

TITLE PAGE

DIVISIBILITY OF QUBIT CHANNELS AND DYNAMICAL MAPS



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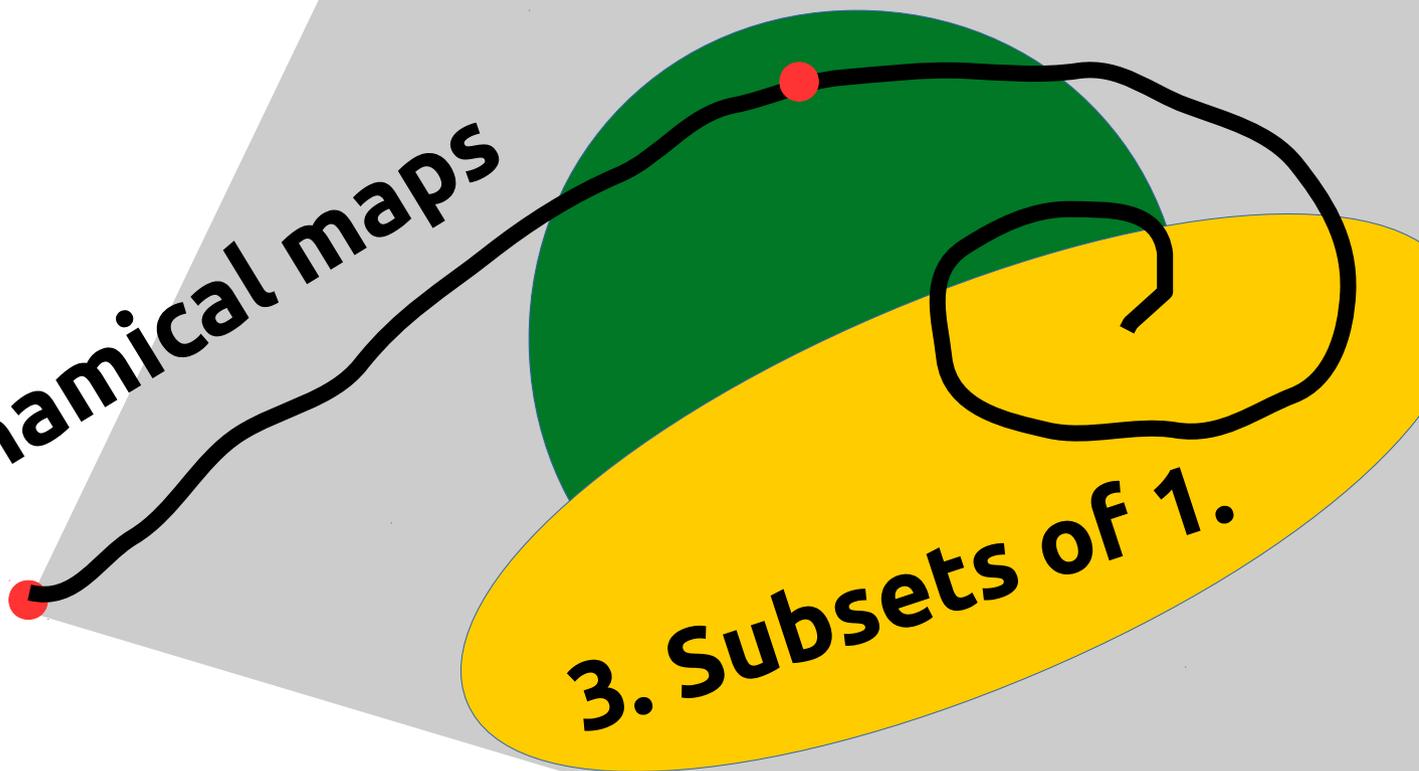
PLAN

1. Channel divisibility

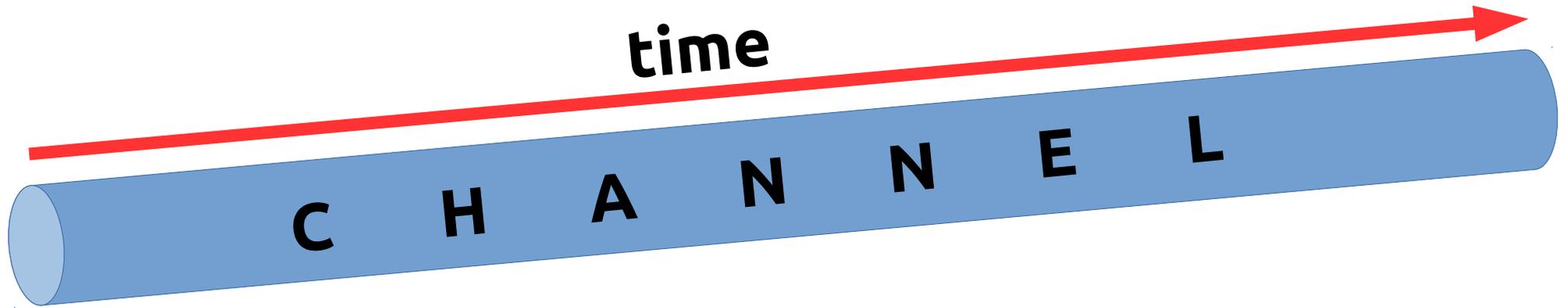
2. dynamical maps

3. Subsets of 1.

4. How 2. passes through 3.

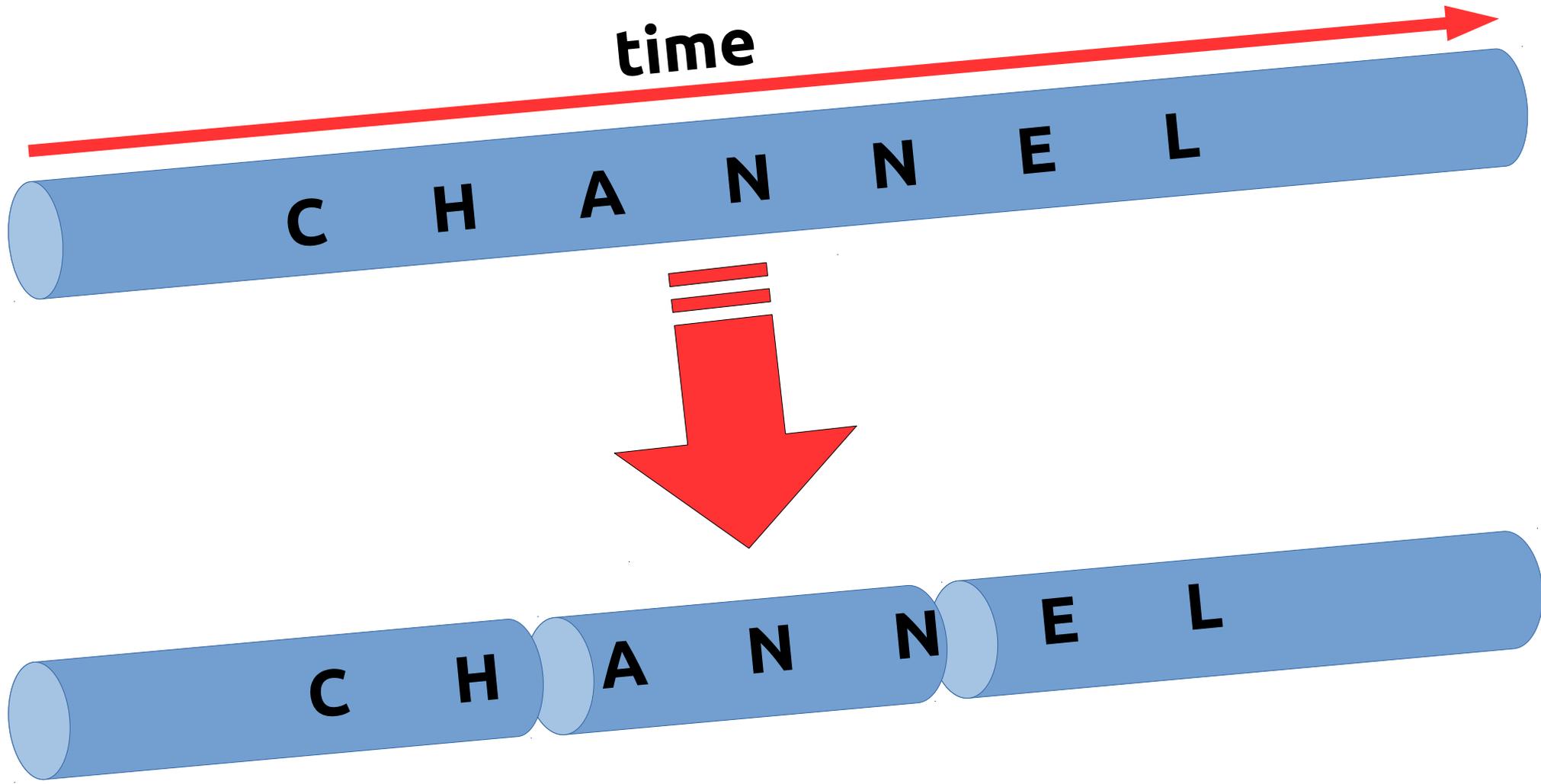


QUANTUM CHANNELS



Φ completely positive
trace-preserving
linear map

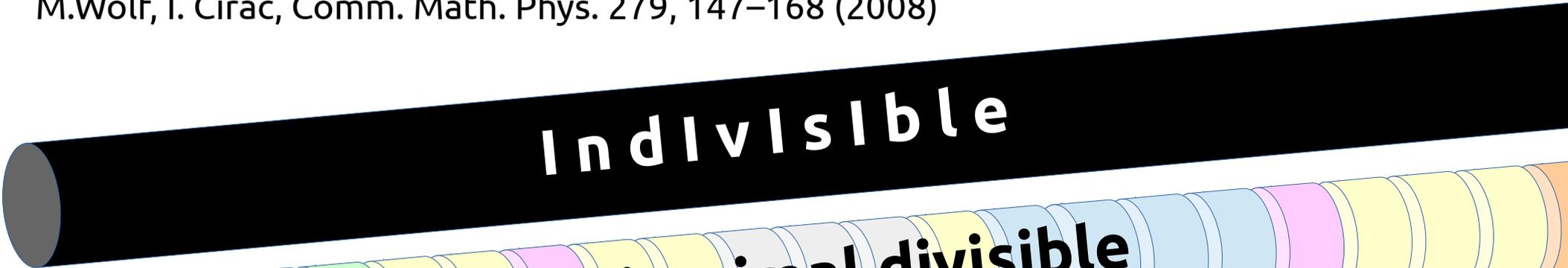
DIVISIBILITY



TYPES OF DIVISIBILITY

in^{finite}ly^{simal} **DIVISIBLE**

M.Wolf, J. Eisert, T. Cubitt, I. Cirac, Phys. Rev. Lett., 101, 150402 (2008)
M.Wolf, I. Cirac, Comm. Math. Phys. 279, 147–168 (2008)



Indivisible



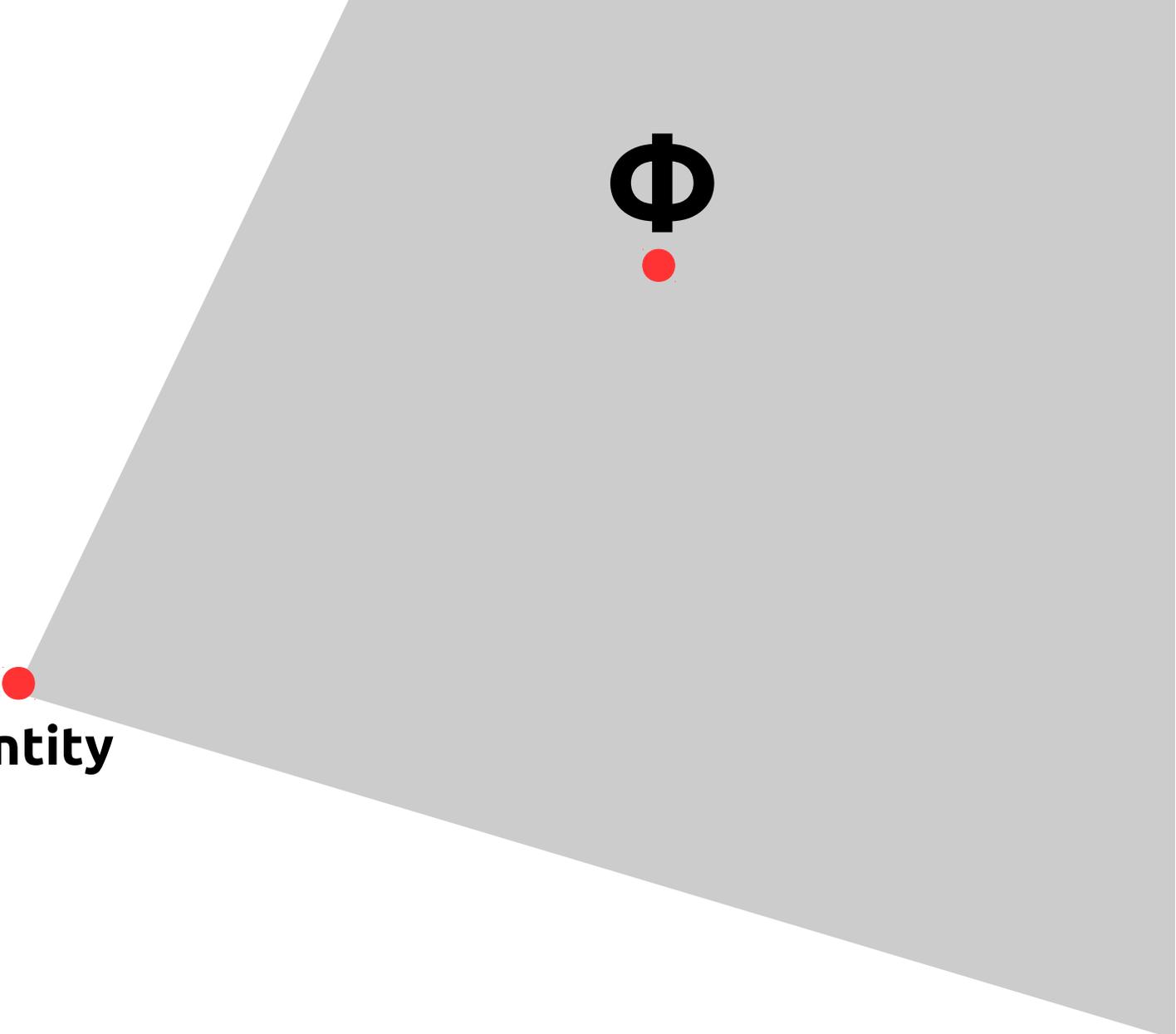
infinitesimal divisible



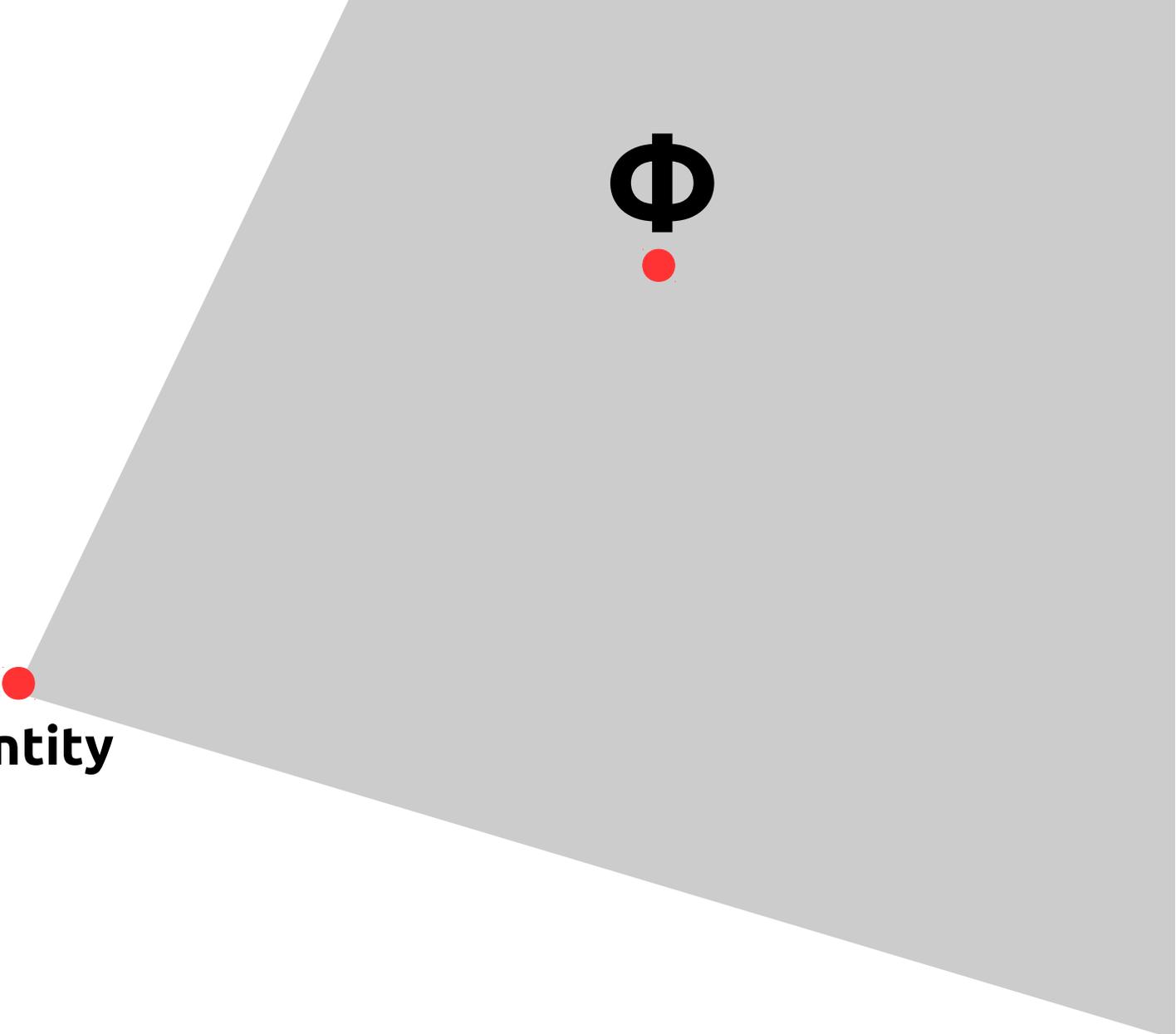
infinitely divisible

SET OF CHANNELS

- convex set



identity



Φ

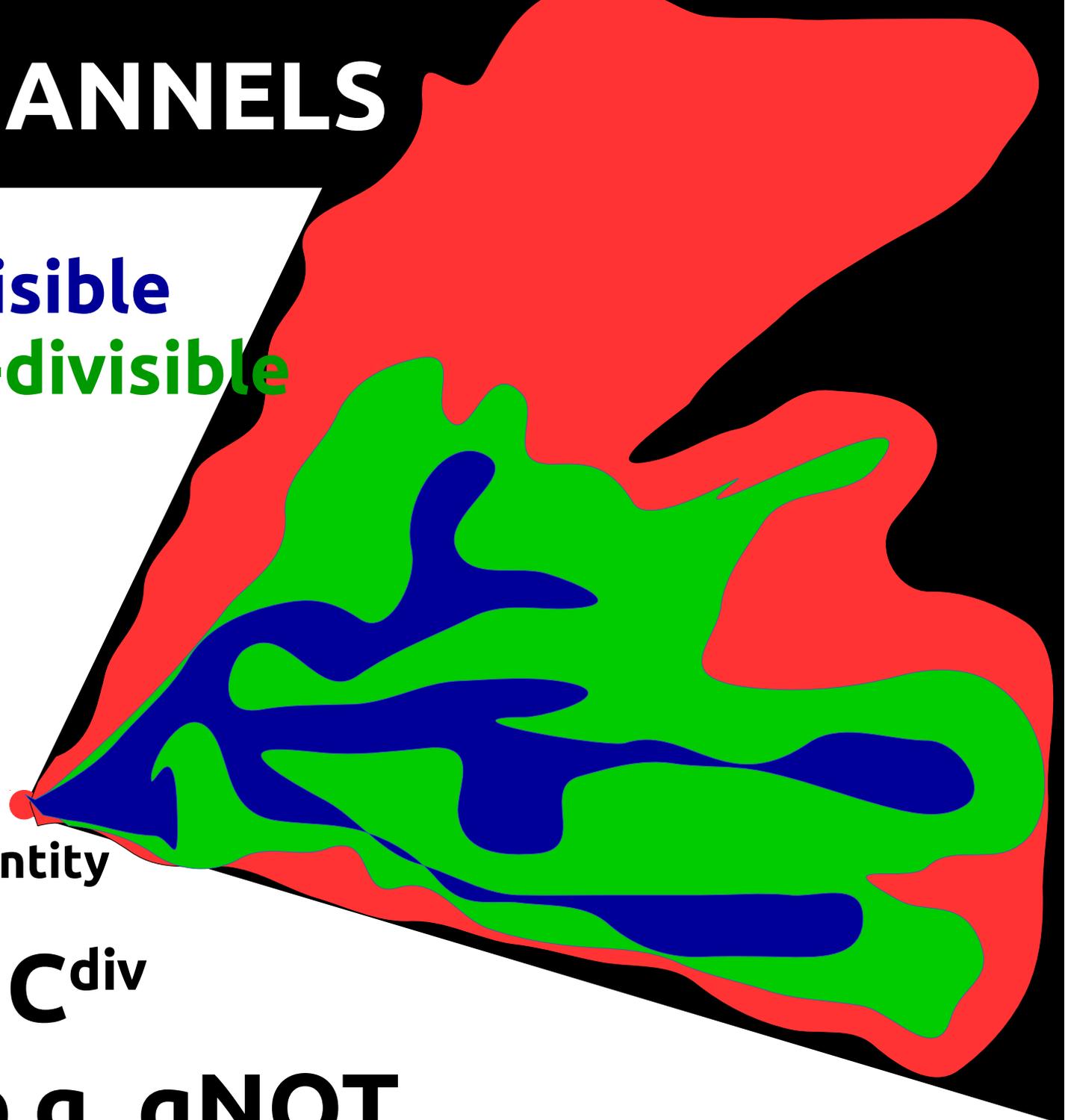
SET OF CHANNELS

infinitely-divisible
infinitesimal-divisible
divisible
indivisible

identity

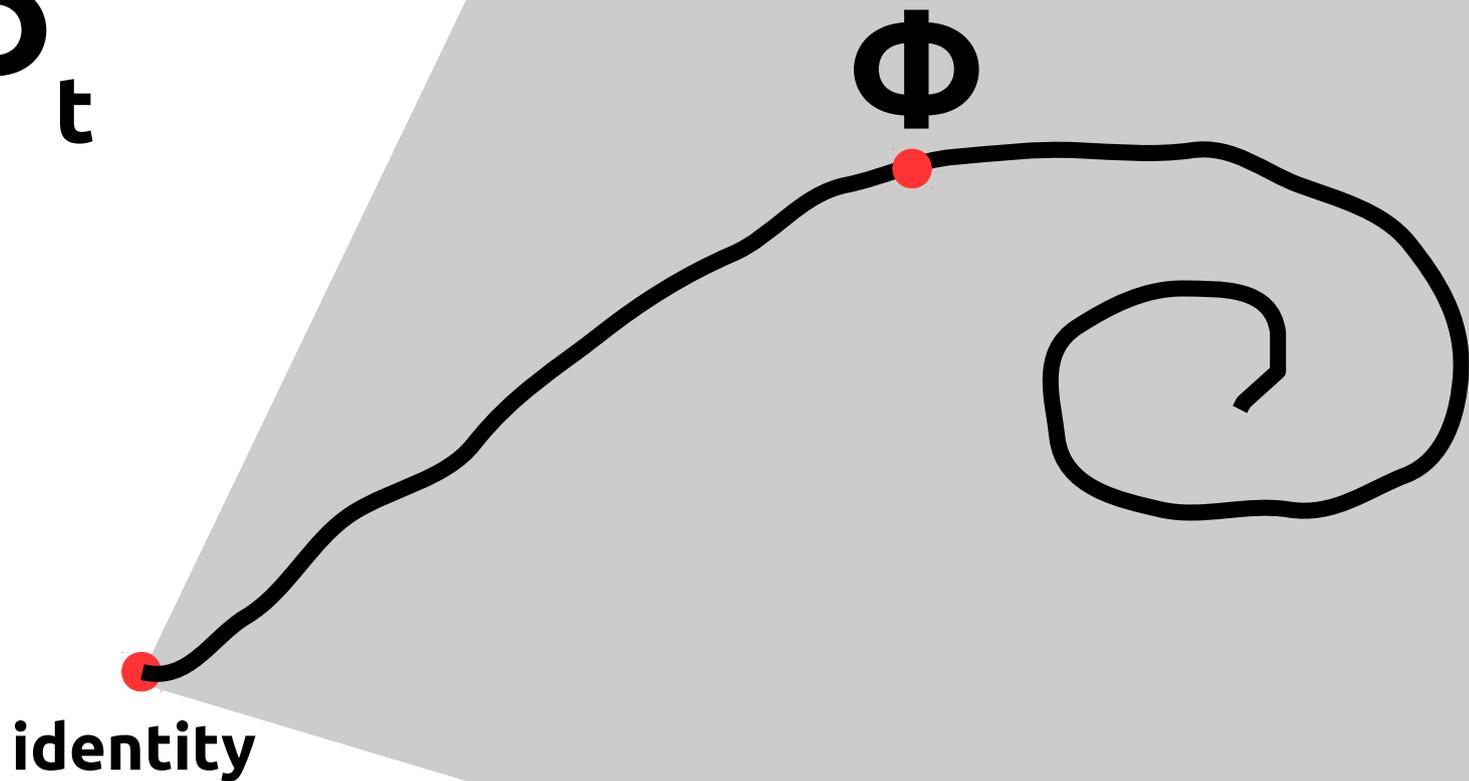
$$\mathbb{C}^\infty \subset \mathbb{C}^{\text{inf}} \subset \mathbb{C}^{\text{div}}$$

$\mathbb{C}^{\text{indivisible}}$, e.g. qNOT



DYNAMICAL MAP

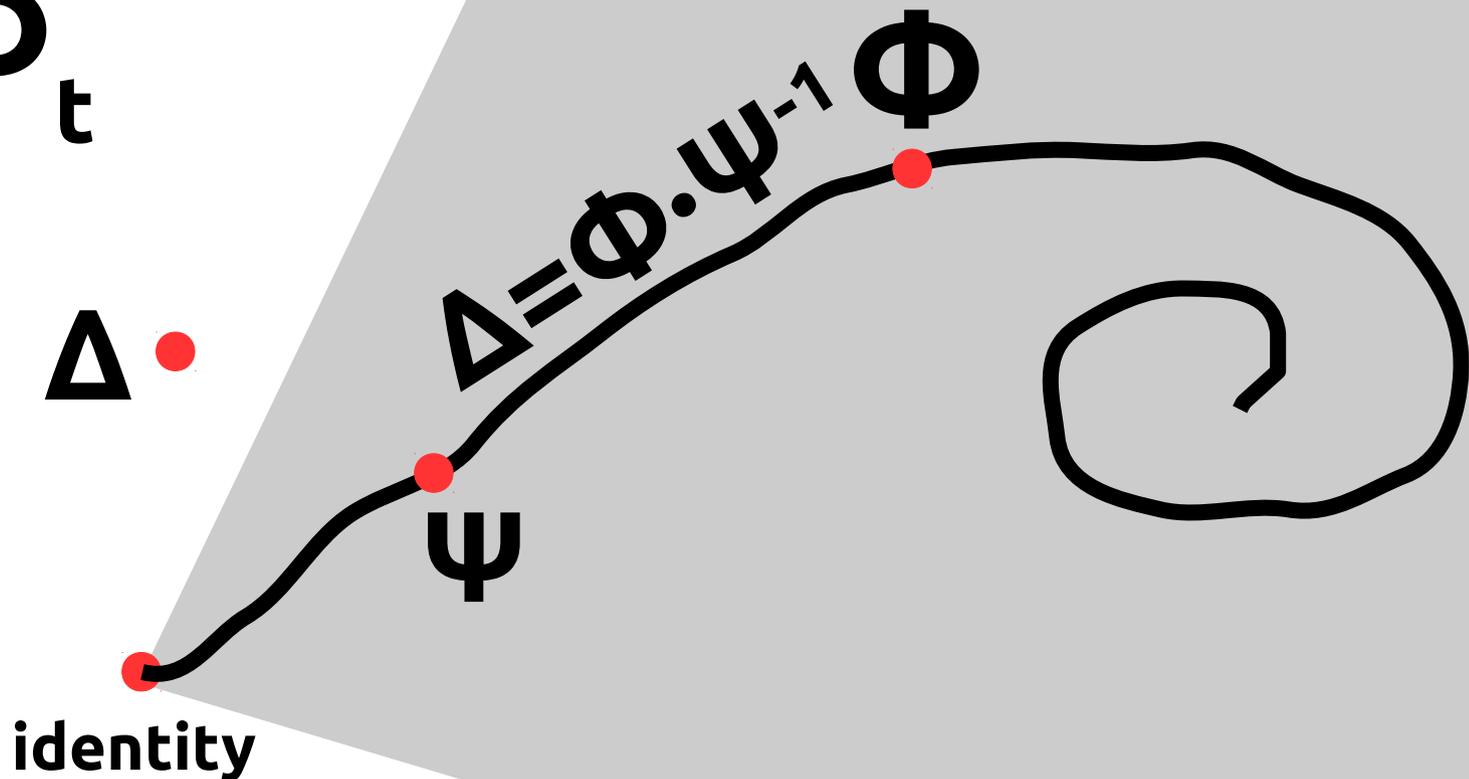
$$t \rightarrow \Phi_t$$



SET OF CHANNELS

DYNAMICAL MAP

$$t \rightarrow \Phi_t$$

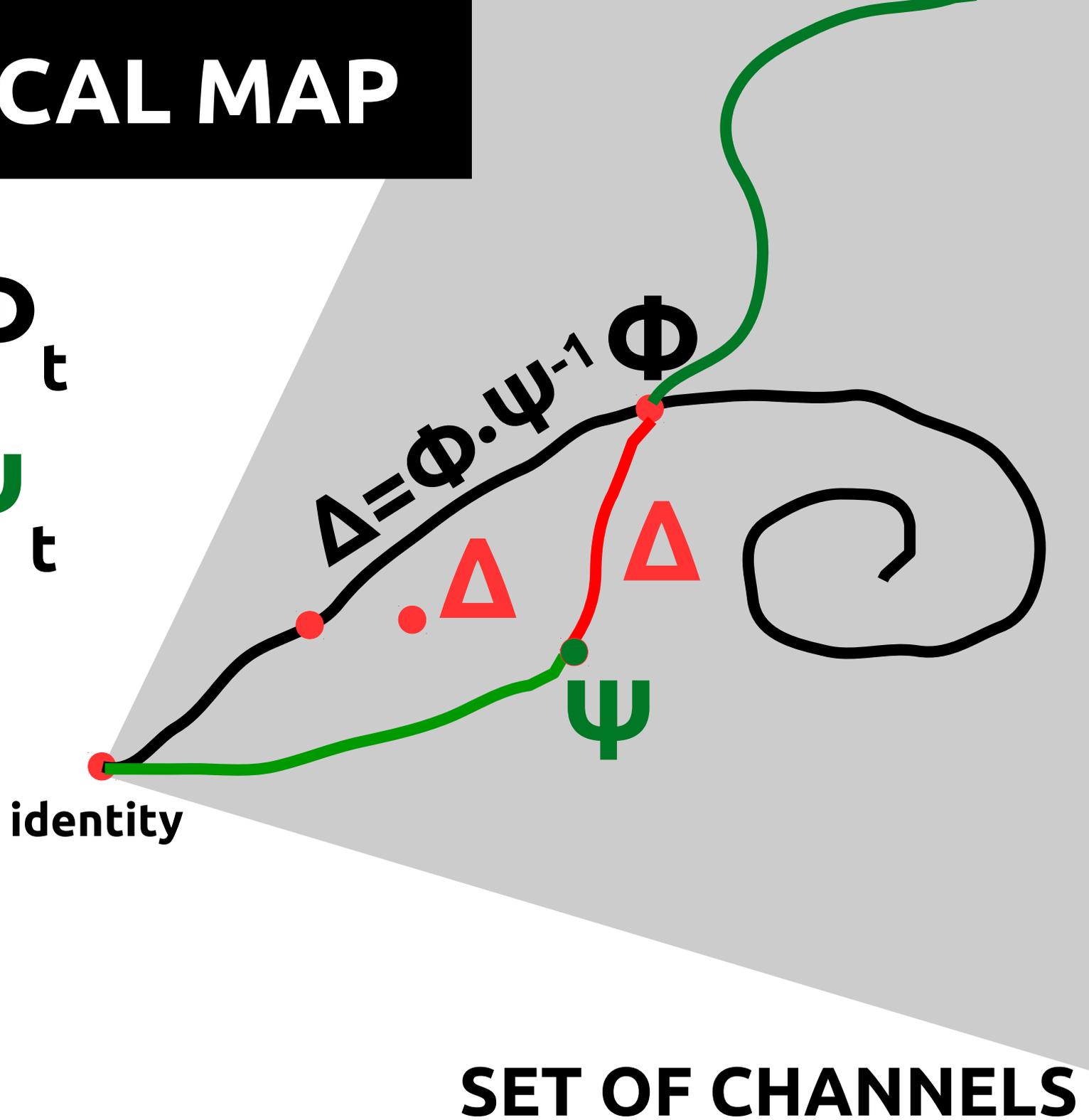


SET OF CHANNELS

DYNAMICAL MAP

$$t \rightarrow \Phi_t$$

$$t \rightarrow \Psi_t$$



DYNAMICAL MAP

$$t \rightarrow \Phi_t$$

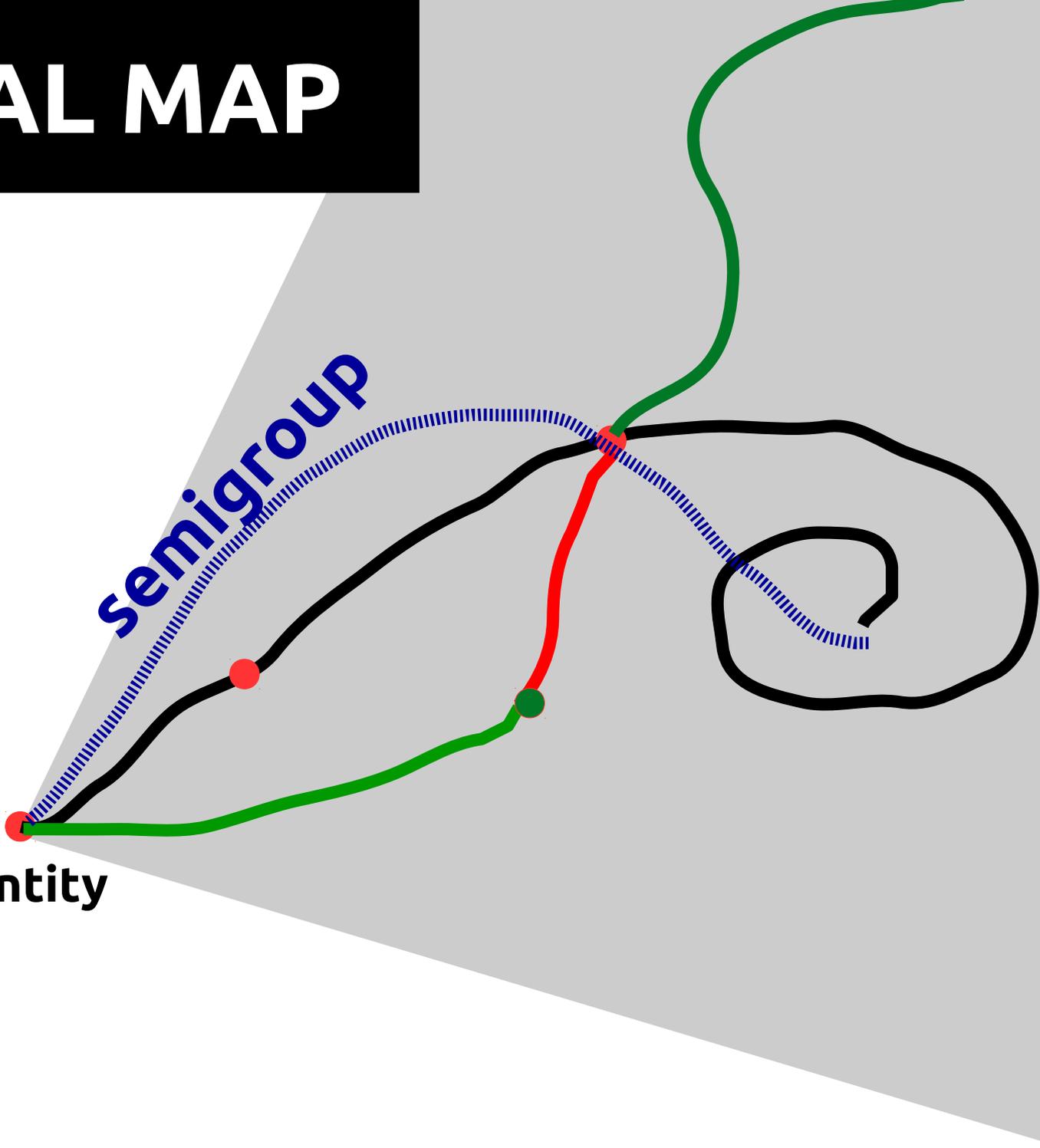
$$t \rightarrow \Psi_t$$

$$t \rightarrow \mathcal{E}_t$$

identity

semigroup

SET OF CHANNELS



DYNAMICAL MAP

$$t \rightarrow \Phi_t$$

$$t \rightarrow \Psi_t$$

$$t \rightarrow \mathcal{E}_t$$

identity

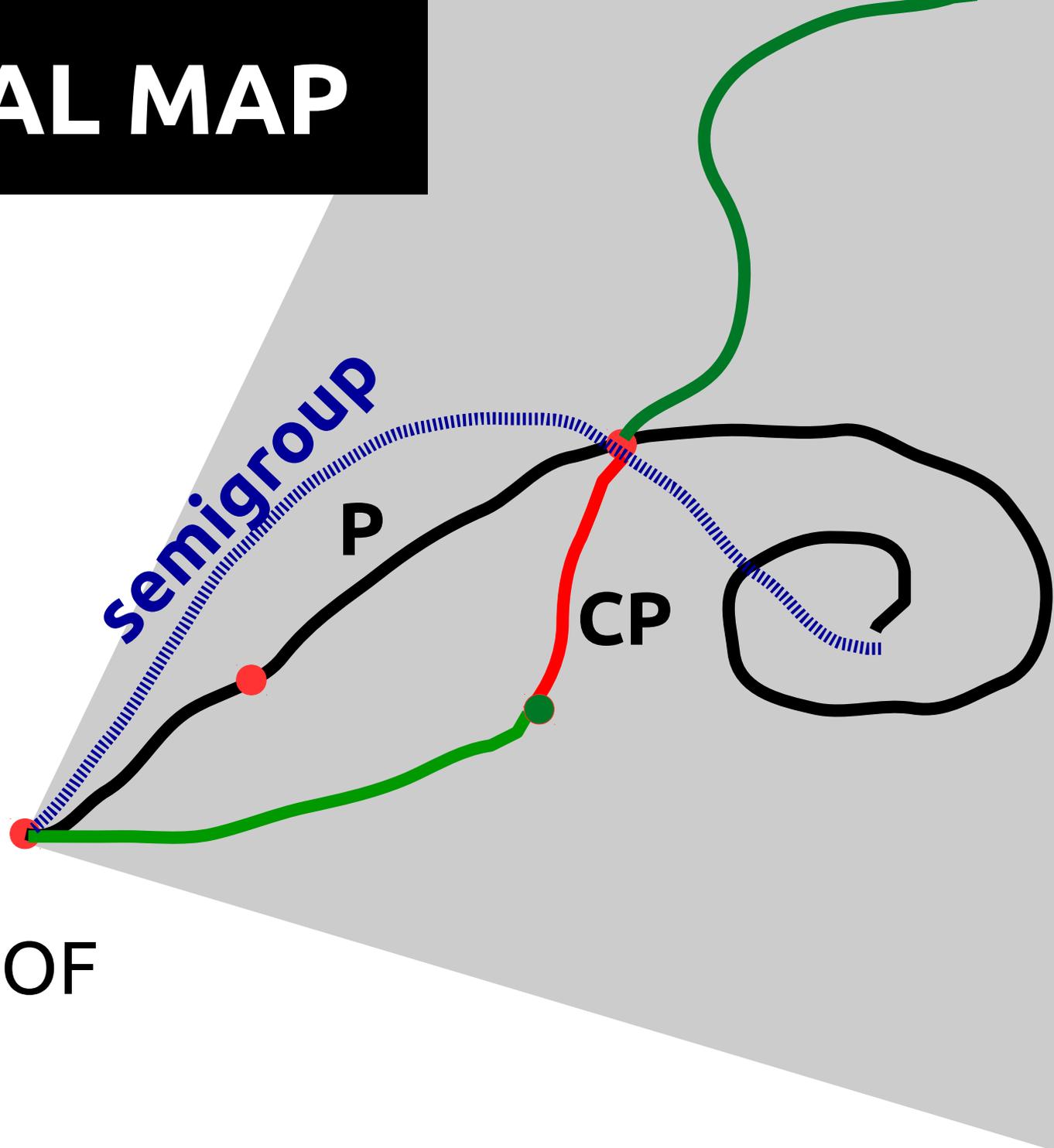
semigroup

P

CP

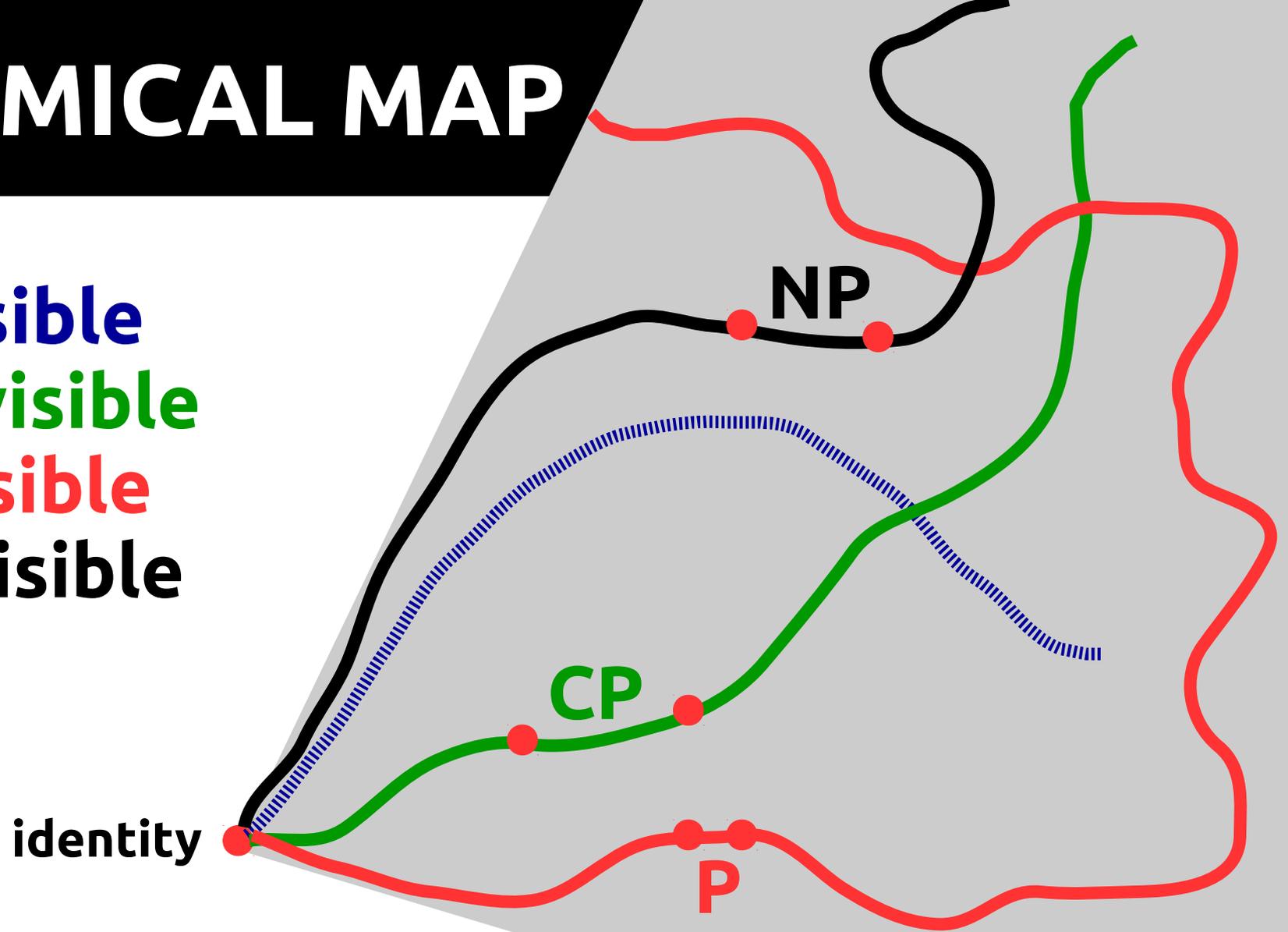
DIVISIBILITY OF
DYNAMICAL
MAPS

SET OF CHANNELS



DYNAMICAL MAP

L-divisible
CP-divisible
P-divisible
IN-divisible



for all time intervals

CHANNEL TYPES

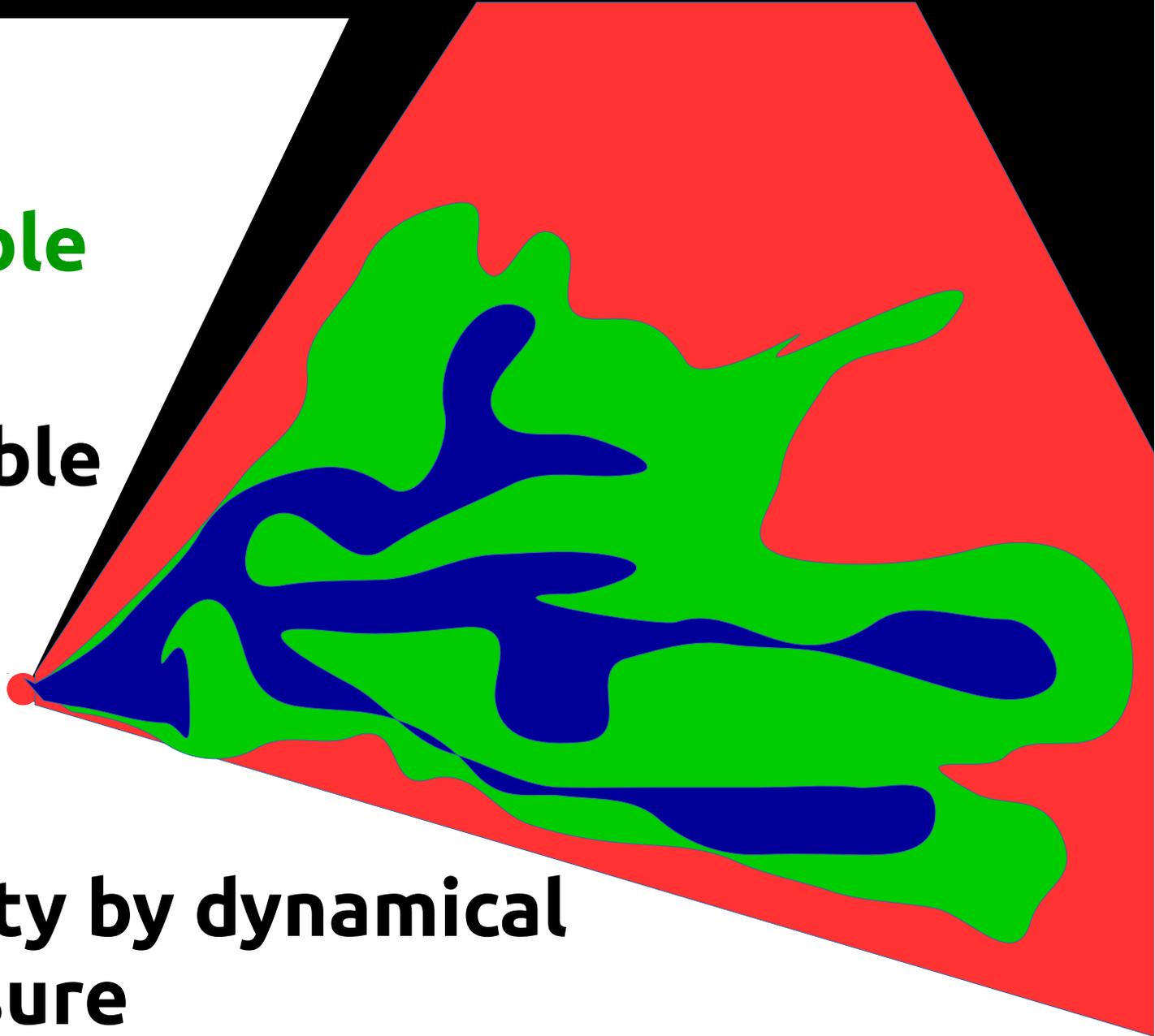
C^L L-divisible

C^{CP} CP-divisible

C^P P-divisible

C^{NP} NP-divisible

identity



Achievability by dynamical maps + closure

CHANNEL TYPES

C^L L-divisible

C^{CP} CP-divisible

C^P P-divisible

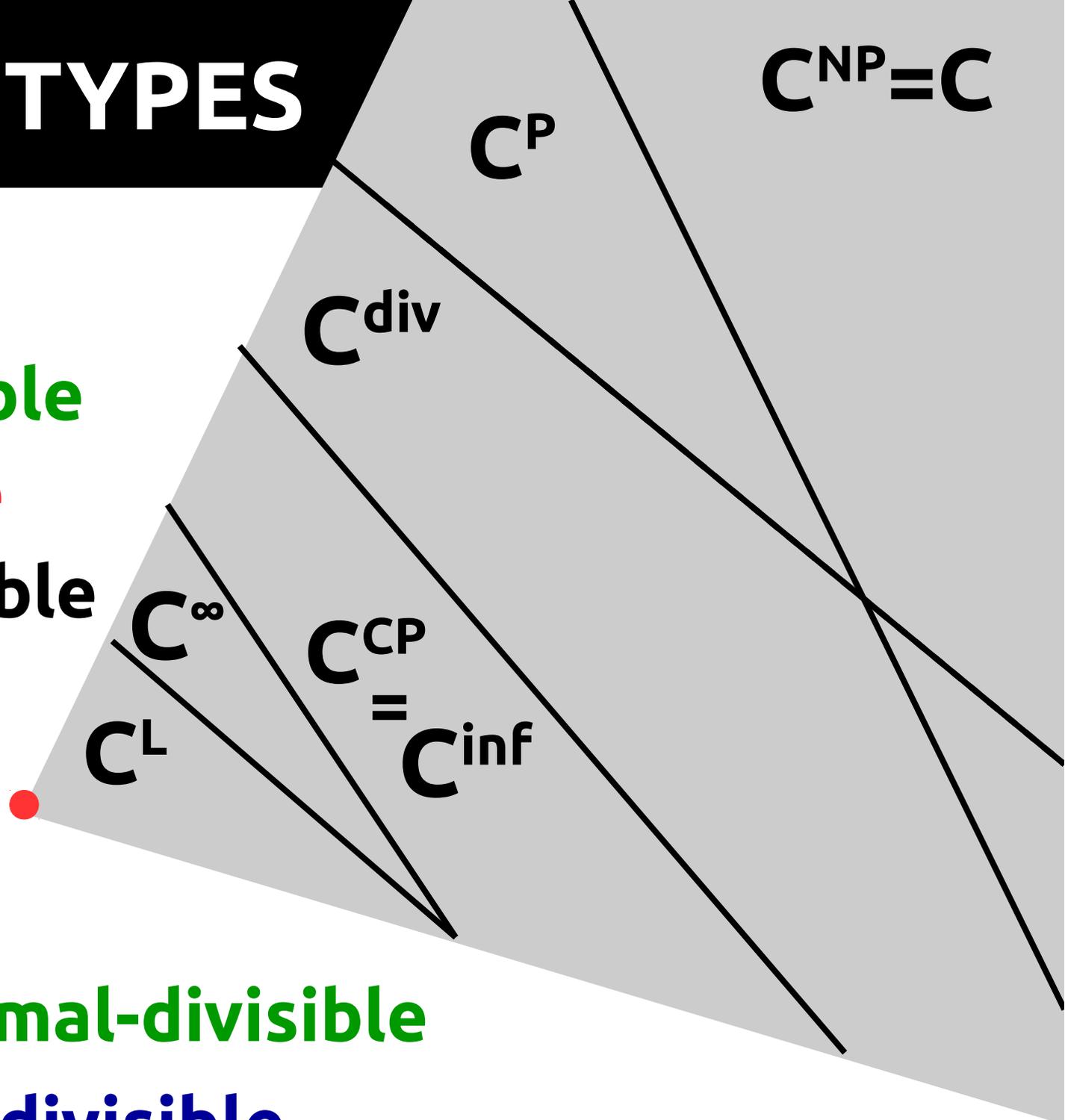
C^{NP} NP-divisible

$C^{indivisible}$

C^{div} divisible

C^{inf} infinitesimal-divisible

C^∞ infinitely-divisible



CHANNEL TYPES

$$C^{NP} = C$$

C^P

C^{div}

C^L L-divisible

C^{CP} CP-divisible

C^P P-divisible

C^{NP} NP-divisible

$$C^\infty \subset C^{inf} \subset C^{div}$$

$$U \quad ||$$

$$C^L \subset C^{CP} \subset C^P$$

$C^{indivisible}$

C^{div} divisible

C^{inf} infinitesimal-divisible

C^∞ infinitely-divisible

QUBIT UNITAL

$$\mathbf{C}^P \Leftrightarrow \det \geq 0$$

$\mathbf{C}^{\text{indivisible}} = \text{faces}$

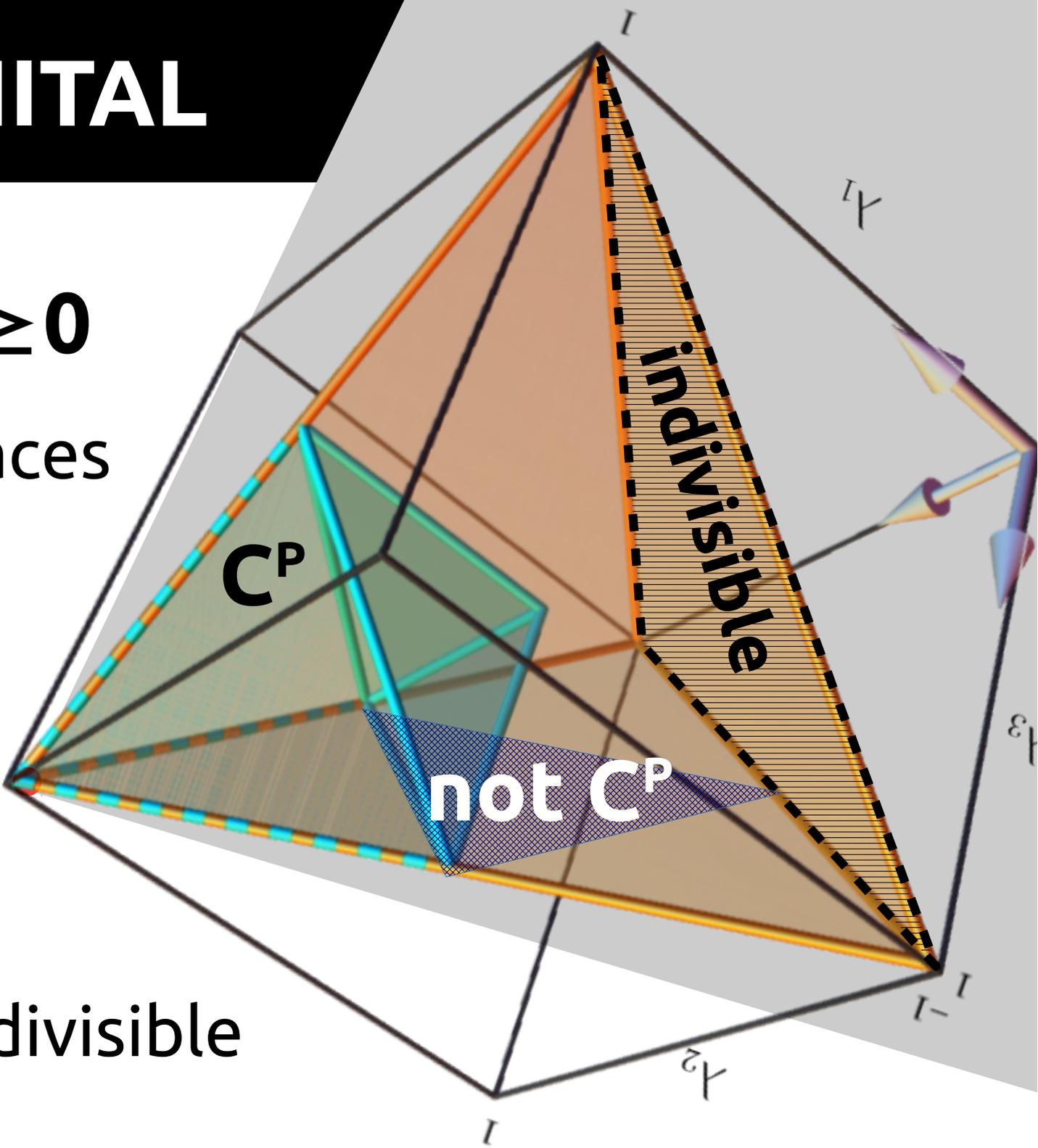
identity

\mathbf{C}^P

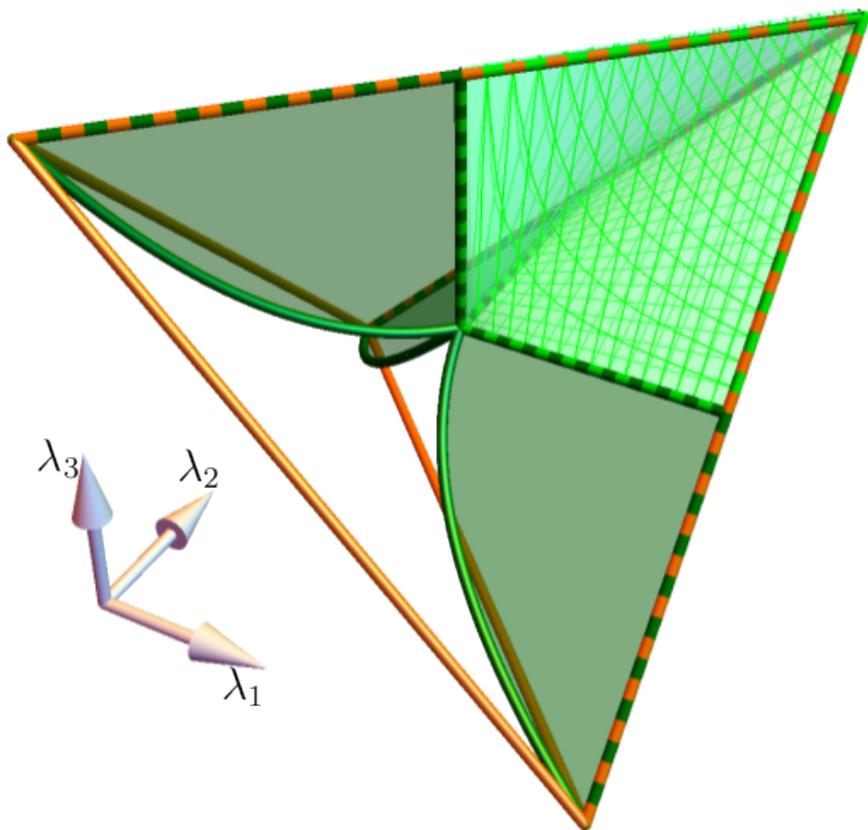
not \mathbf{C}^P

indivisible

\mathbf{C}^P but not divisible

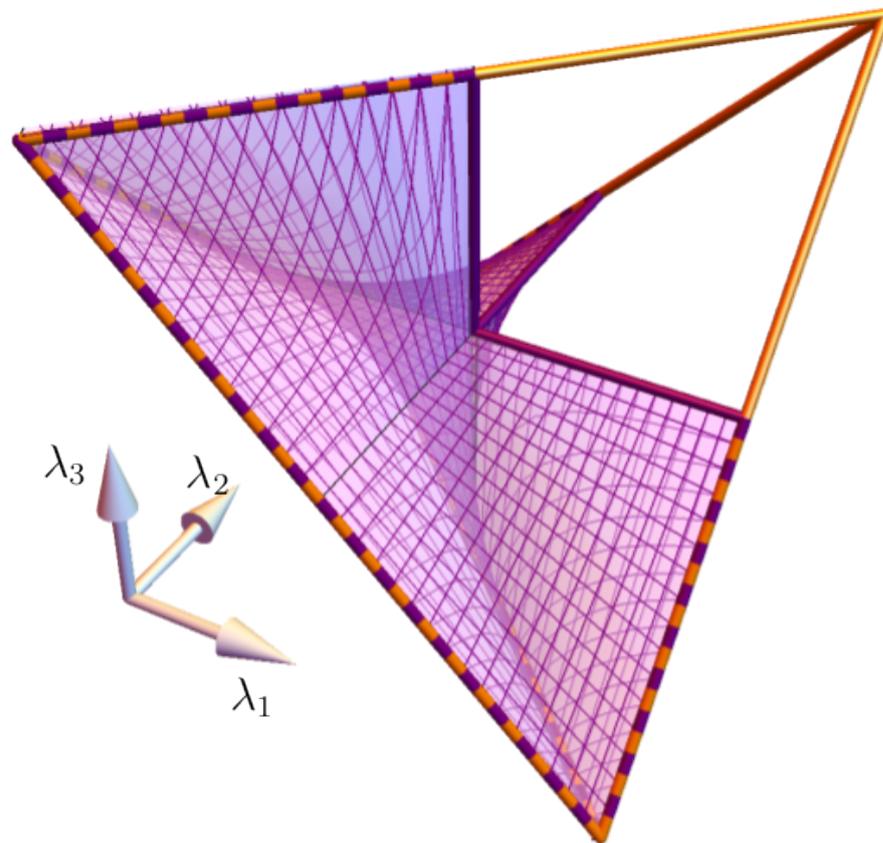


QUBIT UNITAL



\mathbb{C}^L

no tetrahedron symmetries

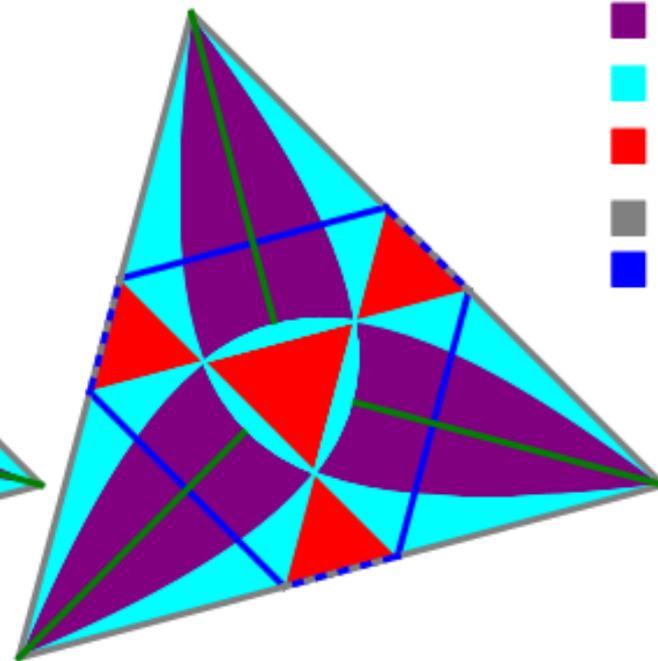
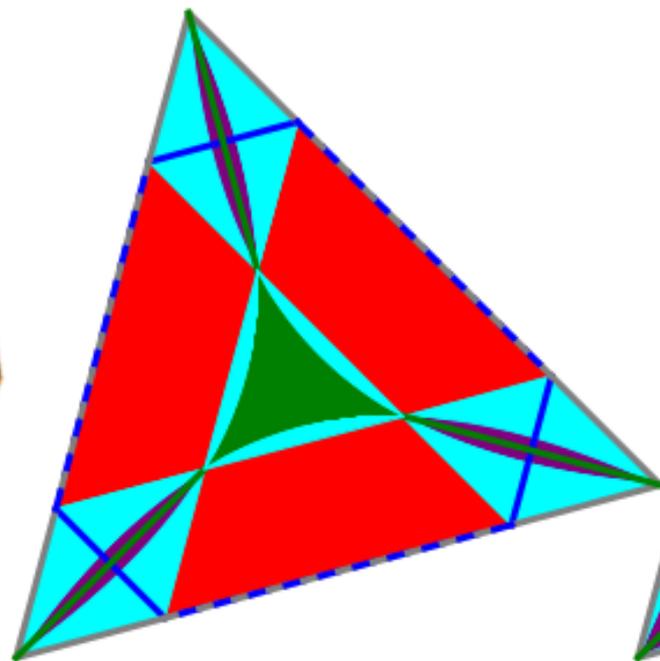
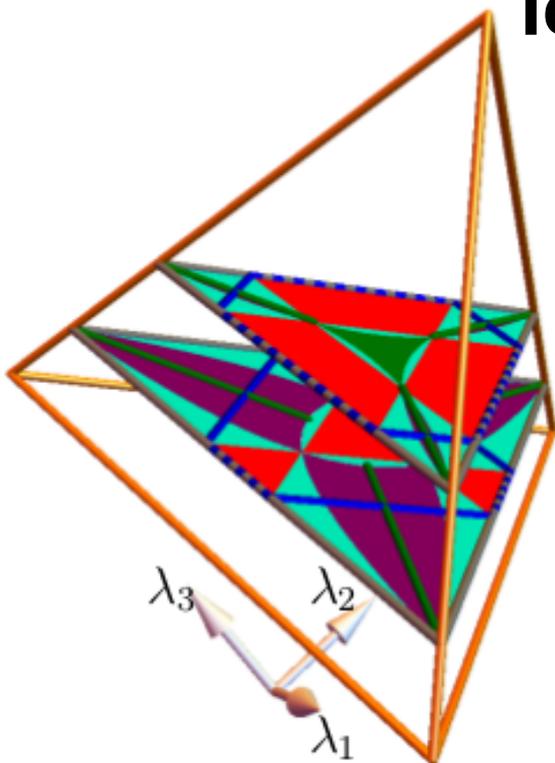


$\mathbb{C}^{CP} \setminus \mathbb{C}^L$

plus tetrahedron symmetries

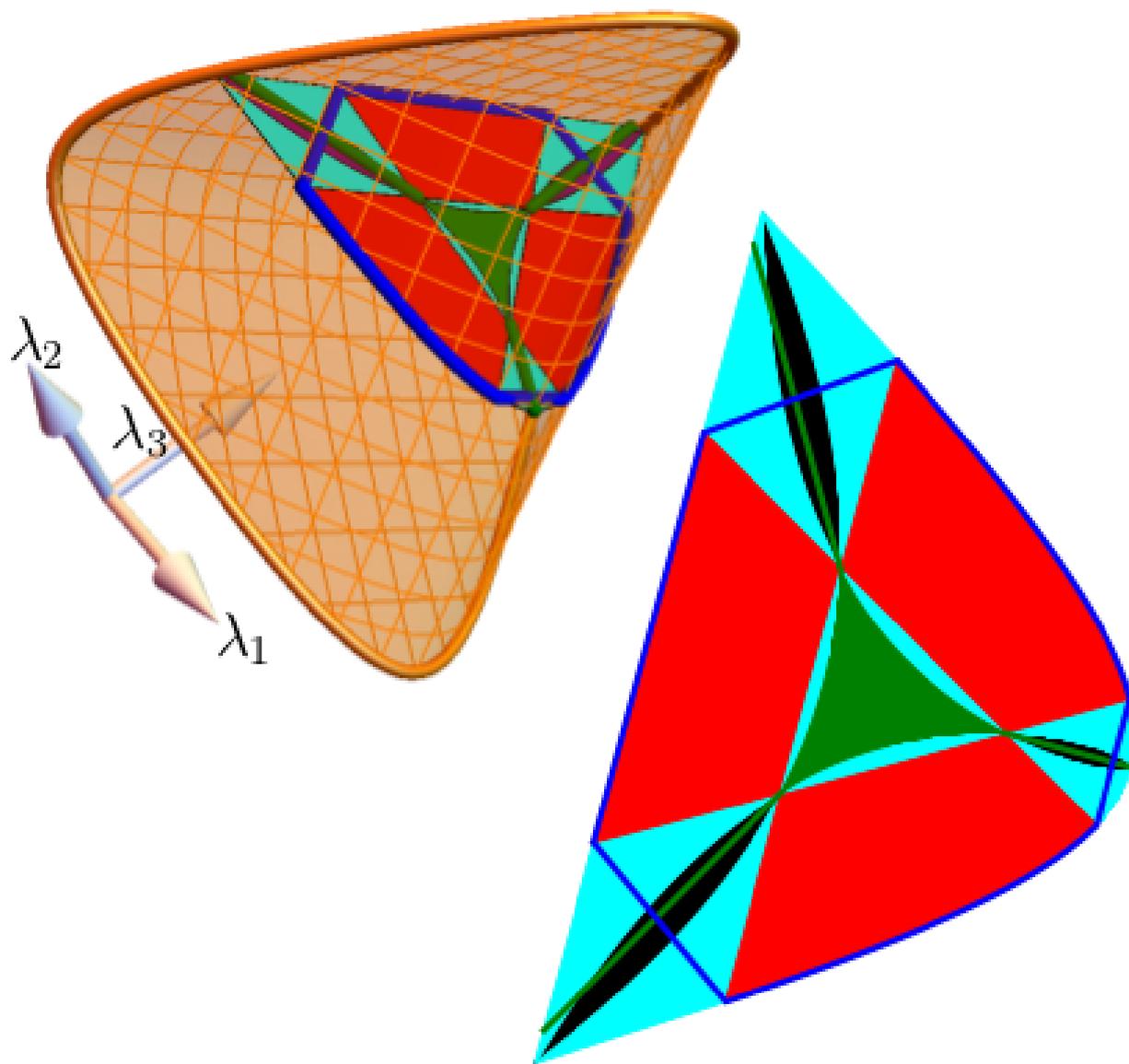
QUBIT UNITAL

identity



- \mathbb{C}^L
- $\mathbb{C}^{CP} \setminus \mathbb{C}^L$
- $\mathbb{C}^P \setminus \mathbb{C}^{CP}$
- $\mathbb{C}^{div} \setminus \mathbb{C}^P$
- $\overline{\mathbb{C}^{div}}$
- EB boundary

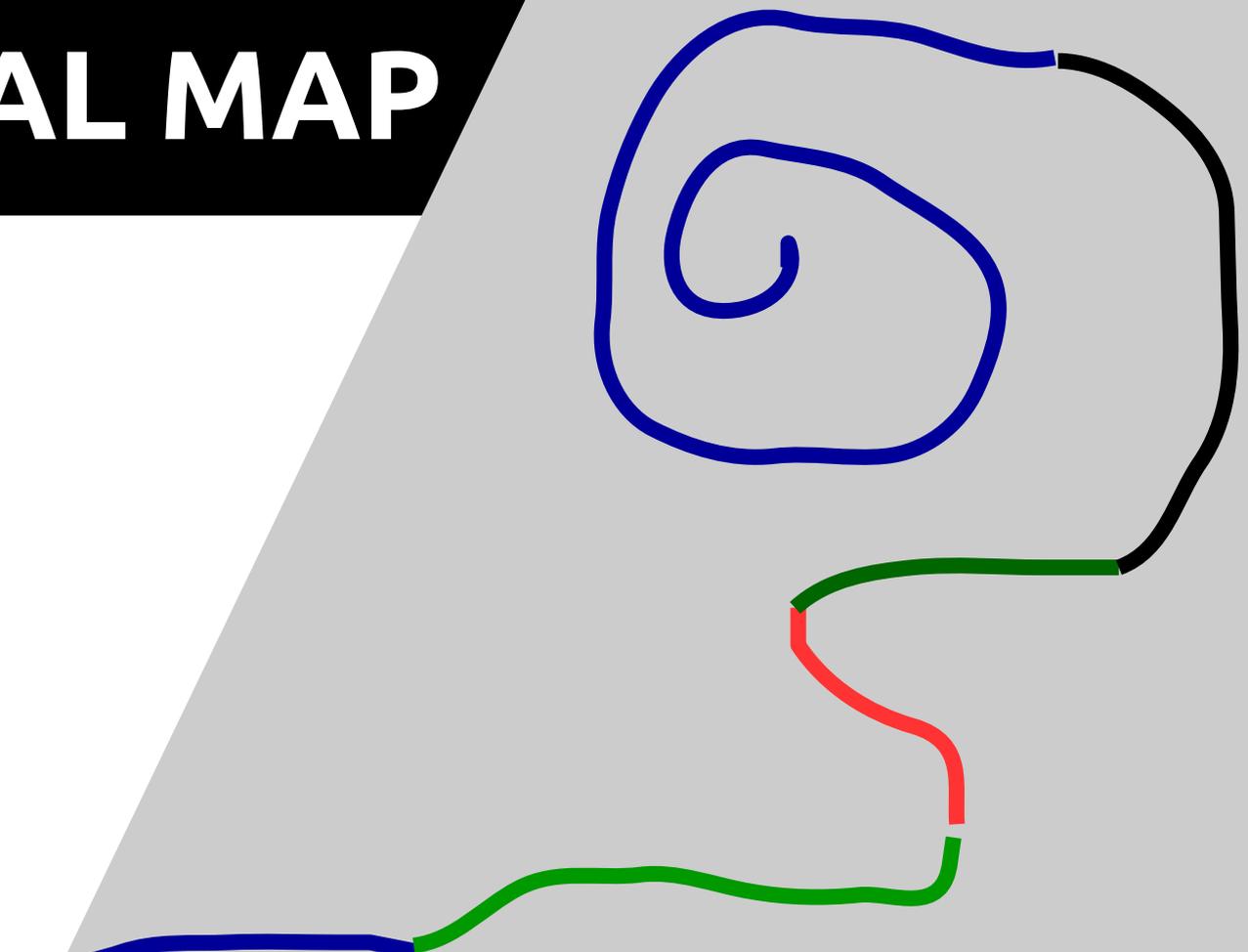
QUBIT NONUNITAL



DYNAMICAL MAP

L-divisible
CP-divisible
P-divisible
NP-divisible

identity ●

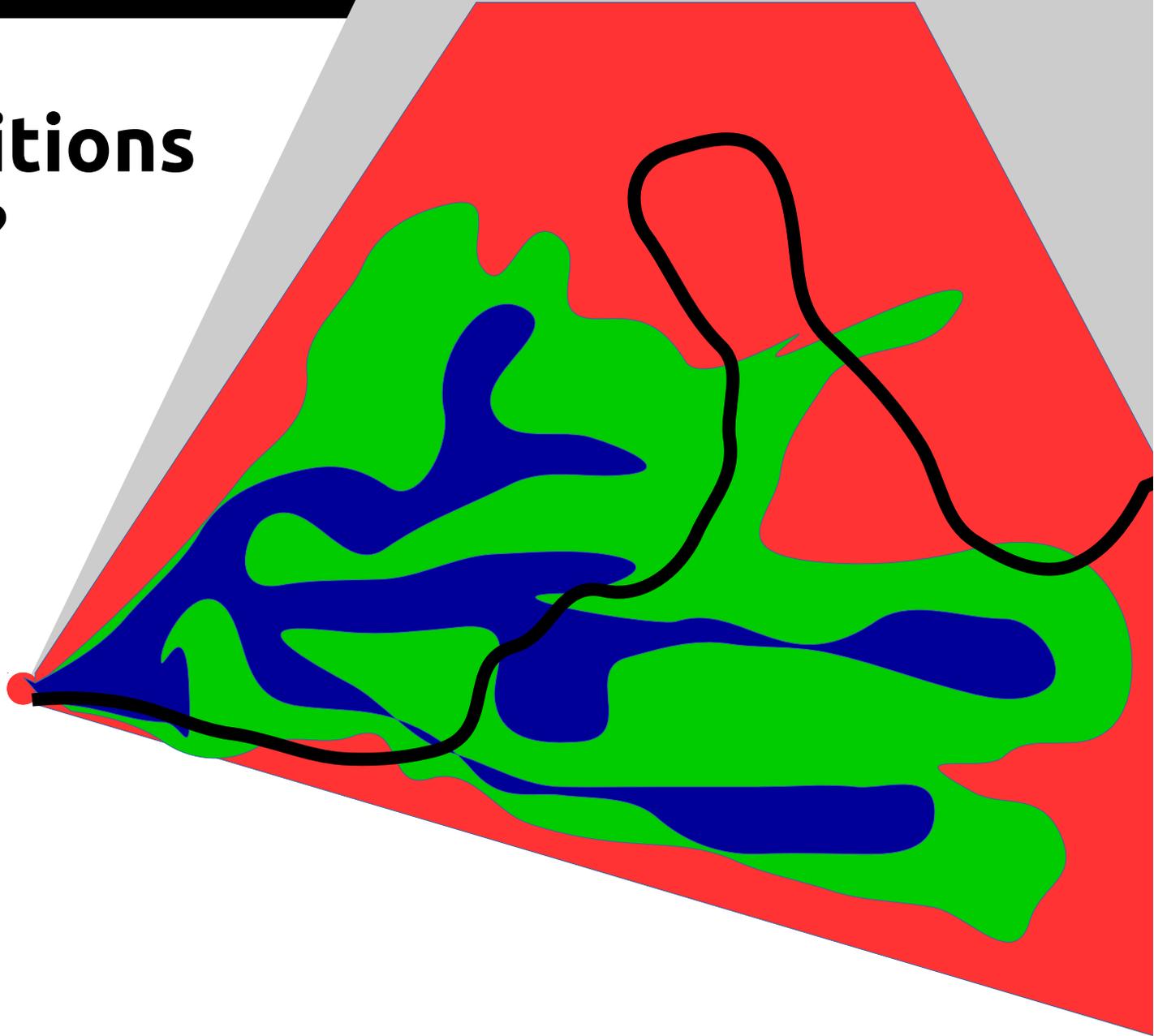


dynamical phases
- intervals

QUESTION

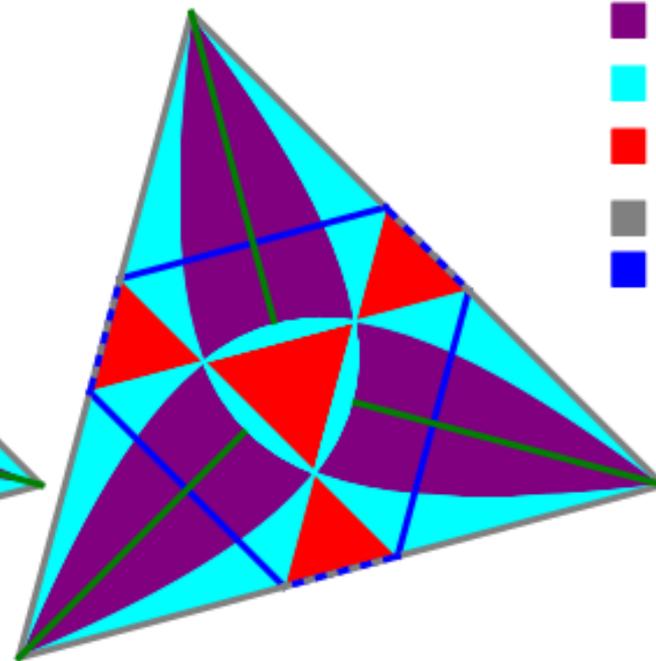
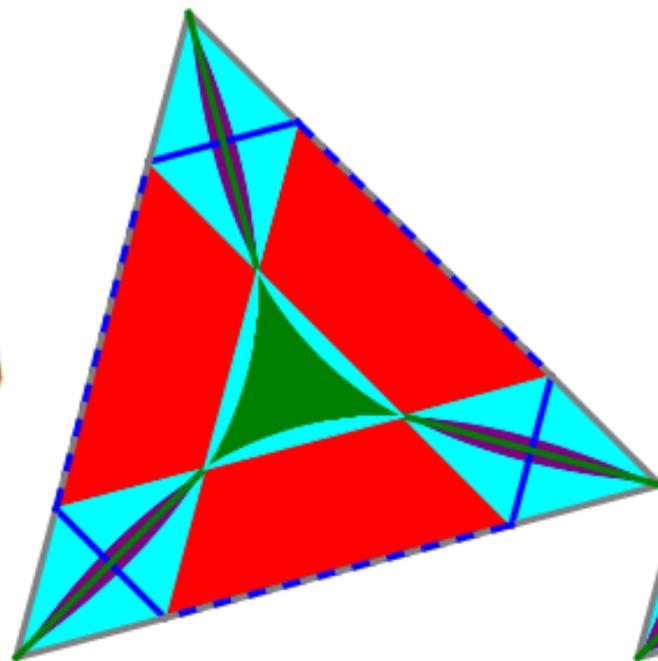
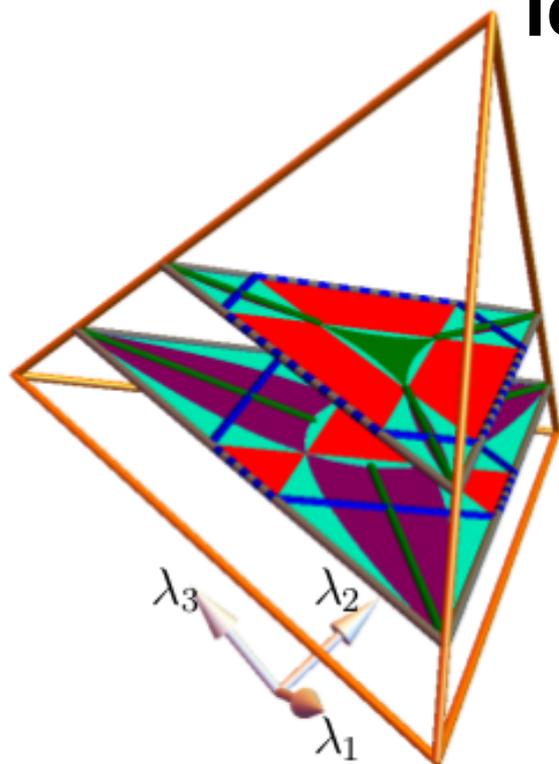
Which transitions are allowed?

identity



QUBIT UNITAL

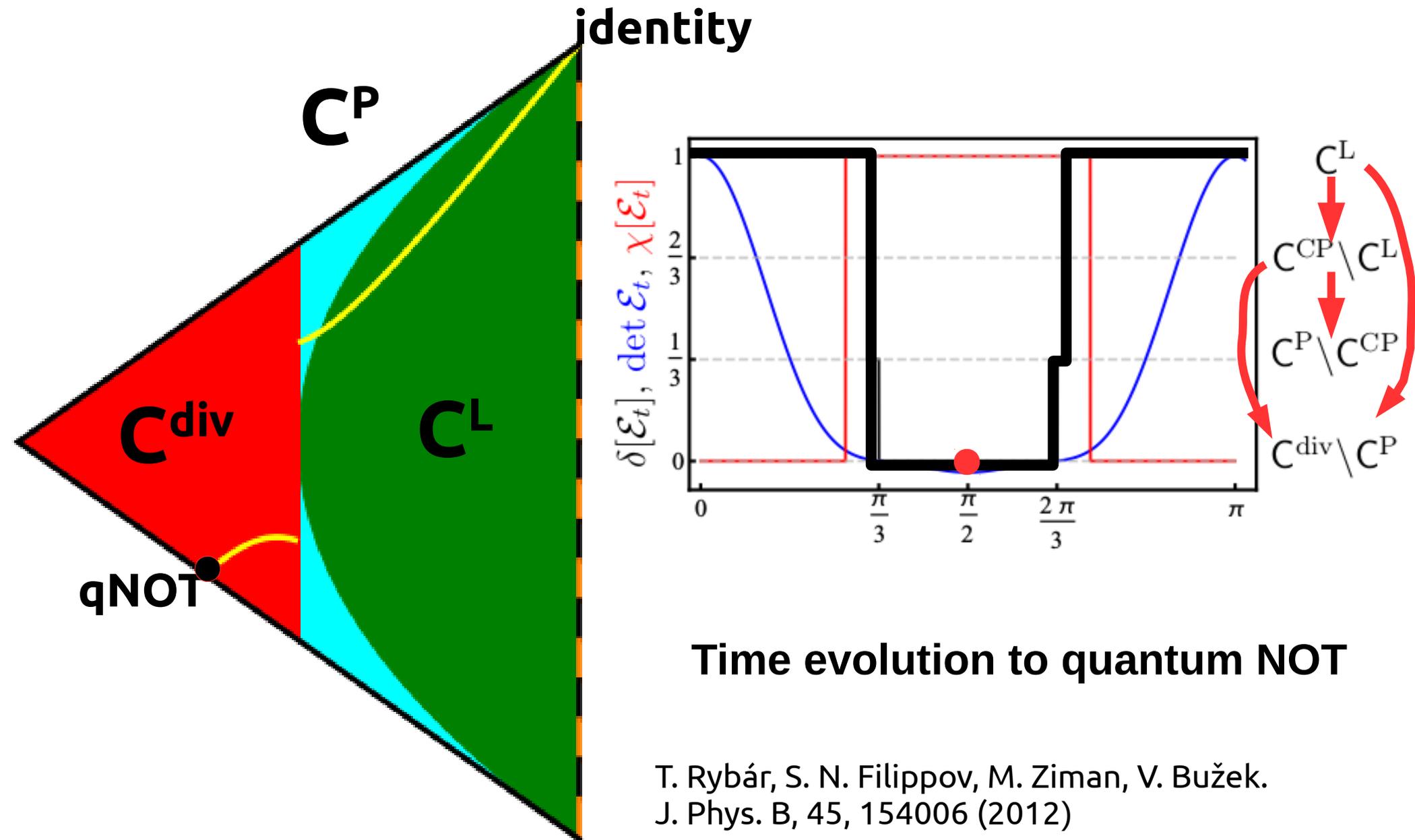
identity



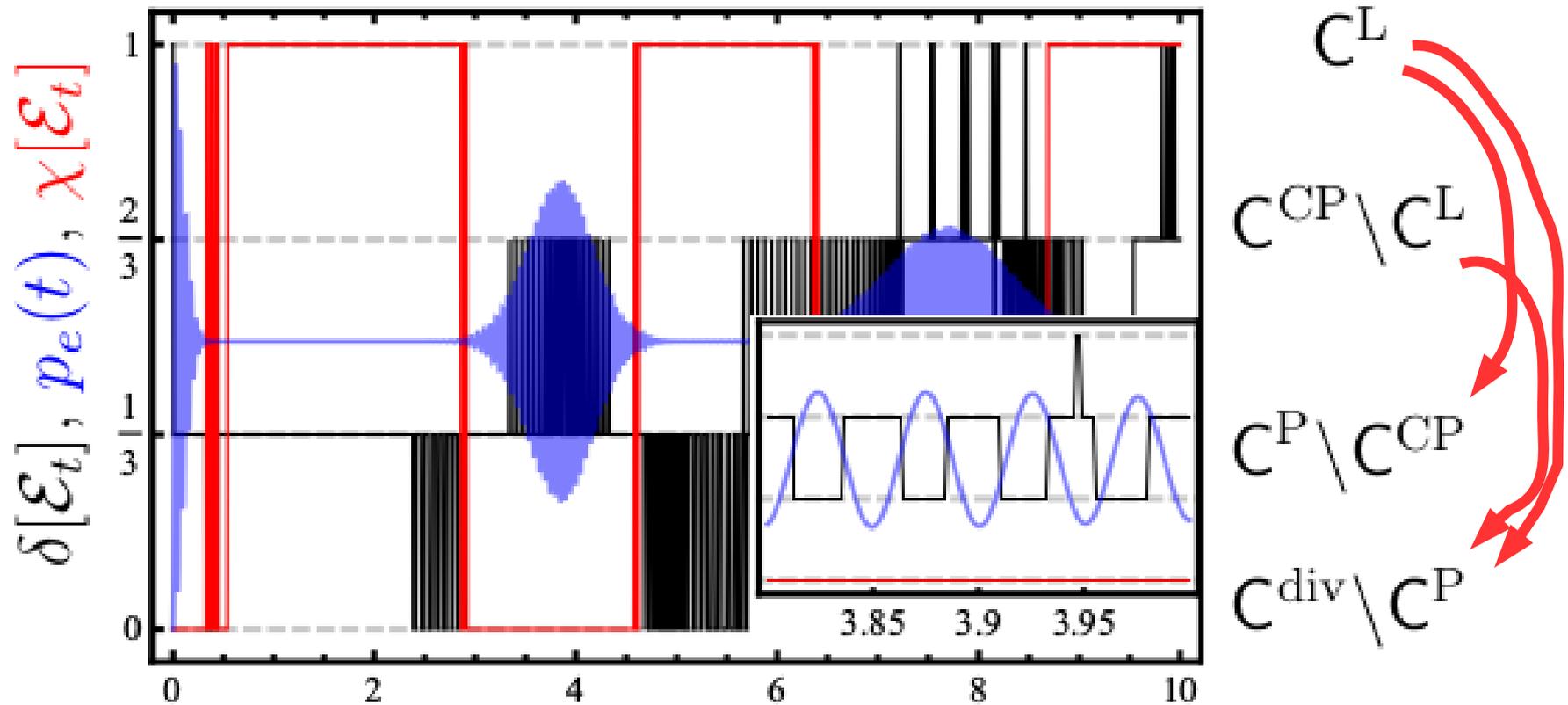
- \mathbb{C}^L
- $\mathbb{C}^{CP} \setminus \mathbb{C}^L$
- $\mathbb{C}^P \setminus \mathbb{C}^{CP}$
- $\mathbb{C}^{div} \setminus \mathbb{C}^P$
- $\overline{\mathbb{C}^{div}}$
- EB boundary

All types of borders exist.

QUBIT DYNAMICAL MAP



QUBIT DYNAMICAL MAP



Jaynes-Cumming model

GAME OVER

**THANK YOU
FOR YOUR
ATTENTION**