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Separability of symmetric states and moment problem

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51 Symposium on Mathematical Physics
Toruń, Poland
June 16 – 18, 2019

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[A. Rutkowski, M. Banacki, M. Marciniak, Phys. Rev A 99 (2019)]

Symmetric states for N qudits

- Let $H = \mathbb{C}^d$ and let us fix a basis $|0\rangle, |1\rangle, \dots, |d-1\rangle$.
- Symmetrizer $P_{\mathrm{S}} \in B\left(H^{\otimes N}\right)$

$$P_{\mathrm{S}}|\mathbf{i}\rangle = \frac{1}{N!} \sum_{\sigma \in S_N} |\sigma(\mathbf{i})\rangle$$

$$|\mathbf{i}\rangle := |i_1, i_2, \dots, i_N\rangle, \quad i_1, i_2, \dots, i_N \in \{0, 1, \dots, d-1\}.$$

$$|\sigma(\mathbf{i})\rangle := |i_{\sigma^{-1}(1)}, i_{\sigma^{-1}(2)}, \dots, i_{\sigma^{-1}(N)}\rangle$$

- Permutationally symmetric state ρ

$$\sigma\rho = \rho\sigma, \qquad \sigma \in S_N$$

- (Bosonic) symmetric state ρ

$$P_{\mathrm{S}}\rho = \rho P_{\mathrm{S}}$$

Problem of separability of symmetric states

In the past decade, the problem of separability of permutationally symmetric states has been intensively analyzed
[O. Guhne and G. Toth, Phys. Rep. 474, 1 (2009)]
[G. Toth and O. Guhne, Phys. Rev. Lett. 102, 170503 (2009)]
[G. Toth and O. Guhne, Appl. Phys. B 98, 617 (2010)]
[E. Wolfe and S. F. Yelin, Phys. Rev. Lett. 112, 140402 (2014)]
[N. Yu, Phys. Rev. A 94, 060101(R) (2016)]

Dicke states for qubits

- Fix $d = 2$.
- Basis of Dicke (unnormalized) states:

$$|D_{N;k}\rangle := \binom{N}{k} P_{\mathrm{S}} |\underbrace{0,\ldots,0}_{N-k},\underbrace{1,\ldots,1}_{k}\rangle, \quad k = 0,1,\ldots,N$$

[Wolfe at al. (2014), Yu (2016)]

- It has been observed by several authors that there is a strong connection between separability and the PPT property for mixtures of Dicke states.

[Toth at al. (2009), Wolfe at al. (2014)]

Separability and PPT property

- $\rho \in B(H_1) \otimes \ldots \otimes B(H_N)$ a (nonnormalized) state i.e. positive semidefinite, $\mathrm{Tr}\rho = 1$ (but not necessarily)
- (Full) separability

$$\rho = \sum_\alpha \lambda_\alpha \rho_\alpha^1 \otimes \ldots \otimes \rho_\alpha^N$$

$$\rho_\alpha^i \in B(H_i), \qquad \lambda_\alpha \geq 0$$

- $(m_1, \ldots, m_N)$-PPT property

$$(T_1^{m_1} \otimes \ldots \otimes T_N^{m_N})\rho \quad \text{is a state,}$$

  - T_j the transposition on $B(H_j)$,
  - $(m_1, \ldots, m_N) \in \{0, 1\}^N$
  - $T_j^0 = \mathrm{id}_j$ and $T_j^1 = T_j$, i.e. all 1's in the system $(m_1, \ldots, m_N)$ mark subsytems which are transposed.

Separability and PPT property

- Clearly, if a state ρ is separable then it has a $(m_1, \ldots, m_n)$-PPT property for every binary system $(m_1, \ldots, m_n)$.
- In general, the converse implication is not true unless $N = 2$ and the pair (H_1, H_2) is one of the following: $(\mathbb{C}^2, \mathbb{C}^2), (\mathbb{C}^2, \mathbb{C}^3), (\mathbb{C}^3, \mathbb{C}^2)$.
- In spite of this general statement, there are classes of states such that the PPT property implies separability within them

States diagonal in Dicke basis (for qubits)

- Dicke diagonal states

$$\rho_{(p_k)} = \sum_{k=0}^{N} p_k |D_{N;k}\rangle\langle D_{N;k}|, \qquad p_k \geq 0$$

- For fixed $m := m_1 + \ldots + m_N$ all $(m_1, \ldots, m_N)$-PPT conditions are equivalent for symmetric states. Thus, it is enough to consider only PPT conditions with first m subsystems transposed, where $m \leq \lfloor N/2 \rfloor$, denoted by m-PPT.

Theorem (Yu (2016))

Let $(p_k)_{0 \leq k \leq N}$ be a sequence of non-negative numbers. Then the following conditiions are equivalent:

1. *The state $\rho_{(p_k)}$ is separable*
2. *The state $\rho_{(p_k)}$ has $\lfloor N/2 \rfloor$-PPT property.*

Qudit case - stright generalization

- Dicke states for qudits, $d \geq 2$ arbitrary

$$|D_{N,d;k_0,k_1,\ldots,k_{d-1}}\rangle = \binom{N}{k_0,\ldots,k_{d-1}} P_S \left(|0\rangle^{\otimes k_0} \otimes \ldots \otimes |d-1\rangle^{\otimes k_{d-1}}\right)$$

$$k_i \geq 0, \qquad k_0 + k_1 + \ldots + k_{d-1} = N$$

[T.-C. Wei at al., Quantum Inf. Comput. 4, 252 (2004)]
[N. Ananth and M. Senthilvelan, Int. J. Theor. Phys. 55, 1854 (2016)]
[J. Tura at al., Quantum 2, 45 (2018)]

- Dicke diagonal states for qudits

$$\rho = \sum p_{k_0,\ldots,k_{d-1}} |D_{N,d;k_0,\ldots,k_{d-1}}\rangle\langle D_{N,d;k_0,\ldots,k_{d-1}}|$$

In general, PPT does not imply separability.
[Tura at al. (2018)]

D-symmetry of states

- D-binomial coefficients

$$\mathbf{i} = (i_1, \ldots, i_N), \quad 0 \le i_1, \ldots, i_n \le d-1, \quad |\mathbf{i}| = i_1 + \ldots + i_N$$

$$\binom{N}{k}_d := \#\{\mathbf{i} : |\mathbf{i}| = k\}, \quad 0 \le k \le N(d-1), \quad \binom{N}{k}_2 = \binom{N}{k}$$

Generalized property of binomial coefficients

$$\binom{N}{k}_d = \sum_{j=0}^{\min\{k,d-1\}} \binom{N-1}{k-j}_d$$

D-symmetry of states

- D-symmetrizer

$$P_{\mathrm{D}}|\mathbf{i}\rangle = \binom{N}{|\mathbf{i}|}_d^{-1} \sum_{\mathbf{j}:\,|\mathbf{j}|=|\mathbf{i}|} |\mathbf{j}\rangle$$

- P_{D} is a projection.
- $P_{\mathrm{D}}P_{\mathrm{S}} = P_{\mathrm{S}}P_{\mathrm{D}} = P_{\mathrm{D}}$,
- $P_{\mathrm{D}}\left((\mathbb{C}^d)^{\otimes N}\right) \subset P_{\mathrm{S}}\left((\mathbb{C}^d)^{\otimes N}\right)$, i.e. D-symmetric vectors are permutationally symmetric.
- D-symmetric states

$$\rho P_{\mathrm{D}} = P_{\mathrm{D}}\rho$$

- Restricted Dicke states

$$|R_{N,d;k}\rangle = |R_k\rangle := \sum_{i_1+i_2+\ldots+i_N=k} |i_1, i_2, \ldots, i_N\rangle$$

Restricted Dicke states

$$|R_k\rangle := \sum_{i_1+i_2+\ldots+i_N=k} |i_1, i_2, \ldots, i_N\rangle$$

Assume that a system is composed of N bosons with d levels of excitation each. We make an assumption that subsequent levels differ by a fixed value. Then $|R_{N,d;k}\rangle$ can be interpreted as such a state of the system that the total number of excitations in all bosons is equal to k. It can be used to model systems of bosons concentrated in a small area which behave as single particle and only total energy can be recognized. Such models were used to explain the notion of superradiance in quantum optics.

[R. H. Dicke, Phys. Rev. 93, 99 (1954)]

[M. Gross and S. Haroche, Phys. Rep. 93, 301 (1982)]

Restricted Dicke diagonal states

$$\rho_{(p_k)} = \sum_{k=0}^{N(d-1)} p_k |R_{N,d;k}\rangle\langle R_{N,d;k}| \qquad p_0, p_1, \ldots, p_{N(d-1)} \geq 0.$$

Problem

What is the relationship between PPT property and separability for resrticted Dicke diagonal states?

Separable S-symmetric states

$$\rho = \sum_{\alpha} \lambda_\alpha \rho_\alpha^1 \otimes \ldots \otimes \rho_\alpha^N, \qquad \rho_\alpha^i = |\xi_\alpha^i\rangle\langle\xi_\alpha^i|$$

Proposition

Assume that ρ is symmetric, i.e. $\rho = P_{\mathrm{S}} \rho P_{\mathrm{S}}$. If all coefficients λ_α are strictly positive then $\rho_\alpha^i = \rho_\alpha^j$ for every $\alpha = 1, \ldots, n$ and $i, j = 1, \ldots, N$.

Can assume $|\xi_\alpha^i\rangle = |\xi_\alpha^j\rangle$ for $i, j = 1, \ldots, N$.

Separable D-symmetric states

$$\rho = \sum_{\alpha} \lambda_\alpha \rho_\alpha \otimes \ldots \otimes \rho_\alpha, \qquad \rho_\alpha = |\xi_\alpha\rangle\langle\xi_\alpha|$$

Proposition

Assume that ρ is D-symmetric, i.e. $\rho = P_{\mathrm{D}} \rho P_{\mathrm{D}}$. Then for each $\alpha = 1, \ldots, n$, either

$$|\xi_\alpha\rangle = |d-1\rangle$$

or there is a number $z \in \mathbb{C}$ such that

$$|\xi_\alpha\rangle = C_z \sum_{i=0}^{d-1} z^i |i\rangle,$$

where C_z is a normalization.

Entanglement witnesses for D-symmetric systems

Definition

A Hermitian operator $W \in B((\mathbb{C}^d)^{\otimes N})$ *is an entaglement witness for the D-symmetric system if*

1. $W = P_{\mathrm{D}} W P_{\mathrm{D}}$
2. $\mathrm{Tr}(W\sigma) \geq 0$ *for all pure separable D-symmetric states*

Proposition

A D-symmetric state ρ *is separable if and only if* $\mathrm{Tr}(W\rho) \geq 0$ *for every entanglement witness* W *for the D-symmetric system.*

A simple consequence of the hyperplane separation theorem.

[Yu (2016)]

Entanglement witnesses for D-symmetric systems

$$|\widetilde{R_k}\rangle = \binom{N}{k}_d^{-1} \sum_{|\mathbf{i}|=k} |\mathbf{i}\rangle = \binom{N}{k}_d^{-1} |R_k\rangle, \qquad \langle \widetilde{R_k}|R_l\rangle = \delta_{kl}.$$

Proposition

Let $n_1 = \left\lfloor \frac{N(d-1)}{2} \right\rfloor$ *and* $n_2 = \left\lfloor \frac{N(d-1)-1}{2} \right\rfloor$. *Let two systems* $(s_k)_{0\leq k\leq n_1}$ *and* $(t_k)_{0\leq k\leq n_2}$ *of complex numbers be given. Define*

$$V_{(s)} = \sum_{k,l=0}^{n_1} s_k \overline{s_l} |\widetilde{R_{k+l}}\rangle\langle\widetilde{R_{k+l}}|$$

$$U_{(t)} = \sum_{k,l=0}^{n_2} t_k \overline{t_l} |\widetilde{R_{k+l+1}}\rangle\langle\widetilde{R_{k+l+1}}|.$$

Then $V_{(s)}$ *and* $U_{(t)}$ *are entanglement witnesses for D-symmetric systems.*

Moment problem – definition

Definition

Let $(p_k)_{k=0}^n$ be a finite sequence of real numbers. We say that the sequence (p_k) is a solution of the generalized moment problem on the interval $[0,\infty)$ if there exists a positive measure σ with support contained in $[0,\infty)$ such that

$$p_k = \begin{cases} \int_0^\infty t^k d\sigma(t), & k = 0, 1, \dots, n-1, \\ \int_0^\infty t^n d\sigma(t) + M, & k = n, \end{cases}$$

where $M \geq 0$. Alternatively, we say that it is a solution of the strict moment problem on the interval $[0,\infty)$ if it is a solution of the generalized moment problem with $M = 0$.

[M. G. Krein and A. A. Nudelman, The Markov Moment Problem and Extremal Problems (AMS, Providence, RI, 1977), Russian ed. in 1973]

Hankel matrices

$$n_0 = \left\lfloor \frac{n}{2} \right\rfloor, \qquad n_1 = \left\lfloor \frac{n-1}{2} \right\rfloor$$

$$(p_{k+l})_{k,l=0}^{n_0} = \begin{pmatrix} p_0 & p_1 & p_2 & \cdots & p_{n_0} \\ p_1 & p_2 & p_3 & \cdots & p_{n_0+1} \\ p_2 & p_3 & p_4 & \cdots & p_{n_0+2} \\ \vdots & \vdots & \vdots & & \vdots \\ p_{n_0} & p_{n_0+1} & p_{n_0+2} & \cdots & p_{2n_0} \end{pmatrix},$$

$$(p_{k+l+1})_{k,l=0}^{n_1} = \begin{pmatrix} p_1 & p_2 & p_3 & \cdots & p_{n_1+1} \\ p_2 & p_3 & p_4 & \cdots & p_{n_1+2} \\ p_3 & p_4 & p_5 & \cdots & p_{n_1+3} \\ \vdots & \vdots & \vdots & & \vdots \\ p_{n_1+1} & p_{n_1+2} & p_{n_1+3} & \cdots & p_{2n_1+1} \end{pmatrix}$$

Moment problem – characterization

Theorem

A sequence $(p_k)_{k=0}^{n}$ is a solution of the generalized moment problem if and only if both Hankel matrices $(p_{k+l})_{k,l=0}^{n_0}$ and $(p_{k+l+1})_{k,l=0}^{n_1}$ are positive semidefinite. If both matrices are strictly positive definite then the sequence is a solution of the strict moment problem.

[M. G. Krein and A. A. Nudelman, The Markov Moment Problem and Extremal Problems (AMS, Providence, RI, 1977)]

Example: $n = 9$, $n_0 = 4$, $n_1 = 4$

$$\begin{pmatrix} p_0 & p_1 & p_2 & p_3 & p_4 \\ p_1 & p_2 & p_3 & p_4 & p_5 \\ p_2 & p_3 & p_4 & p_5 & p_6 \\ p_3 & p_4 & p_5 & p_6 & p_7 \\ p_4 & p_5 & p_6 & p_7 & p_8 \end{pmatrix} \quad \begin{pmatrix} p_1 & p_2 & p_3 & p_4 & p_5 \\ p_2 & p_3 & p_4 & p_5 & p_6 \\ p_3 & p_4 & p_5 & p_6 & p_7 \\ p_4 & p_5 & p_6 & p_7 & p_8 \\ p_5 & p_6 & p_7 & p_8 & p_9 \end{pmatrix}$$

Restricted Dicke diagonal states with PPT

Theorem

Let $m \leq N/2$. *The state* $\rho_{(p_k)}$ *is* m*-PPT if and only if*

(a) *matrices* $(p_{i+j})_{i,j=0}^{m(d-1)}$ *and* $(p_{i+j+1})_{i,j=0}^{m(d-1)-1}$ *are positive definite, when* $N = 2m$,

(b) *matrices* $(p_{i+j+l})_{i,j=0}^{m(d-1)}$, $l = 0, \ldots, (N-2m)(d-1)$, *are positive definite, when* $2m < N$.

Restricted Dicke diagonal states with PPT

Theorem

Let $m \leq N/2$. The state $\rho_{(p_k)}$ is m-PPT if and only if

a. *matrices $(p_{i+j})_{i,j=0}^{m(d-1)}$ and $(p_{i+j+1})_{i,j=0}^{m(d-1)-1}$ are positive definite, when $N = 2m$,*
b. *matrices $(p_{i+j+l})_{i,j=0}^{m(d-1)}$, $l = 0, \ldots, (N-2m)(d-1)$, are positive definite, when $2m < N$.*

Example: $d = 3$, $N = 3$, $m = 1$

$$\rho_{(p_k)} = \sum_{k=0}^{6} p_k |R_k\rangle\langle R_k|$$

$$\begin{pmatrix} p_0 & p_1 & p_2 \\ p_1 & p_2 & p_3 \\ p_2 & p_3 & p_4 \end{pmatrix} \quad \begin{pmatrix} p_1 & p_2 & p_3 \\ p_2 & p_3 & p_4 \\ p_3 & p_4 & p_5 \end{pmatrix} \quad \begin{pmatrix} p_2 & p_3 & p_4 \\ p_3 & p_4 & p_5 \\ p_4 & p_5 & p_6 \end{pmatrix}$$

Restricted Dicke diagonal states with PPT

Theorem

Let $m \leq N/2$. The state $\rho_{(p_k)}$ is m-PPT if and only if

- (a) *matrices $(p_{i+j})_{i,j=0}^{m(d-1)}$ and $(p_{i+j+1})_{i,j=0}^{m(d-1)-1}$ are positive definite, when $N = 2m$,*
- (b) *matrices $(p_{i+j+l})_{i,j=0}^{m(d-1)}$, $l = 0, \dots, (N-2m)(d-1)$, are positive definite, when $2m < N$.*

Example: $d = 3$, $N = 4$, $m = 2$, $\rho_{(p_k)} = \sum_{k=0}^{9} p_k |R_k\rangle\langle R_k|$

$$\begin{pmatrix} p_0 & p_1 & p_2 & p_3 & p_4 \\ p_1 & p_2 & p_3 & p_4 & p_5 \\ p_2 & p_3 & p_4 & p_5 & p_6 \\ p_3 & p_4 & p_5 & p_6 & p_7 \\ p_4 & p_5 & p_6 & p_7 & p_8 \end{pmatrix} \quad \begin{pmatrix} p_1 & p_2 & p_3 & p_4 & p_5 \\ p_2 & p_3 & p_4 & p_5 & p_6 \\ p_3 & p_4 & p_5 & p_6 & p_7 \\ p_4 & p_5 & p_6 & p_7 & p_8 \\ p_5 & p_6 & p_7 & p_8 & p_9 \end{pmatrix}$$

PPT vs moment problem

Corollary

Assume that N is even and let $(p_k)_{0\leq k\leq N(d-1)}$ be a sequence of nonnegative numbers. The following are equivalent:

1. $\rho_{(p_k)}$ *is* $N/2$*-PPT,*
2. *The sequence* (p_k) *is a solution of generalized moment problem*

Moreover, if $d=2$ and N is odd, then the following are equivalent

1. $\rho_{(p_k)}$ *is* $(N-1)/2$*-PPT,*
2. *The sequence* (p_k) *is a solution of generalized moment problem*

PPT vs moment problem

$d = 2,\ N = 5,\ \rho_{(p_k)} = \sum_{k=0}^{5} p_k |R_k\rangle\langle R_k|$

- 2-PPT:

$$\begin{pmatrix} p_0 & p_1 & p_2 \\ p_1 & p_2 & p_3 \\ p_2 & p_3 & p_4 \end{pmatrix} \quad \begin{pmatrix} p_1 & p_2 & p_3 \\ p_2 & p_3 & p_4 \\ p_3 & p_4 & p_5 \end{pmatrix}$$

- Moment problem: $n = 5,\ n_0 = 2,\ n_1 = 2.$ The above matrices are precisely the two Hankel matrices from the theorem.

Separability vs moment problem

Proposition

If $(p_k)_{k=0,1,\dots,N(d-1)}$ *is a geometric sequence then* $\rho_{(p_k)}$ *separable.*

Proof. Let $p_k = t^k$ for some $t > 0$.

$$\omega = \exp\left(\frac{2\pi i}{N(d-1)+1}\right)$$

$$|\hat{\alpha}\rangle = \sum_{j=0}^{d-1} t^{j/2}\omega^{\alpha j}|j\rangle, \quad \alpha = 0, 1, \dots, N(d-1).$$

Then

$$\rho_{(t^k)} = \frac{1}{N(d-1)+1} \sum_{\alpha=0}^{N(d-1)} |\hat{\alpha}\rangle\langle\hat{\alpha}|^{\otimes N}$$

Separability vs moment problem

Theorem

Let $d \geq 2$ and N be arbitrary. The state $\rho_{(p_k)}$ is fully separable if and only the sequence $(p_k)_{k=0}^{N(d-1)}$ is a solution of the generalized moment problem.

Proof of necessity. Since $\rho_{(p_k)}$ is separable, $\mathrm{Tr}(\rho_{(p_k)} W) \geq 0$ for every entanglement witness for D-symmetric systems. In particular, for any sequence $(s_k)_{0 \leq k \leq \lfloor N(d-1)/2 \rfloor}$

$$\sum_{k,l=0}^{\lfloor N(d-1)/2 \rfloor} s_k \overline{s_l} p_{k+l} = \mathrm{Tr}(\rho_{(p_k)} V_{(s)}) \geq 0$$

what means that $(p_{k+l})_{0 \leq k,l \leq n_0}$ is positive semidefinte. Similarly, for the second Hankel matrix. Hence (p_k) is a solution of moment problem.

Separability vs moment problem

Theorem

Let $d \geq 2$ and N be arbitrary. The state $\rho_{(p_k)}$ is fully separable if and only the sequence $(p_k)_{k=0}^{N(d-1)}$ is a solution of the generalized moment problem.

Proof of sufficiency. Since (p_k) is a solution of the generalized moment problem, there are a positive measure σ supported on $[0,\infty)$ and $M \geq 0$ such that

$$p_k = \int_0^\infty t^k d\sigma(t) + \delta_{k,N(d-1)} M.$$

Then

$$\rho_{(p_k)} = \int_0^\infty \rho_{(t^k)} d\sigma(t) + M |R_{N(d-1)}\rangle\langle R_{N(d-1)}|.$$

$|R_{N(d-1)}\rangle\langle R_{N(d-1)}| = |d-1\rangle\langle d-1|^{\otimes N}$, so it is separable. According to Proposition from the previous slide, each $\rho_{(t^k)}$ is also a separable state. Consequently, $\rho_{(p_k)}$ is separable too.

Main result

Theorem (A.R, M.Marciniak, M.Banacki)

Assume that $d \geq 2$ is arbitrary and N is even. Let $(p_k)_{0 \leq k \leq N(d-1)}$ be a sequence of nonnegative numbers. The following conditions are equivalent:

(a) $\rho_{(p_k)}$ *is fully separable.*
(b) $\rho_{(p_k)}$ *is $N/2$-PPT*
(c) *The sequence (p_k) is a solution of the generalized moment problem.*

Moreover, if $d = 2$ and N is odd the following conditions are equivalent:

(a) $\rho_{(p_k)}$ *is fully separable.*
(b) $\rho_{(p_k)}$ *is $(N-1)/2$-PPT*
(c) *The sequence (p_k) is a solution of the generalized moment problem.*

Let us note that for $d = 2$, i.e. for qubits, the above equivalence was proved in [Yu (2016)].

The case $d \geq 3$ and N odd

On the contrary to the case $d = 2$, if N is odd then $\frac{N-1}{2}$-PPT property does not imply separability of $\rho_{(p_k)}$ for $d \geq 3$.
Let $N = 3$ and $d = 3$ and let

$$(p_k)_{0\leq k\leq 6} = (1, 1/4, 1/8, 1/9, 1/8, 1/4, 1)\,.$$

ρ is a 1-PPT state. Indeed, one can easily check that matrices

$$\begin{pmatrix} 1 & 1/4 & 1/8 \\ 1/4 & 1/8 & 1/9 \\ 1/8 & 1/9 & 1/8 \end{pmatrix} \quad \begin{pmatrix} 1/4 & 1/8 & 1/9 \\ 1/8 & 1/9 & 1/8 \\ 1/9 & 1/8 & 1/4 \end{pmatrix} \quad \begin{pmatrix} 1/8 & 1/9 & 1/8 \\ 1/9 & 1/8 & 1/4 \\ 1/8 & 1/4 & 1 \end{pmatrix}$$

are positive semidefinite. On the other hand the determinant of the Hankel matrix ($n_0 = 3$)

$$(p_{k+l})_{0\leq k,l\leq 3} = \begin{pmatrix} 1 & 1/4 & 1/8 & 1/9 \\ 1/4 & 1/8 & 1/9 & 1/8 \\ 1/8 & 1/9 & 1/8 & 1/4 \\ 1/9 & 1/8 & 1/4 & 1 \end{pmatrix}$$

is negative, hence it is not positive semidefinite.

Conclusions

- We introduced the notion of D-symmetry for multipartite states which is stronger then bosonic symmetry.
- We considered D-symmetric analogs of Dicke states: restricted Dicke states.
- We proved that for even number N of systems $N/2$-PPT property is equivalent to separability. It was done using classical results on moment problem.

Conclusions

- We introduced the notion of D-symmetry for multipartite states which is stronger then bosonic symmetry.
- We considered D-symmetric analogs of Dicke states: restricted Dicke states.
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THANK YOU!