No purification ontology, no quantum paradoxes

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Multiverse Theories Are Bad for Science

New books by a physicist and science journalist mount aggressive but ultimately unpersuasive defenses of multiverses

By John Horgan on November 25, 2019 16

In his book “Something Deeply Hidden”, Carroll asserts that quantum mechanics describes not just very small things but everything, including us. “As far as we currently know,” he writes, “quantum mechanics isn’t just an approximation to the truth; it is the truth.” And however preposterous it might seem, a multiverse, Carroll argues, is an inescapable consequence of quantum mechanics.

……

Carroll proposes furthermore that because quantum mechanics is falsifiable, the many-worlds hypothesis “is the most falsifiable theory ever invented”—even if we can never directly observe any of those many worlds.

Nonsense!

We will see that the Multiverse, more than being an interpretation of the theory, originates from its axiomatic formulation (von Neumann), containing not essential elements.

We will see that this is the case of:

unitarity (regarded as mandatory realization of transformations)

purification of mixed states (i.e. believing that all states are “actually pure”, and “we are part of a huge entangled state”).

To explain what I mean, compare the following two axiomatizations:

Customary mathematical axiomatisation of Quantum Theory		
system	A	$\mathcal{H}_{\mathrm{A}}$
system composition	AB	$\mathcal{H}_{\mathrm{AB}} = \mathcal{H}_{\mathrm{A}} \otimes \mathcal{H}_{\mathrm{B}}$
deterministic pure state	$\sigma \in \mathrm{PurSt}_1(\mathrm{A})$	$\sigma = \|\psi\rangle\langle\psi\|,\ \psi \in \mathcal{H}_{\mathrm{A}},\ \\|\psi\\| = 1$
reversible transf.	$\mathcal{U} \in \mathrm{RevTrn}(\mathrm{A})$	$\mathcal{U}\sigma = U\|\psi\rangle\langle\psi\|U^\dagger,\ U \in \mathbb{U}(\mathrm{A})$
von Neumann-Lüders transformation	$\sigma \to \mathcal{Z}_i\sigma := Z_i \sigma Z_i$	$\{Z_i\}_{i\in\mathrm{X}} \subset \mathrm{Bnd}(\mathcal{H}_{\mathrm{A}})$ PVM
Born rule	$p(i\|\psi) = \langle\psi\|Z_i\|\psi\rangle$	

Minimal mathematical axiomatisation of Quantum Theory		
system	A	$\mathcal{H}_{\mathrm{A}}$
system composition	AB	$\mathcal{H}_{\mathrm{AB}} = \mathcal{H}_{\mathrm{A}} \otimes \mathcal{H}_{\mathrm{B}}$
transformation	$\mathcal{T} \in \mathrm{Trn}(\mathrm{A} \to \mathrm{B})$	$\mathcal{T} \in \mathrm{CP}_{\leq}(\mathrm{T}(\mathcal{H}_{\mathrm{A}}) \to \mathrm{T}(\mathcal{H}_{\mathrm{B}}))$
Born rule	$p(\mathcal{T}) = \mathrm{Tr}\,\mathcal{T}$	$\mathcal{T} \in \mathrm{Trn}(\mathrm{I} \to \mathrm{A})$

Customary mathematical axiomatisation of Quantum Theory

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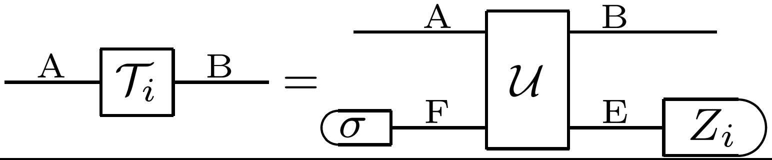
Theorems

trivial system	I	$\mathcal{H}_{\mathrm{I}} = \mathbb{C}$
deterministic states	$\rho \in \mathrm{St}_1(\mathrm{A}) \equiv \mathsf{Conv}(\mathrm{PurSt}_1(\mathrm{A}))$	$\rho \in \mathrm{T}^{+}_{=1}(\mathcal{H}_{\mathrm{A}})$
states	$\rho \in \mathrm{St}(\mathrm{A}) \equiv \mathsf{Cone}_{\leqslant 1}(\mathrm{PurSt}_1(\mathrm{A}))$	$\rho \in \mathrm{T}^{+}_{\leqslant 1}(\mathcal{H}_{\mathrm{A}})$
Transformation as unitary interaction + von Neumann observable on "meter"		$\mathcal{T}_i\rho = \mathrm{Tr}_{\mathrm{E}}[U(\rho\otimes\sigma)U^{\dagger}(I_{\mathrm{B}}\otimes Z_i)]$
transformation	$\mathcal{T} \in \mathrm{Trn}(\mathrm{A}\to\mathrm{B})$	$\mathcal{T} \in \mathrm{CP}_{\leqslant}(\mathrm{T}(\mathcal{H}_{\mathrm{A}}) \to \mathrm{T}(\mathcal{H}_{\mathrm{B}}))$
parallel composition	$\mathcal{T}_1 \in \mathrm{Trn}(\mathrm{A}\to\mathrm{B}),\ \mathcal{T}_2 \in \mathrm{Trn}(\mathrm{C}\to\mathrm{D})$	$\mathcal{T}_1 \otimes \mathcal{T}_2$
sequential composition	$\mathcal{T}_1 \in \mathrm{Trn}(\mathrm{A}\to\mathrm{B}),\ \mathcal{T}_2 \in \mathrm{Trn}(\mathrm{B}\to\mathrm{C})$	$\mathcal{T}_2\mathcal{T}_1$
effects	$\epsilon \in \mathrm{Eff}(\mathrm{A}) \equiv \mathrm{Trn}(\mathrm{A}\to\mathrm{I})$	$\epsilon(\cdot) = \mathrm{Tr}_{\mathrm{A}}[\cdot E],\ 0 \leqslant E \leqslant I_A$
	$\epsilon \in \mathrm{Eff}_1(\mathrm{A}) \equiv \mathrm{Trn}_1(\mathrm{A}\to\mathrm{I})$	$\epsilon = \mathrm{Tr}_{\mathrm{A}}$

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Theorems		
trivial system	I	$\mathcal{H}_\mathrm{I} = \mathbb{C}$
reversible transf.	$\mathcal{U} = U \cdot U^\dagger$	$U \in \mathbb{U}(\mathcal{H}_\mathrm{A})$
determ. transformation	$\mathcal{T} \in \mathrm{Trn}_1(\mathrm{A} \to \mathrm{B})$	$\mathcal{T} \in \mathrm{CP}_{=}(\mathrm{T}(\mathcal{H}_\mathrm{A}) \to \mathrm{T}(\mathcal{H}_\mathrm{B}))$
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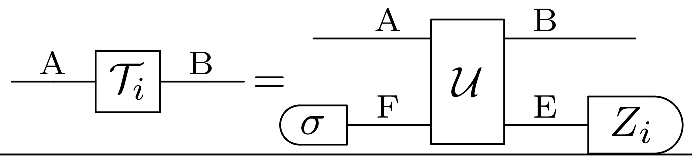
states	$\rho \in \mathrm{St}(\mathrm{A}) \equiv \mathrm{Trn}(\mathrm{I} \to \mathrm{A})$	$\rho \in \mathrm{T}^+_{\leq 1}(\mathcal{H}_\mathrm{A})$
	$\rho \in \mathrm{St}_1(\mathrm{A}) \equiv \mathrm{Trn}_1(\mathrm{I} \to \mathrm{A})$	$\rho \in \mathrm{T}^+_{=1}(\mathcal{H}_\mathrm{A})$
	$\rho \in \mathrm{St}(\mathrm{I}) \equiv \mathrm{Trn}(\mathrm{I} \to \mathrm{I})$	$\rho \in [0,1]$
	$\rho \in \mathrm{St}_1(\mathrm{I}) \equiv \mathrm{Trn}(\mathrm{I} \to \mathrm{I})$	$\rho = 1$
effects	$\epsilon \in \mathrm{Eff}(\mathrm{A}) \equiv \mathrm{Trn}(\mathrm{A} \to \mathrm{I})$	$\epsilon(\cdot) = \mathrm{Tr}_\mathrm{A}[\cdot E],\ 0 \leq E \leq I_A$
	$\epsilon \in \mathrm{Eff}_1(\mathrm{A}) \equiv \mathrm{Trn}_1(\mathrm{A} \to \mathrm{I})$	$\epsilon = \mathrm{Tr}_\mathrm{A}$
Transformations as unitary interaction + von Neumann-Lüders		$\mathcal{T}_i\rho = \mathrm{Tr}_\mathrm{E}[U(\rho \otimes \sigma)U^\dagger(I_\mathrm{B} \otimes Z_i)]$

We observe irreversible processes:
are they obtained through a unitary?

Arguments in favor of optionality of unitarity

- unitarity fails in quantum gravity (Hawking)
- unitarity can be temporarily violated during the black hole evaporation process, accommodating violations of monogamy of entanglement and the no-cloning principle, and allowing assumptions (1), (2), and (3) to be reconciled

  S. Lloyd and J. Preskill, JHEP 08 2014 126

  (1) An evaporating black hole scrambles quantum information without destroying it.
  (2) A freely falling observer encounters nothing unusual upon crossing the event horizon of a black hole.
  (3) An observer who stays outside a black hole detects no violations of relativistic effective quantum field theory.

- Violation of unitarity by Hawking radiation does not violate energy-momentum conservation

  H. Nikolic(Boskovic Inst., Zagreb) Feb 15, 2015

- This is the essence of the black hole information paradox (BHIP): unlike any other classical or quantum system, black holes may not conserve information, thus violating unitarity.

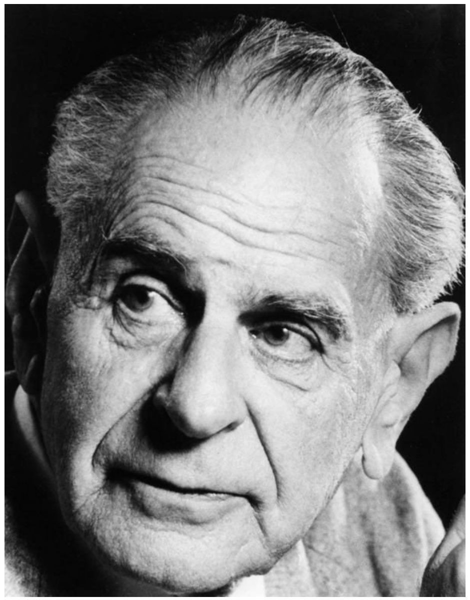
  The Black Hole Information Paradox, S. Antonini, J. Martyn, G. Nambiar,14/10/2018

- Unitarity? Non consistent with AdS/CFT

  Joe Polchinski, Simons Symposium, Caneel Bay 2/5/13

Unitarity of an unknown transformation
and
purity of an unknown state
are not falsifiable!

Quantum falsification tests



The falsification test

Definition 1 (Falsifier). *The event F is a* falsifier *of hypothesis* Hyp *if F cannot happen for* Hyp $=$ TRUE.

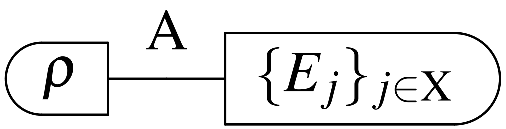
Accordingly we will call the binary test $\{F, F_?\}$ *a falsification test* for hypothesis Hyp, $F_?$ denoting the *inconclusive event*.[2]

Practically one is interested in *effective* falsification tests $\{F, F_?\}$ which are not singleton–the two singleton tests corresponding to $F = 0$ and $F_? = 0$ being the *inconclusive falsification test* and the *logical falsification*, respectively.

Suppose now that one wants to falsify a proposition about the state $\rho \in \mathrm{St}(\mathrm{A})$ of system A. In such case any effective falsification test can be achieved as a binary *observation test* of the form

$$\boxed{\{F, F_?\} \subset \mathrm{Eff}(A), \quad F_? := I_{\mathrm{A}} - F, \quad F > 0, F_? \geq 0,} \tag{0.1}$$

where with the symbol F ($F_?$) we denote both the event and its corresponding positive operator.



Example of falsification test

Consider the proposition

$$\mathrm{Hyp}: \quad \mathsf{Supp}\,\rho = \mathcal{K} \subset \mathcal{H}_{\mathrm{A}},\ \rho \in \mathrm{St}(\mathrm{A}), \qquad \dim \mathcal{H}_{\mathrm{A}} \geq 2 \tag{4.2}$$

where $\mathsf{Supp}\,\rho$ denotes the support of ρ. Then, any operator of the form

$$0 < F \leq I_{\mathrm{A}}, \quad \mathsf{Supp}\,F \subseteq \mathcal{K}^{\perp} \tag{4.3}$$

would have zero expectation for a state ρ satisfying Hyp (4.2), which means that occurrence of F would be a falsification of Hyp, namely

$$\mathrm{Tr}[\rho F] > 0 \Rightarrow \mathrm{Hyp} = \mathsf{FALSE}. \tag{4.4}$$

In this example we can see how the falsification test is not dichotomic, namely the occurrence of $F_?$ does not mean that $\mathrm{Hyp} = \mathsf{TRUE}$, since $F_?$ occurs if $\mathsf{Supp}\,F_? \cap \mathcal{K} \neq 0$. Eq. (4.3) provides the most general falsification test of Hyp (4.2), and the choice $\mathsf{Supp}\,F = \mathcal{K}^{\perp}$ provides the most efficient test since it maximises the falsification chance.

Unfalsifiability of purity of quantum states

Theorem 1 (Unfalsifiability of state-purity)**.** *There exists no test falsifying purity of an unknown state of a given system* A*.*

Proof. In order to falsify the hypothesis

$$\mathrm{Hyp}: \ \rho \in \mathrm{PurSt}(\mathrm{A}), \tag{4.1}$$

we need a falsifier $F \in \mathrm{Eff}(\mathrm{A})$ satisfying

$$\mathrm{Tr}[F\rho] = 0, \ \forall \rho \in \mathrm{PurSt}(\mathrm{A}), \tag{4.2}$$

which means that

$$\forall \psi \in \mathcal{H}_{\mathrm{A}}: \ \langle \psi | F | \psi \rangle = 0, \tag{4.3}$$

namely $F = 0$, which means that the test is inconclusive. ■

By the same argument one can easily prove the impossibility of falsifying purity even when $N > 1$ copies of the state are available.

Notation

In the following we will use maximally entangled pure bipartite states in $\mathcal{H}_{\mathrm{A}} \otimes \mathcal{H}_{\mathrm{B}}$ generally with non equal dimensions $d_{\mathrm{A}} \geq d_{\mathrm{B}}$ and Schmidt number equal to d_{B}. A maximally entangled state of this kind has the general form

$$|V\rangle = \sum_{n=1}^{d_{\mathrm{A}}} \sum_{m=1}^{d_{\mathrm{B}}} V_{nm} |n\rangle \otimes |m\rangle, \tag{10}$$

where the matrix of coefficients V_{nm} correspond to the isometry

$$V = \sum_{n=1}^{d_{\mathrm{A}}} \sum_{m=1}^{d_{\mathrm{B}}} V_{nm} |n\rangle\langle m|, \tag{11}$$

satisfying $V^{\dagger} V = I_{\mathrm{B}}$. We are now in position to prove the following theorem.

Unfalsifiability of max entanglement of a pure bipartite state

Theorem 3 (Unfalsifiability of max-entanglement of a pure state of systems AB) *There exists no test falsifying max-entanglement of a pure bipartite state.*

Proof W.l.g. we consider the case of $d_{\mathrm{A}} \geq d_{\mathrm{B}}$, as in Eq. (10). Falsification of max-entanglement of state $|V\rangle\langle V|$ needs a falsifier $F \in \mathrm{Eff}(\mathrm{AB})$ satisfying

$$\mathrm{Tr}[F|V\rangle\langle V|] = 0, \ \forall |V\rangle \text{ maximally entangled.} \tag{12}$$

In particular, since unitary transformations on either $\mathcal{H}_{\mathrm{A}}$ or $\mathcal{H}_{\mathrm{B}}$ preserve max-entanglement, one has

$$\mathrm{Tr}[F(\mathcal{U} \otimes \mathcal{I}_{\mathrm{B}})|V\rangle\langle V|] = 0, \ \forall \mathcal{U} = U \cdot U^{\dagger}, U \in \mathbb{U}(\mathcal{H}_{\mathrm{A}}). \tag{13}$$

It follows that the average over the unitary group $G_{\mathrm{A}} = SU(d_{\mathrm{A}})$ must be zero, corresponding to[5]

$$\begin{aligned} 0 &= \int_{G_{\mathrm{A}}} d\mathcal{U}\, \mathrm{Tr}[F(\mathcal{U} \otimes \mathcal{I}_{\mathrm{B}})|V\rangle\langle V|] = \mathrm{Tr}[F(I_{\mathrm{A}} \otimes \mathrm{Tr}_{\mathrm{A}}[|V\rangle\langle V|])] \\ &= \mathrm{Tr}[F(I_{\mathrm{A}} \otimes (V^{\dagger}V)^{*})] = \mathrm{Tr}[F(I_{\mathrm{A}} \otimes I_{\mathrm{B}})] = \mathrm{Tr}[F], \end{aligned} \tag{14}$$

where the complex conjugation is w.r.t. the chosen basis in Eq.(10). Eq. (14) implies that $F = 0$, which contradicts the falsification effectiveness condition $F > 0$. $\square$

Unfalsifiability of isometricity of a quantum transformation

Theorem 4 (Unfalsifiability of isometricity of a transformation from B **to** A **with** $d_{\mathrm{A}} \geq d_{\mathrm{B}}$**)** *There exists no test falsifying isometricity of a transformation* $\mathcal{V} \in \mathrm{Trn}(\mathrm{B} \to \mathrm{A})$ *with* $\dim \mathcal{H}_{\mathrm{A}} \geq \dim \mathcal{H}_{\mathrm{B}}$*.*

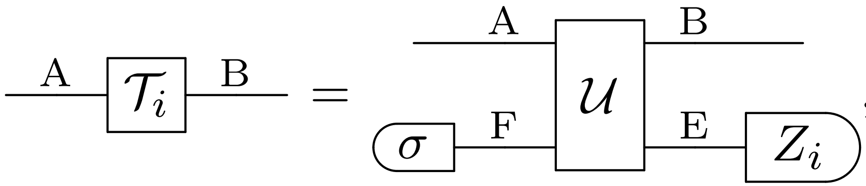
Proof The application of the operator to a fixed maximally-entangled state puts isometricity transformations in one-to-one correspondence with maximally entangled states. Thus, falsifying maximal isometricity on a transformation is equivalent to falsifying maximal entanglement of a state, which is impossible. $\square$

Corollary 1 (It is not possible to falsify unitarity of a transformation)

Proof Obviously Theorem 4 exclude the possibility of falsifying unitarity of a transformation, since it is a special case of isometricity.

Unfalsifiability of unitary realization of a transformation

The impossibility of establishing the unitariety of transformation (Theorem 4) with input and output systems under our control excludes the possibility of falsifying that a transformation is actually achieved unitarily, according to the scheme

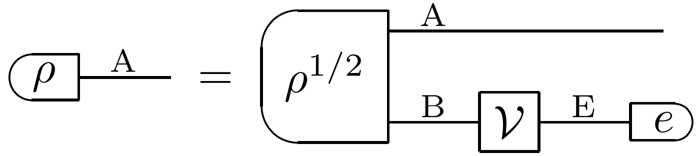


, (5.10)

with $\{Z_i\}$ von Neuman measurement over the output environment E, and the input environment F is prepared in a state σ. Systems E, F, state σ, measurement Z_i, and unitary $\mathcal{U}$ are all not unique and unknown, hence the testing resorts to falsifying unitarity of $\mathcal{U}$, which is impossible, not even with control of input-output systems AF and BE.

Unfalsifiability of a mixed state being the marginalization of a pure one

Any purification of the mixed state $\rho \in \mathrm{St}(\mathrm{A})$ can be written in the following diagrammatic form



, (15)

with $d_\mathrm{B} = d_\mathrm{A} \leqslant d_\mathrm{E}$ and e denoting the deterministic effect, corresponding to discarding system E, and $\mathcal{V}$ being any map isometric on $\mathsf{Supp}\, \rho$. We thus resort to the falsifiability of being a pure state of the form $(I_\mathrm{A} \otimes V)|\rho^{1/2}\rangle_{\mathrm{AA}}$.

Unfalsifiability of a mixed state being the marginalization of a pure one

Theorem 5 (Unfalsifiability of mixed state in $\mathrm{St(A)}$ being the marginalisation of a pure state of AE with $d_\mathrm{E} \geq d_\mathrm{A}$) *There exists no test falsifying the assertion that a mixed state in $\mathrm{St(A)}$ is actually the marginal of a pure state of AE with $\dim \mathcal{H}_\mathrm{E} \geq \dim \mathcal{H}_\mathrm{A}$.*

Proof Consider the general purification scheme in Eq. (15). Upon denoting by $\mathcal{V} \in \mathrm{Trn(A \to E)}$ an isometric transformation with $d_\mathrm{E} \geq d_\mathrm{B} = d_\mathrm{A}$, a falsifier $F \in \mathrm{Bnd}^+\mathrm{(AE)}$ should satisfy the following identity

$$\mathrm{Tr}[F(\mathcal{I}_\mathrm{A} \otimes \mathcal{V})|\rho^{1/2}\rangle\langle\rho^{1/2}||] = 0, \tag{16}$$

and by unitarily connecting all the possible isometries $\mathcal{V}$ with fixed support, one has

$$\mathrm{Tr}[F(\mathcal{I}_\mathrm{A} \otimes \mathcal{U}\mathcal{V})|\rho^{1/2}\rangle\langle\rho^{1/2}||] = 0, \quad \forall \mathcal{U},\, \mathcal{U} = U \cdot U^\dagger,\, U \in \mathbb{U}(\mathcal{H}_\mathrm{E}). \tag{17}$$

Unfalsifiability of a mixed state being the marginalization of a pure one

It follows that the average over the unitary group $G_{\mathrm{E}} = SU(d_{\mathrm{E}})$ must be zero, corresponding to

$$\begin{aligned} 0 &= \int_{G_{\mathrm{E}}} d\mathcal{U}\, \mathrm{Tr}[F(\mathcal{I}_{\mathrm{A}} \otimes \mathcal{U}\mathcal{V})|\rho^{1/2}\rangle\langle\rho^{1/2}|] = \mathrm{Tr}[F(\mathrm{Tr}_{\mathrm{E}}[|\rho^{1/2}V^T\rangle\langle\rho^{1/2}V^T|] \otimes I_{\mathrm{E}}])] \\ &= \mathrm{Tr}[F(\rho \otimes I_{\mathrm{E}})] = \mathrm{Tr}[F_{\mathrm{A}}\rho], \end{aligned} \tag{18}$$

where T denotes the transpose w.r.t. the basis for the representation of the $\rho^{1/2}$ purification, and $F_{\mathrm{A}} = \mathrm{Tr}_{\mathrm{E}}\, F$. For ρ full-rank one has $F_{\mathrm{A}} = 0$, implying $\mathrm{Tr}\, F = 0$, namely $F = 0$, proving the statement. For $\mathsf{Rnk}\,\rho < d_{\mathrm{A}}$, F_{A} becomes a falsifier of $\mathsf{Supp}\,\rho$, which is known a priori. $\square$

This excludes the possibility of falsifying that a knowingly mixed state of a quantum system A is actually the marginal of a pure entangled state with an environment system E. Moreover, the system E is unknown (we just know that it must have dimension $d_{\mathrm{E}} \geq d_{\mathrm{A}}$).

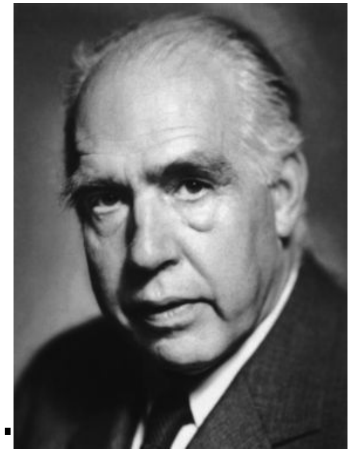
Conclusions:

Bye bye many-worlds, Rovelli's, … interpretations…

These are actually <u>interpretations of non-mandatory postulates</u>

However, they remain helpful tools for reasoning

…<u>but the interpretation of the strict-theory is Copenhagen's</u>



Purification and unitariety
make a powerful and elegant symmetry of the theory

They simplify the theoretical evaluations, but …

Quantum Theory

is intrinsically a *theory of irreversible phenomena*

which we strive to explain in a deterministic fashion.

“This is more or less what I wanted to say”

Thank you for your attention