Entanglement negativity as a universal non-Markovianity witness

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Symposium on Mathematical Physics, Toruń
June 18, 2019

European Funds Smart Growth
Republic of Poland
FNP Foundation for Polish Science
European Union European Regional Development Fund

Outline

1. Witnessing non-Markovianity with contractive functions
2. Witnessing non-Markovianity with entanglement
3. Example: eternally non-Markovian dynamics

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Witnessing non-Markovianity with contractive functions

- Consider two-state function $f(\rho, \sigma)$ such that

$$f(\Lambda[\rho], \Lambda[\sigma]) \leq f(\rho, \sigma)$$

for all CPTP maps and all states ρ, σ

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- Examples: trace distance, quantum relative entropy
- Markovian dynamics $\Lambda_t = V_{t,s} \circ \Lambda_s$ lead to monotonic decrease of f for all $0 \leq s \leq t$:

$$\begin{aligned} f(\Lambda_t[\rho], \Lambda_t[\sigma]) &= f(V_{t,s} \circ \Lambda_s[\rho], V_{t,s} \circ \Lambda_s[\sigma]) \\ &\leq f(\Lambda_s[\rho], \Lambda_s[\sigma]) \end{aligned}$$

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- **Witness of non-Markovianity:** $\frac{\mathrm{d}}{\mathrm{d}t} f(\Lambda_t[\rho], \Lambda_t[\sigma]) > 0$

Witnessing non-Markovianity with contractive functions

Consider P-divisible dynamics $\Lambda_t = V_{t,s} \circ \Lambda_t$ such that

$$V_{t,s}[\rho] = p\mathcal{E}_1[\rho] + (1-p)\mathcal{E}_2[\rho^T] \quad (1)$$

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Theorem: For any non-Markovian evolution $\Lambda_t = V_{t,s} \circ \Lambda_s$ with $V_{t,s}$ fulfilling Eq. (1) it holds that:

$$\frac{\mathrm{d}}{\mathrm{d}t} f(\Lambda_t[\rho], \Lambda_t[\sigma]) \leq 0$$

for any contractive function $f(\rho, \sigma)$ and any single-qubit states ρ and σ.

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for any contractive function $f(\rho, \sigma)$ and any single-qubit states ρ and σ.

$\rightarrow$contractive functions of two single-qubit states cannot witness all non-Markovianity

Proof of the theorem

- For any two single-qubit states ρ and σ there exists a unitary U such that
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where
$$V_{t,s}[\rho] = p\mathcal{E}_1[\rho] + (1-p)\mathcal{E}_2[\rho^T],$$
$$\Phi_{t,s}[\rho] = p\mathcal{E}_1[\rho] + (1-p)\mathcal{E}_2\left[U\rho U^\dagger\right].$$

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- Combining these results, we obtain:

$$\begin{aligned} f\left(\Lambda_t[\rho], \Lambda_t[\sigma]\right) &= f\left(V_{t,s} \circ \Lambda_s[\rho], V_{t,s} \circ \Lambda_s[\sigma]\right) \\ &= f\left(\Phi_{t,s} \circ \Lambda_s[\rho], \Phi_{t,s} \circ \Lambda_s[\sigma]\right) \\ &\leq f\left(\Lambda_s[\rho], \Lambda_s[\sigma]\right) \end{aligned}$$

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Quantifying entanglement[a]

Postulates on entanglement monotones E:

[a] Vedral, Plenio, Rippin, Knight, Phys. Rev. Lett. **78**, 2275 (1997)
[b] Vidal and Werner, Phys. Rev. A **65**, 032314 (2002)

Quantifying entanglement[a]

Postulates on entanglement monotones E:

- $E^{A|B}(\rho^{AB}) \geq 0$ with equality on non-entangled (separable) states $\rho_{\text{sep}}^{AB} = \sum_i p_i \rho_i^A \otimes \rho_i^B$

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- Entanglement negativity[b]:

$$E^{A|B}(\rho^{AB}) = \frac{\|\rho^{T_B}\|_1 - 1}{2}$$

with trace norm $\|M\|_1 = \mathrm{Tr}\sqrt{M^\dagger M}$ and partial transpose T_B

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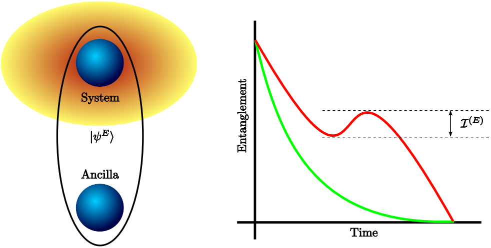
Witnessing non-Markovianity with entanglement[a]



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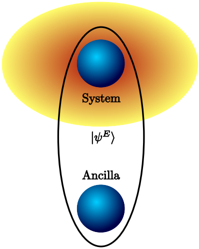

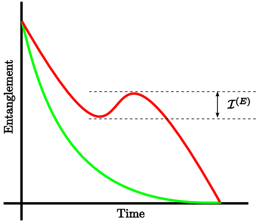

Local Markovian dynamics $\Lambda_t = V_{t,s} \circ \Lambda_s$ lead to monotonic decrease of entanglement:

$$E^{A|B}(\Lambda_t^A \otimes \mathbb{1}^B[\rho^{AB}]) = E^{A|B}(V_{t,s}^A \circ \Lambda_s^A \otimes \mathbb{1}^B[\rho^{AB}])$$
$$\leq E^{A|B}(\Lambda_s^A \otimes \mathbb{1}^B[\rho^{AB}])$$

for $0 \leq s \leq t$

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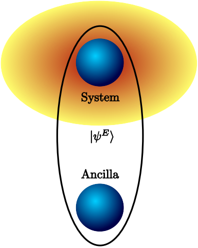

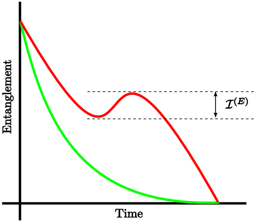

Witness of non-Markovianity[a]:

$$\frac{\mathrm{d}}{\mathrm{d}t} E^{A|B}(\Lambda_t^A \otimes \mathbb{1}^B[\rho^{AB}]) > 0$$

for some entanglement monotone E and some t

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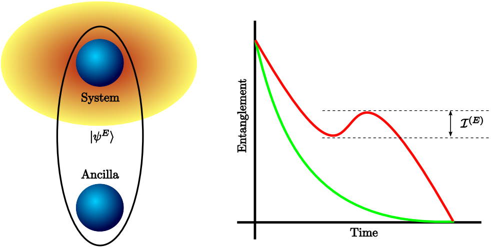

- Witness not universal[b]: there exist non-Markovian evolutions with $\frac{\mathrm{d}}{\mathrm{d}t} E^{A|B}(\Lambda_t^A \otimes \mathbb{1}^B[\rho^{AB}]) \leq 0$ for all t
- Example: $\Lambda_t = \begin{cases} \mathcal{E}_t & \text{for } \; t \leq 1 \\ \tilde{\mathcal{E}}_{t-1} \circ \mathcal{E}_1 & \text{for } \; t > 1 \end{cases}$
  with Markovian evolution $\mathcal{E}_t$ s.t. $\mathcal{E}_1$ is entanglement breaking

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Witnessing non-Markovianity with entanglement

Extension to tripartite setting:

$$\frac{\mathrm{d}}{\mathrm{d}t} E^{AB|C}(\Lambda_t^A \otimes \mathbb{1}^{BC}[\rho^{ABC}]) > 0$$

potentially universal witness of non-Markovianity?

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Theorem[a]**:** *For any invertible non-Markovian evolution Λ_t there exists a quantum state ρ^{ABC} such that*

$$\frac{\mathrm{d}}{\mathrm{d}t} E^{AB|C}(\Lambda_t^A \otimes \mathbb{1}^{BC}[\rho^{ABC}]) > 0$$

for some $t > 0$. For single-qubit evolutions the statement also holds for non-invertible dynamics.

[a] Kołodyński, Rana, Streltsov, arXiv:1903.08663

Proof of the theorem[a]

- Consider the initial state

$$\rho^{ABC} = p_1 \rho_1^{AB_1} \otimes |\Psi^+\rangle\langle\Psi^+|^{B_2C} + p_2 \rho_2^{AB_1} \otimes |\Psi^-\rangle\langle\Psi^-|^{B_2C}$$

with $|\Psi^\pm\rangle = (|01\rangle \pm |10\rangle)/\sqrt{2}$

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- The time-evolved state takes the form

$$\tau_t^{ABC} = p_1 \Lambda_t^A \left[\rho_1^{AB_1}\right] \otimes |\Psi^+\rangle\langle\Psi^+|^{B_2C} + p_2 \Lambda_t^A \left[\rho_2^{AB_1}\right] \otimes |\Psi^-\rangle\langle\Psi^-|^{B_2C}$$

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- Partially transposed state:

$$\tau_t^{T_C} = \frac{1}{2}\Lambda_t^A\left[p_1\rho_1^{AB_1} + p_2\rho_2^{AB_1}\right]\otimes\left(|01\rangle\langle 01|^{B_2C} + |10\rangle\langle 10|^{B_2C}\right) + \frac{1}{2}\Lambda_t^A\left[p_1\rho_1^{AB_1} - p_2\rho_2^{AB_1}\right]\otimes\left(|\Phi^+\rangle\langle\Phi^+|^{B_2C} - |\Phi^-\rangle\langle\Phi^-|^{B_2C}\right)$$

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- Evaluating trace norm of $\tau_t^{T_C}$:

$$\left\|\tau_t^{T_C}\right\|_1 = 1 + \left\|\Lambda_t^A\left[p_1\rho_1^{AB_1} - p_2\rho_2^{AB_1}\right]\right\|_1$$

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- Negativity of τ_t^{ABC}:

$$E^{AB|C}\left(\tau_t^{ABC}\right) = \frac{\|\tau_t^{T_C}\|_1 - 1}{2} = \frac{1}{2}\left\|\Lambda_t^A\left[p_1\rho_1^{AB_1} - p_2\rho_2^{AB_1}\right]\right\|_1$$

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- For any invertible non-Markovian dynamics Λ_t there exist probabilities p_i and states ρ_i such that[bc]

$$\frac{\mathrm{d}}{\mathrm{d}t}\left\|\Lambda_t^A\left[p_1\rho_1^{AB_1} - p_2\rho_2^{AB_1}\right]\right\|_1 > 0$$

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- For any invertible non-Markovian dynamics Λ_t there exist probabilities p_i and states ρ_i such that[b,c]

$$\frac{\mathrm{d}}{\mathrm{d}t}\left\|\Lambda_t^A\left[p_1\rho_1^{AB_1} - p_2\rho_2^{AB_1}\right]\right\|_1 > 0$$

- For single-qubit dynamics Λ_t the statement holds also for non-invertible evolutions[d]

[a] Kołodyński, Rana, Streltsov, arXiv:1903.08663
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Example: eternally non-Markovian dynamics

- Eternally non-Markovian dynamics[a]:

$$\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} = \sum_{i=1}^{3} \gamma_i(t) \left[\sigma_i \rho(t) \sigma_i - \rho(t)\right] \qquad (2)$$

with $\gamma_1 = \gamma_2 = \alpha \frac{c}{2}$, $\gamma_3(t) = -\alpha \frac{c}{2} \tanh(ct)$ with $\alpha \geq 1$ and $c > 0$

[a] Hall, Cresser, Li, Andersson, Phys.Rev. A **89**, 042120 (2014)
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- Evolution (2) can be written as $\Lambda_t = V_{t,s} \circ \Lambda_s$ with

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Non-Markovianity of (2) cannot be witnessed by contractive functions $f(\rho, \sigma)$ of single-qubit states and bipartite negativity[b]:

$$\frac{\mathrm{d}}{\mathrm{d}t} E^{A|B}(\Lambda_t^A \otimes \mathbb{1}^B[\rho^{AB}]) \leq 0$$

[a] Hall, Cresser, Li, Andersson, Phys.Rev. A **89**, 042120 (2014)

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Example: eternally non-Markovian dynamics

Lemma[a]**:** Negativity is monotonic under local maps of the form

$$P^A \otimes \mathbb{1}^B[\rho] = p\mathcal{E}_1^A \otimes \mathbb{1}^B[\rho] + (1-p)\mathcal{E}_2^A \otimes \mathbb{1}^B\left[\rho^{T_A}\right]. \qquad (3)$$

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Proof:

$$E^{A|B}\left(P^A \otimes \mathbb{1}^B[\rho]\right) = \frac{1}{2}\left(\left\|P^A \otimes \mathbb{1}^B\left[\rho^{T_B}\right]\right\|_1 - 1\right)$$
$$\leq \frac{1}{2}\left(p\left\|\mathcal{E}_1^A \otimes \mathbb{1}^B\left[\rho^{T_B}\right]\right\|_1 + (1-p)\left\|\mathcal{E}_2^A \otimes \mathbb{1}^B\left[\rho^{T_{AB}}\right]\right\|_1 - 1\right)$$
$$= \frac{p}{2}\left(\left\|\mathcal{E}_1^A \otimes \mathbb{1}^B\left[\rho^{T_B}\right]\right\|_1 - 1\right) = pE^{A|B}\left(\mathcal{E}_1^A \otimes \mathbb{1}^B[\rho]\right) \leq E^{A|B}(\rho)$$

[a] Kołodyński, Rana, Streltsov, arXiv:1903.08663

Example: eternally non-Markovian dynamics

Lemma[a]**:** Negativity is monotonic under local maps of the form

$$P^A \otimes \mathbb{1}^B[\rho] = p\mathcal{E}_1^A \otimes \mathbb{1}^B[\rho] + (1-p)\mathcal{E}_2^A \otimes \mathbb{1}^B\left[\rho^{T_A}\right]. \qquad (3)$$

Proof:

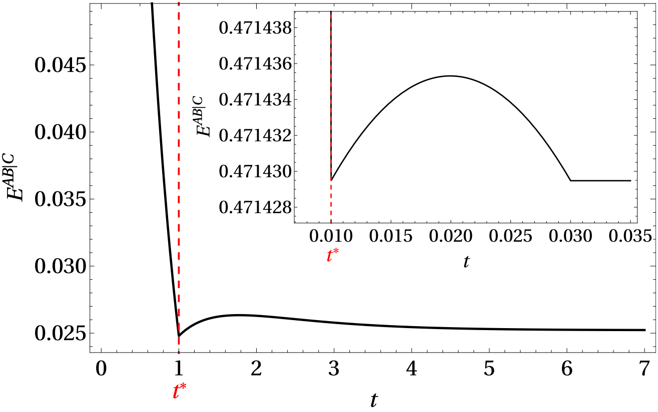
$$E^{A|B}\left(P^A \otimes \mathbb{1}^B[\rho]\right) = \frac{1}{2}\left(\left\|P^A \otimes \mathbb{1}^B\left[\rho^{T_B}\right]\right\|_1 - 1\right)$$
$$\leq \frac{1}{2}\left(p\left\|\mathcal{E}_1^A \otimes \mathbb{1}^B\left[\rho^{T_B}\right]\right\|_1 + (1-p)\left\|\mathcal{E}_2^A \otimes \mathbb{1}^B\left[\rho^{T_{AB}}\right]\right\|_1 - 1\right)$$
$$= \frac{p}{2}\left(\left\|\mathcal{E}_1^A \otimes \mathbb{1}^B\left[\rho^{T_B}\right]\right\|_1 - 1\right) = pE^{A|B}\left(\mathcal{E}_1^A \otimes \mathbb{1}^B[\rho]\right) \leq E^{A|B}(\rho)$$

$\Rightarrow$ Any local dynamics $\Lambda_t^A = V_{t,s}^A \circ \Lambda_s^A$ with $V_{t,s}^A$ of the form (3) fulfills

$$\frac{\mathrm{d}}{\mathrm{d}t} E^{A|B}(\Lambda_t^A \otimes \mathbb{1}^B[\rho^{AB}]) \leq 0$$

[a] Kołodyński, Rana, Streltsov, arXiv:1903.08663

Example: eternally non-Markovian dynamics[a]



Non-Markovianity of eternally non-Markovian dynamics can be witnessed in the tripartite setting by $E^{AB|C}(\Lambda_t^A \otimes \mathbb{1}^{BC}[\rho^{ABC}])$

[a] Kołodyński, Rana, Streltsov, arXiv:1903.08663

Summary

Entanglement negativity in tripartite setting[a]**:**
universal non-Markovianity witness for

- invertible dynamics in any dimension
- all (also non-invertible) qubit dynamics

[a]Kołodyński, Rana, Streltsov, arXiv:1903.08663

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- any contractive function of qubit states $f(\rho, \sigma)$

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Open question:
can entanglement monotones universally witness non-Markovianity of all evolutions, incuding non-invertible dynamics beyond qubits?

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[a] Kołodyński, Rana, Streltsov, arXiv:1903.08663