Bound states in the continuum for an array of quantum emitters

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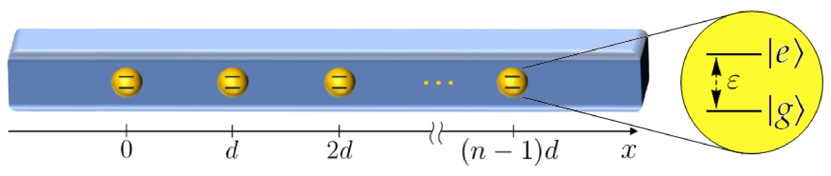
Physical system



- n two-level emitters, spacing d;
- structured one-dimensional photon continuum with $\omega(k)=\sqrt{k^2+m^2}$.

Hamiltonian

The evolution preserves the number of excitations, according to the dipolar interaction:

$$H=\varepsilon\sum_{j=1}^{n}|e_j\rangle\langle e_j|+\int \mathrm{d}k\,\omega(k)b^\dagger(k)b(k)+\sum_{j=1}^{n}\int \mathrm{d}k\,\left[F(k)\,e^{\mathrm{i}(j-1)kd}|e_j\rangle\langle g_j|\,b(k)+h.c.\right],$$

with $[b(k),b^\dagger(k')]=\delta(k-k')$ and form factor $F(k)=\sqrt{\frac{\gamma}{2\pi\omega(k)}}$.

Hilbert space

For the one-excitation sector:

$$|\Psi\rangle=\sum_{j=1}^{n}a_j\,|E_j^{(n)}\rangle\otimes|\mathrm{vac}\rangle+|G^{(n)}\rangle\otimes\int \mathrm{d}k\;\xi(k)b^\dagger(k)\,|\mathrm{vac}\rangle\,,$$

with $|E_j^{(n)}\rangle=\bigotimes_{l=1}^{j-1}|g_l\rangle\otimes|e_j\rangle\bigotimes_{l=j+1}^{n}|g_l\rangle$ and $|G^{(n)}\rangle=\bigotimes_{l=1}^{n}|g_l\rangle$.

Bound states in the continuum

Inverse propagator

It is defined for the emitters reduced system as $G^{-1}(E)=(\varepsilon-E)\mathbb{1}-\Sigma(E)$, where $\Sigma(E)$ represents the self-energy matrix, and its kernel determines the eigensystem:

$$\ker\{G^{-1}(E)\}=\{\boldsymbol{a}\in\mathbb{C}^n : G^{-1}(E)\cdot\boldsymbol{a}=0\}.$$

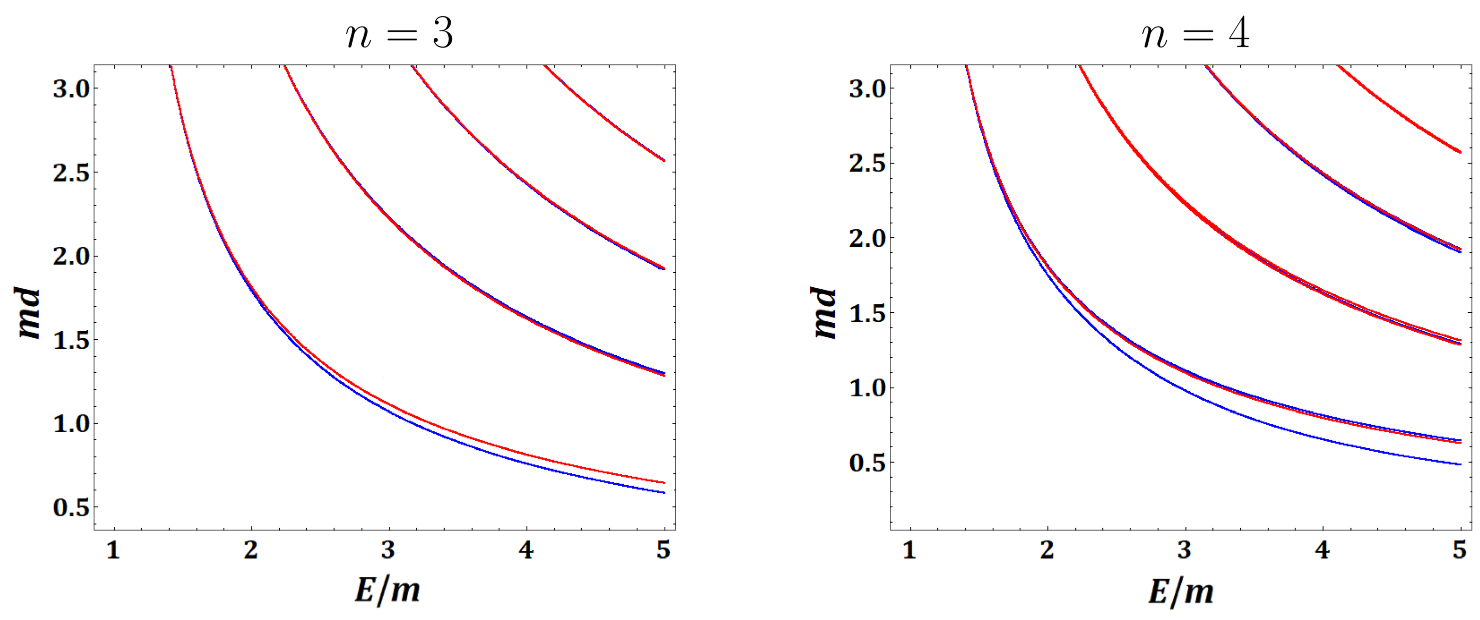
Resonance condition

A linearized dispersion relation yields an eigenspace $(n-1)$-times degenerate corresponding to

$$E_\nu(d)=\sqrt{\frac{\nu^2\pi^2}{d^2}+m^2},$$

with $\nu\in\mathbb{N}$, while considering the full form factor this degeneracy is lifted.

The two panels show spectral lines, in blue for symmetric eigenstates, red for antisymmetric ones:



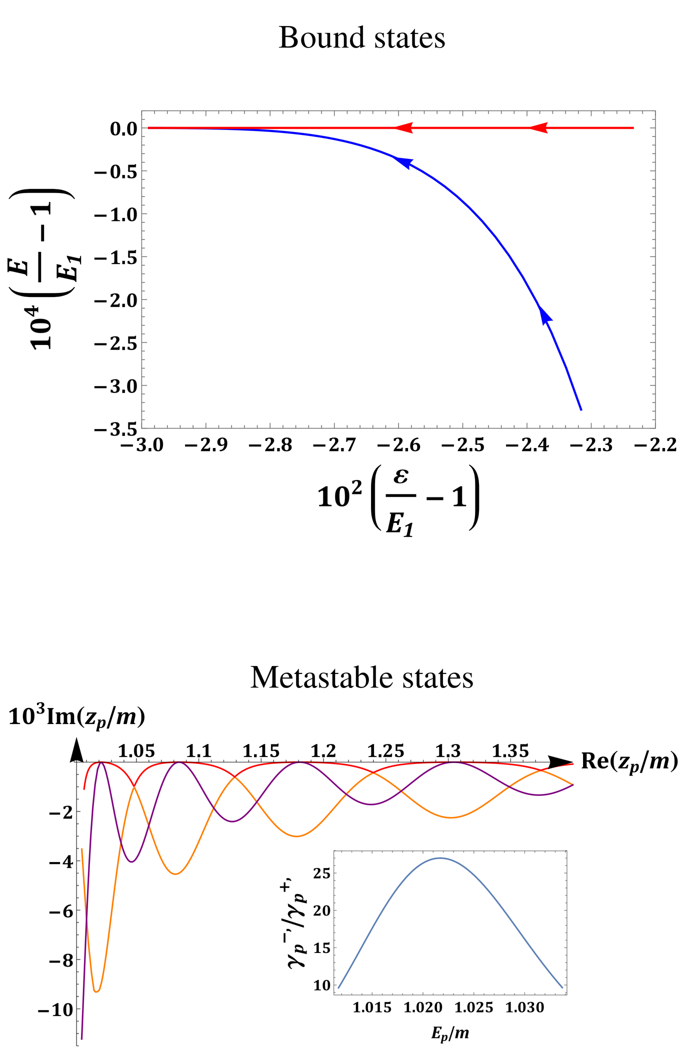
Metastable states

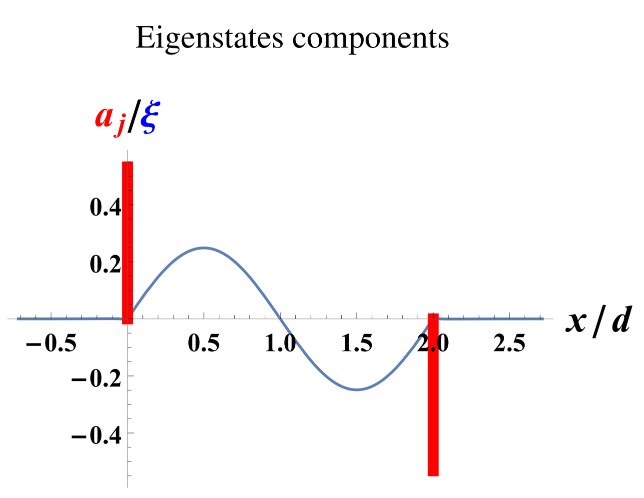
The analytic continuation of the inverse propagator to the second Riemann sheet:

$$G^{(\mathrm{II})-1}(z)=(\varepsilon-z)\mathbb{1}-\Sigma^{(\mathrm{II})}(z),$$

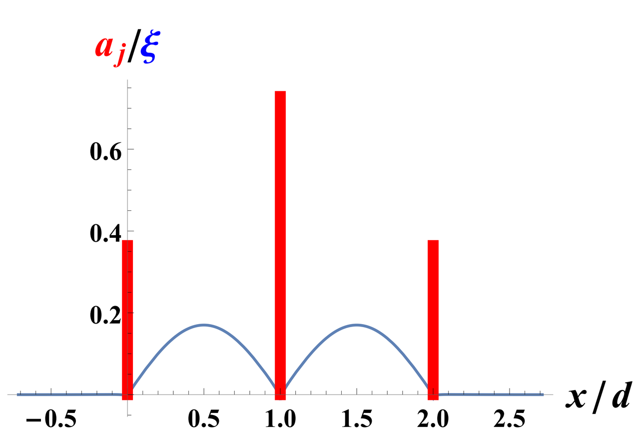
allows us to characterize the singularity condition in the complex energy plane $z_p=E_p-\mathrm{i}\gamma_p/2$.

Three emitters system



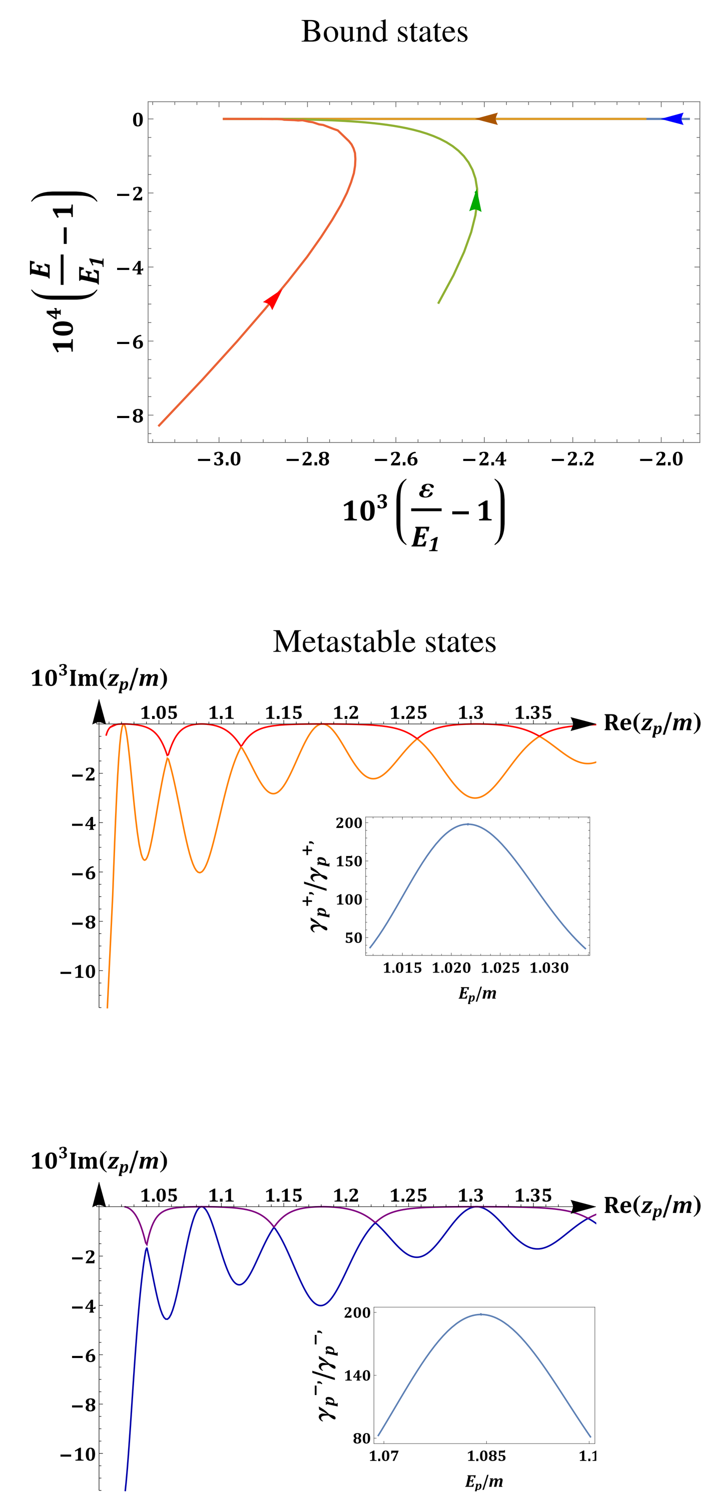


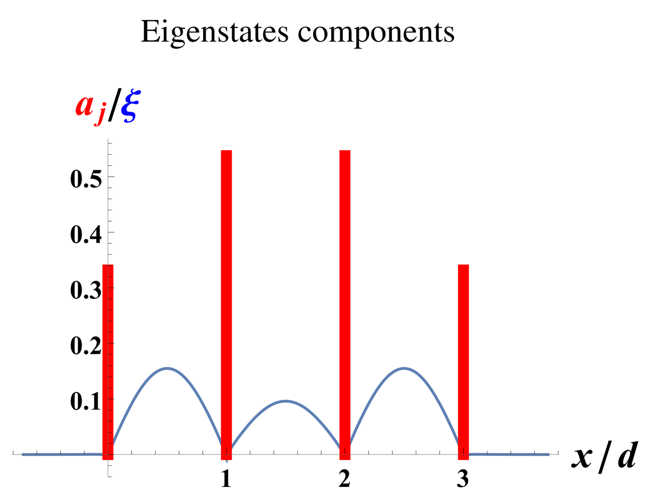
$\frac{a_1}{a_3}=-1,\ a_2=0$



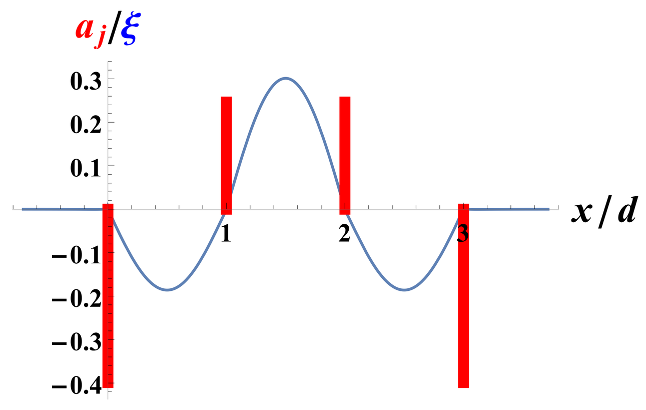
$\frac{a_1}{a_3}=1,\ a_2\simeq 2$

Four emitters system

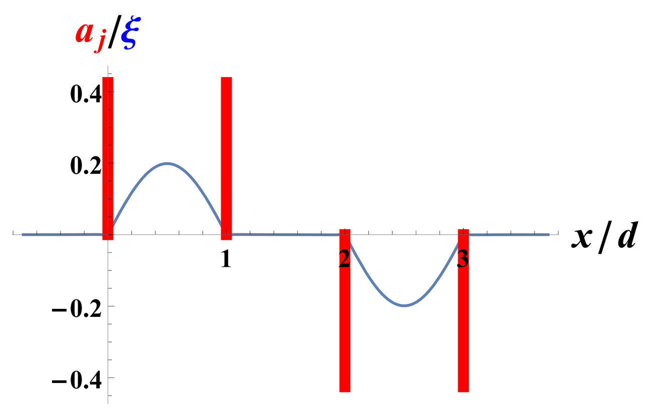




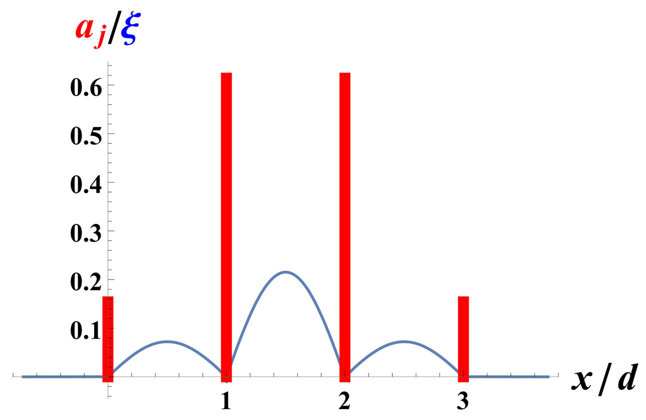
$\frac{a_1}{a_4}=\frac{a_2}{a_3}=1,\ \frac{a_1}{a_2}\simeq\frac{\sqrt{5}-1}{2}$



$\frac{a_1}{a_4}=\frac{a_2}{a_3}=1,\ \frac{a_1}{a_2}\simeq-\frac{\sqrt{5}+1}{2}$



$\frac{a_1}{a_2}=\frac{a_3}{a_4}=-\frac{a_2}{a_3}=1$



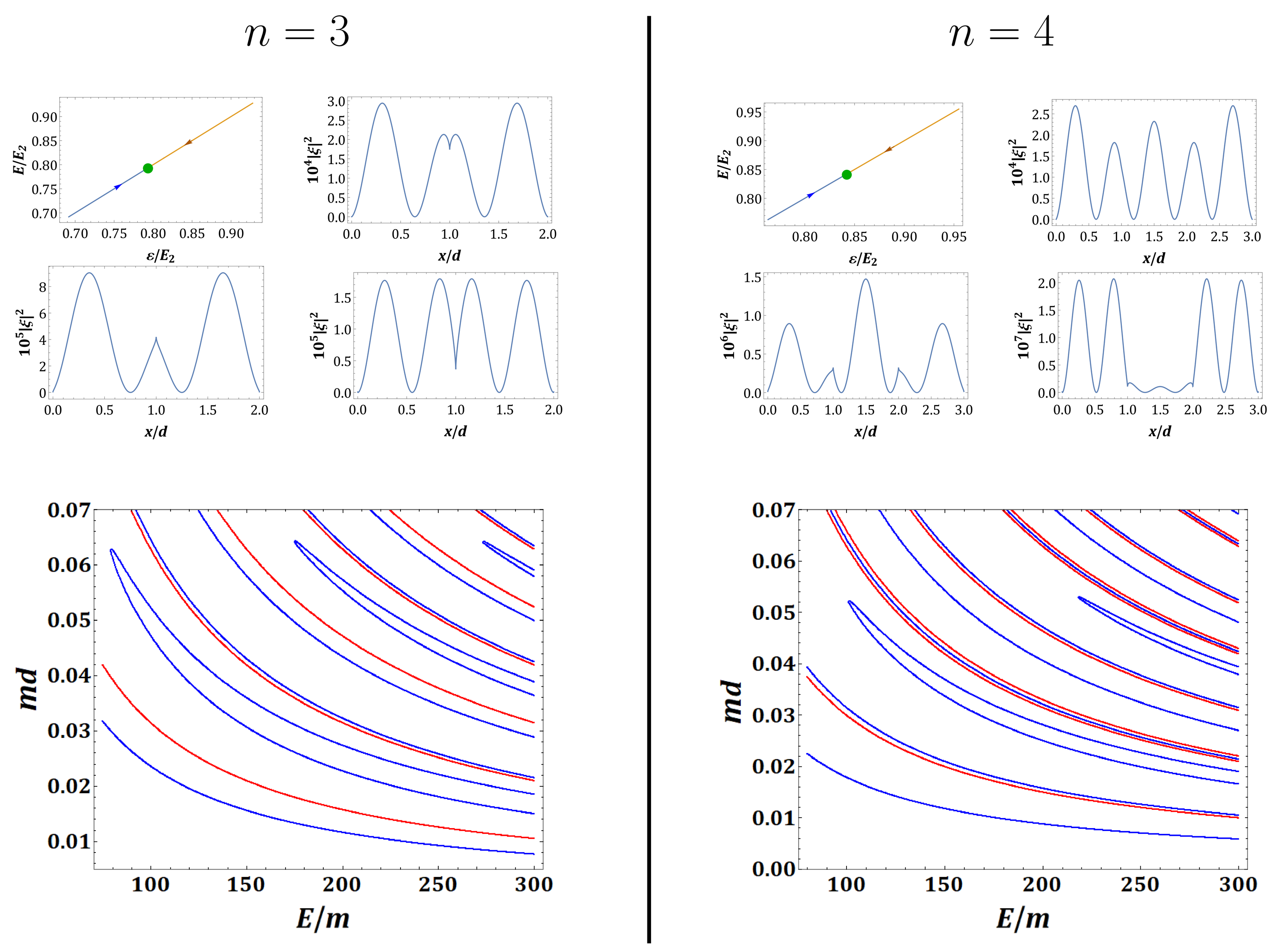
$\frac{a_1}{a_4}=\frac{a_2}{a_3}=1,\ \frac{a_1}{a_2}\simeq 0.25$

New lowest-energy state

The energy of the emerged state for $n=4$ is shifted with respect to the resonant values. In particular, one of the symmetric states is characterized by an eigenvalue slighlty smaller than $E_1(d)$.

Pair formation of high-energy eigenstates

These new solutions at finite d cannot be connected by continuity to the resonant eigenvalues and eigenspaces, with a photon half-wavelength that is far from multiple integers of the spacing:



n	4	6	8	10	12
d_c	0.05	0.18	0.26	0.30	0.33
E_c	101	28	20	16	15

The critical energy decreases to an order 10 for larger systems, thus leaving an open possibility to observe these states experimentally.

References

[1] P. Facchi, M. S. Kim, S. Pascazio, F. V. Pepe, D. Pomarico, and T. Tufarelli, "Bound states and entanglement generation in waveguide QED," Phys. Rev. A 94, 043839 (2016).

[2] P. Facchi, S. Pascazio, F. V. Pepe, and D. Pomarico, "Correlated photon emission by two excited atoms in a waveguide," Phys. Rev. A 98, 063823 (2018).

[3] P. Facchi, D. Lonigro, S. Pascazio, F. V. Pepe, and D. Pomarico, "Bound states in the continuum for an array of quantum emitters," arXiv:1904.13004 (2019).