) = \frac{\gamma(t)}{2m|\xi_k|\varepsilon^2}, \quad L_k = \mathbb{1} + \varepsilon\xi_k L,$$

where $\xi_k \in \mathbb{C}$, $\xi_k + \xi_{k+M} = 0$, $1 \leq k \leq 2M$ and $\varepsilon > 0$.

- Transformation above does not leave NMQJ invariant.
  1. $2M$ Poisson processes
  2. $L_k = \mathbb{1} + \varepsilon\xi_k L$ invertible if $||\epsilon\xi_k L|| < 1$.
  3. State change due to a jump is small: $L_k - \mathbb{1} = \mathcal{O}(\varepsilon)$.
  4. Jump rate scales $\sim \varepsilon^{-2}$.

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Diffusive limit of non-Markovian quantum jumps

- In $\mathcal{P}(\mathcal{H})$ the linear transformed NMQJ is

$$\partial_t P_t[\psi] = i \sum_k \partial_k(\langle k|K(t)|\psi\rangle P_t[\psi]) - \partial_k^*(\langle\psi|K^\dagger(t)|k\rangle P_t[\psi])$$
$$\int \mathrm{d}\phi\, R_t[\psi|\phi]P_t[\phi] - R[\phi|\psi]P_t[\psi],$$

where

$$R_t[\phi|\psi] = \sum_{k=1}^{2M} \frac{\gamma_+(t)}{M\varepsilon^2|\xi_k|^2}\delta(\phi - L_k\psi) + \frac{\gamma_-(t)}{M\varepsilon^2|\xi_k|^2}\frac{P_t[\phi]}{P_t[\psi]}\delta(\psi - L_k\phi).$$

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Diffusive limit of non-Markovian quantum jumps

Perturbative NMQSD in $\mathcal{P}(\mathcal{H})$

$$\partial_t P_Q = \sum_k \partial_k c_k(\psi) P_Q + \partial_k^* c_k(\psi)^* P_Q + \sum_{kl} \partial_k \partial_l^* d_{kl}(\psi) P_Q.$$

1. Clearly: $c_k = i\langle k|K(t)|\psi\rangle$.
2. Proceed by choosing $\varepsilon > 0$ such that $L_k = \mathbb{1} + \varepsilon\xi_k L$ is invertible and $\xi_k + \xi_{k+M} = 0$, then $c_k = i\langle k|K'(t)|\psi\rangle$.
3. Compute $F[\psi] = \int \mathrm{d}\phi\, R_t[\psi|\phi]P_t[\phi] - R[\phi|\psi]P_t[\psi]$
4. Expand $F[\psi]$ to <u>second order in ε</u> and take the limit $\varepsilon \to 0$. Suitable ξ_k can be found such that $F[\psi] \to \partial_k \partial_l^* d_{kl}(\psi)$ as $\varepsilon \to 0$.

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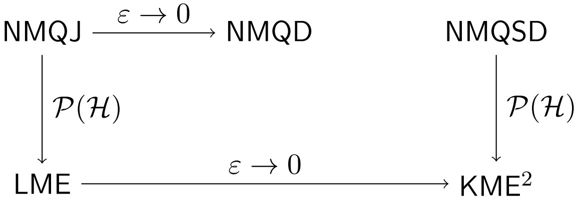
Diffusive limit of non-Markovian quantum jumps



- Diffusive limit of NMQJ in $\mathcal{P}(\mathcal{H})$ leads to the corresponding perturbative NMQSD in $\mathcal{P}(\mathcal{H})$.
- The same diffusive limit leads to a new unraveling in $\mathcal{H}$ which we named non-Markovian quantum diffusion NMQD.

Diffusive limit of non-Markovian quantum jumps

- Observations
  1. NMQSD driven by complex valued process z_t^* and NMQJ by real valued Poisson processes $\mathrm{d}N_t$
     $\Longrightarrow$ we need at least 2 Poisson processes in the diffusive limit.
  2. Correspondence between SDE's and FP equation is many to one
     $\Longrightarrow$ construct diffusive limit in projective Hilbert space.
- Markov limit: commutative diagram

$$\begin{array}{ccc} \text{QJ} & \xrightarrow{\varepsilon \to 0} & \text{QSD} \\ \Big\downarrow \mathcal{P}(\mathcal{H}) & & \Big\downarrow \mathcal{P}(\mathcal{H}) \\ \text{LME} & \xrightarrow{\varepsilon \to 0} & \text{FPE} \end{array}$$

Markov limit

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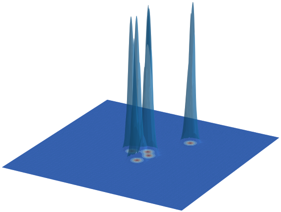
Example: Driven TLA

- Smoothing $P_t[\psi] = \frac{1}{\nu}\sum_{i=1}^{N}\delta(\psi - \psi_\nu(t))$ by

$$P_t[\psi] \approx \frac{1}{\nu\sigma^2\pi^{d+1}}\sum_{i=1}^{\nu} F[(\psi - \psi_i(t))/\sigma], \quad F[\psi] = \frac{1}{\pi^{d+1}}e^{-||\psi||^2}.$$

- $P_t[\psi]$ can be evaluated everywhere in $\mathcal{P}(\mathcal{H})$ and in combination with invertible L_k NMQJ can be used for driven systems, for example.



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Example: Driven TLA

- Master equation

$$\dot{\rho} = -i[\overbrace{\frac{\omega}{2}\sigma_z + \frac{\Omega}{2}\sigma_x}^{=H_S} + S(t)\sigma_+\sigma_-, \rho] + 2\gamma(t)\left(\sigma_-\rho\sigma_+ - \frac{1}{2}\{\sigma_+, \sigma_-, \rho\}\right)$$

- NMQSD

$$\partial_t|\psi_t(z^*)\rangle = \left(-iH_S + z_t^*\sigma_- - F(t)\sigma_+\sigma_-\right)|\psi_t(z^*)\rangle$$

- NMQJ

$$\mathrm{d}\psi = -i\overbrace{\left(H_S - iF(t)\right)\sigma_+\sigma_-}^{K(t)}|\psi\rangle\mathrm{d}t + \sum_{k=1}^{4}\varepsilon\xi_k\sigma_-\left(\mathrm{d}M_k^+ - \mathrm{d}M_k^-\right)|\psi\rangle$$

- NMQD

$$\begin{aligned}\mathrm{d}|\psi\rangle = &\left(-iK(t) + 2\gamma(t)\langle\psi|\sigma_+|0\rangle\partial_0^*\ln P[\psi, t]\right)|\psi\rangle\mathrm{d}t \\ &+ \sigma_-|\psi\rangle(\mathrm{d}Z_+ - \mathrm{d}Z_-)\end{aligned}$$

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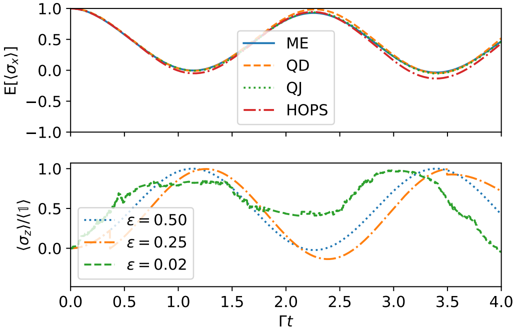
Example: Driven TLA



Top: Ensemble average over 3000 stochastic trajectories of $\langle \sigma_x \rangle$ computed with LNMQJ (dotted), NMQD (dashed) with $\varepsilon = \frac{1}{2}$ and HOPS (dash dotted) with comparison to the master equation solution (ME). **Bottom**: Normalized expectation value for σ_z along a single stochastic trajectory for different values of ε using LNMQJ. The initial state is $|+\rangle = \sqrt{\frac{1}{2}}(|0\rangle + |1\rangle)$.

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Summary and outlook

$$
\begin{array}{ccccc}
\text{NMQJ} & \xrightarrow{\varepsilon \to 0} & \text{NMQD} & & \text{NMQSD} \\
\Big\downarrow \mathcal{P}(\mathcal{H}) & & & & \Big\downarrow \mathcal{P}(\mathcal{H}) \\
\text{LME} & & \xrightarrow{\varepsilon \to 0} & & \text{KME}^2
\end{array}
$$

- Generalization to non-linear version.

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Thank you for your attention!

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