Classical simulation of linear optics subject to (nonuniform) losses

Michał Oszmaniec and Daniel Brod
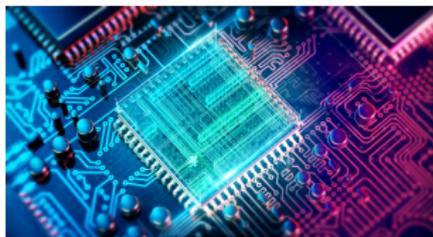
51st Symposium on Mathematical Physics, Toruń, 16 June 2019

⟨KC|K⟩

FNP
Foundation for Polish Science
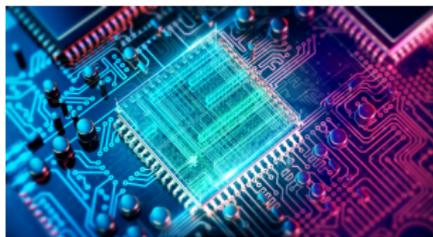
UNIVERSITY OF GDANSK

Daniel Brod (UFF Niteroi)

- Building a working quantum computer is **hard**[1] because of the noise and errors inevitably affecting quantum systems.

- Error correction and very clean physical qubits are needed. This results in **gigantic overheads** (> 1000) and a poses great technological challenges.

- An intermediate step: quantum machines of **restricted purpose** that (hopefully) can demonstrate **quantum computational supremacy**[2].

- Possible advantage: **smaller requirements, no error correction needed**.

---

[1]If not impossible: R. Alicki (2013), G. Kalai (2016)
[2]A. Harrow and A. Montanaro, Nature **549**, 203-209 (2017)
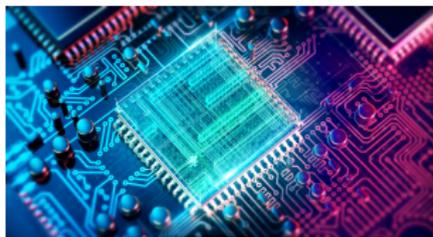
Motivation: Quantum Computational Supremacy

- Building a working quantum computer is **hard**[1] because of the noise and errors inevitably affecting quantum systems.
- Error correction and very clean physical qubits are needed. This results in **gigantic overheads** (> 1000) and a poses great technological challenges.
- An intermediate step: quantum machines of **restricted purpose** that (hopefully) can demonstrate **quantum computational supremacy**[2].
- Possible advantage: **smaller requirements, no error correction needed**.

---

[1]If not impossible: R. Alicki (2013), G. Kalai (2016)
[2]A. Harrow and A. Montanaro, Nature **549**, 203-209 (2017)

- Building a working quantum computer is **hard**[1] because of the noise and errors inevitably affecting quantum systems.

- Error correction and very clean physical qubits are needed. This results in **gigantic overheads** (> 1000) and a poses great technological challenges.

- An intermediate step: quantum machines of **restricted purpose** that (hopefully) can demonstrate **quantum computational supremacy**[2].

- Possible advantage: **smaller requirements, no error correction needed**.

---

[1]If not impossible: R. Alicki (2013), G. Kalai (2016)

[2]A. Harrow and A. Montanaro, Nature **549**, 203-209 (2017)
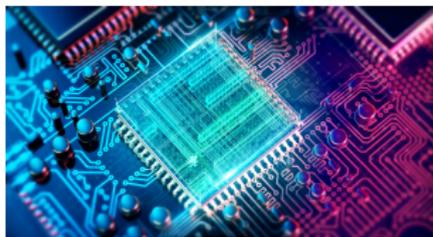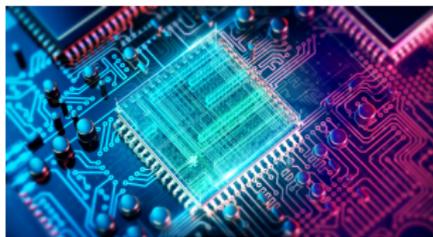
Motivation: Quantum Computational Supremacy

- Building a working quantum computer is **hard**[1] because of the noise and errors inevitably affecting quantum systems.

- Error correction and very clean physical qubits are needed. This results in **gigantic overheads** (> 1000) and a poses great technological challenges.

- An intermediate step: quantum machines of **restricted purpose** that (hopefully) can demonstrate **quantum computational supremacy**[2].

- Possible advantage: **smaller requirements, no error correction needed**.

---

[1]If not impossible: R. Alicki (2013), G. Kalai (2016)

[2]A. Harrow and A. Montanaro, Nature **549**, 203-209 (2017)

- Building a working quantum computer is **hard**[1] because of the noise and errors inevitably affecting quantum systems.

- Error correction and very clean physical qubits are needed. This results in **gigantic overheads** (> 1000) and a poses great technological challenges.

- An intermediate step: quantum machines of **restricted purpose** that (hopefully) can demonstrate **quantum computational supremacy**[2].

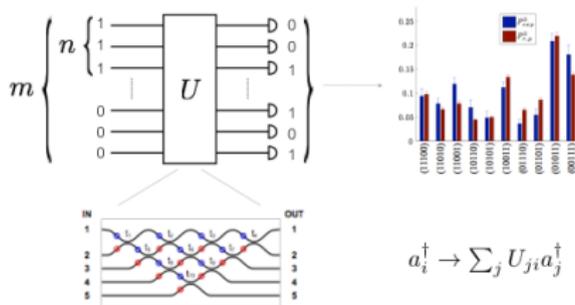- Possible advantage: **smaller requirements, no error correction needed**.

---

[1]If not impossible: R. Alicki (2013), G. Kalai (2016)
[2]A. Harrow and A. Montanaro, Nature **549**, 203-209 (2017)

Boson Sampling (I)

Boson sampling[3] is one of the proposals to attain **quantum advantage** using photonic linear optical circuit (with Fock states and particle-number detectors).

$$a_i^\dagger \to \sum_j U_{ji} a_j^\dagger$$

- **Task**: sample from the distribution p_U^{BS} for typical $U \in \mathrm{SU}(m)$.
- It is **hard** to classically sample from a distribution $\tilde{p}_U$ satisfying

$$\mathrm{TV}(\tilde{p}_U, p_U^{\mathrm{BS}}) \leq \epsilon \ \text{ in time } T = \mathrm{poly}(n, \frac{1}{\epsilon}) \ ,$$

where TV - **total variation distance** ($\sim$ distinguishing probability).

---

[3]S. Aaronson, A. Arkhipov, Proceedings of STOC'11 (2010)

Boson sampling[3] is one of the proposals to attain **quantum advantage** using photonic linear optical circuit (with Fock states and particle-number detectors).

$$a_i^\dagger \to \sum_j U_{ji} a_j^\dagger$$

- **Task**: sample from the distribution p_U^{BS} for typical $U \in \mathrm{SU}(m)$.
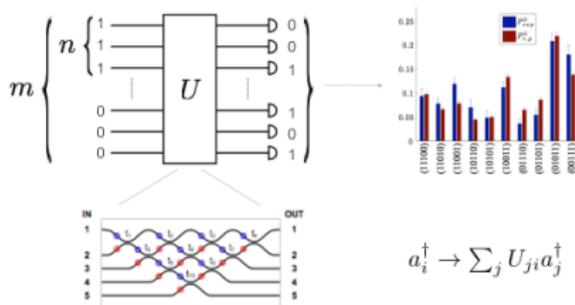- It is **hard** to classically sample from a distribution $\tilde{p}_U$ satisfying

$$\mathrm{TV}(\tilde{p}_U, p_U^{\mathrm{BS}}) \le \epsilon \ \text{ in time } T = \mathrm{poly}(n, \frac{1}{\epsilon}) \ ,$$

where TV - **total variation distance** ($\sim$ distinguishing probability).

---

[3]S. Aaronson, A. Arkhipov, Proceedings of STOC'11 (2010)

Boson Sampling (I)

Boson sampling[3] is one of the proposals to attain **quantum advantage** using photonic linear optical circuit (with Fock states and particle-number detectors).

$$a_i^\dagger \to \sum_j U_{ji} a_j^\dagger$$

- **Task**: sample from the distribution p_U^{BS} for typical $U \in \mathrm{SU}(m)$.
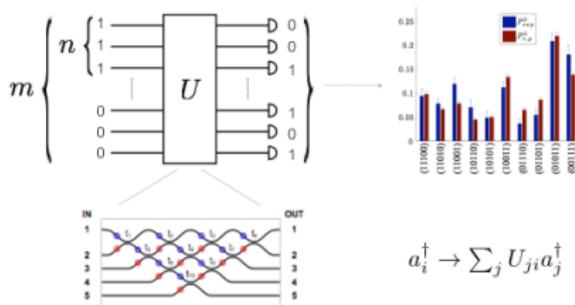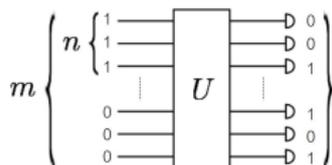- It is **hard** to classically sample from a distribution $\tilde{p}_U$ satisfying

$$\mathrm{TV}(\tilde{p}_U, p_U^{\mathrm{BS}}) \le \epsilon \text{ in time } T = \mathrm{poly}(n, \frac{1}{\epsilon}) ,$$

where TV - **total variation distance** ($\sim$ distinguishing probability).

---

[3]S. Aaronson, A. Arkhipov, Proceedings of STOC'11 (2010)

- Arguments for hardness: difficulty of computation of **matrix permanent**,

$$p_U^{\mathrm{BS}}(\mathbf{n}) \propto \left|\mathrm{Perm}\left(U_{\mathbf{n},\mathbf{s}}\right)\right|^2 \ ,$$

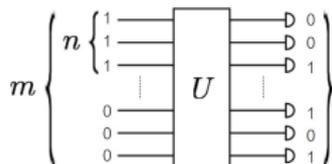 non-collapse of **Polynomial Hierarchy**, other conjectures.

- Interests due to recent developments in **integrated photonics**.

- State of the art: classical simulation for **up to 50 photons**[4] and *seven* photons[5] in experiments. It is not Boson-Sampling **scalable**?

[4]A. Neville *et al.*, Nature Physics **13**, 1153-1157 (2017)

[5]Hui Wang *et al.*, Phys. Rev. Lett. **120**, 230502 (2018)

- Arguments for hardness: difficulty of computation of **matrix permanent**,

$$p_U^{\mathrm{BS}}(\mathbf{n}) \propto \left|\mathrm{Perm}\left(U_{\mathbf{n},\mathbf{s}}\right)\right|^2 \ ,$$

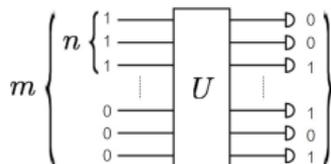 non-collapse of **Polynomial Hierarchy**, other conjectures.

- Interests due to recent developments in **integrated photonics**.

- State of the art: classical simulation for **up to 50 photons**[4] and *seven* photons[5] in experiments. It is not Boson-Sampling **scalable**?

---

[4]A. Neville *et al.*, Nature Physics **13**, 1153-1157 (2017)
[5]Hui Wang *et al.*, Phys. Rev. Lett. **120**, 230502 (2018)

- Arguments for hardness: difficulty of computation of **matrix permanent**,

$$p_U^{\mathrm{BS}}(\mathbf{n}) \propto \left| \mathrm{Perm}\left(U_{\mathbf{n},\mathbf{s}}\right) \right|^2 \ ,$$
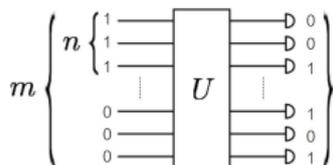
  non-collapse of **Polynomial Hierarchy**, other conjectures.

- Interests due to recent developments in **integrated photonics**.

- State of the art: classical simulation for **up to 50 photons**[4] and *seven* photons[5] in experiments. It is not Boson-Sampling **scalable**?

---

[4]A. Neville *et al.*, Nature Physics **13**, 1153-1157 (2017)

[5]Hui Wang *et al.*, Phys. Rev. Lett. **120**, 230502 (2018)

- Arguments for hardness: difficulty of computation of **matrix permanent**,

$$p_U^{\mathrm{BS}}(\mathbf{n}) \propto |\mathrm{Perm}\,(U_{\mathbf{n},\mathbf{s}})|^2 \ ,$$

  non-collapse of **Polynomial Hierarchy**, other conjectures.

- Interests due to recent developments in **integrated photonics**.

- State of the art: classical simulation for **up to 50 photons**[4] and *seven* photons[5] in experiments. It is not Boson-Sampling **scalable**?
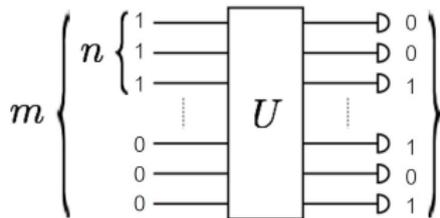
THIS WORK: EFFICIENT CLASSICAL SIMULATION OF BOSON SAMPLING UNDER (NONUNIFORM) PHOTON LOSSES
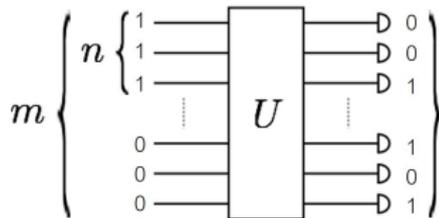
---

[4]A. Neville *et al.*, Nature Physics **13**, 1153-1157 (2017)

[5]Hui Wang *et al.*, Phys. Rev. Lett. **120**, 230502 (2018)

- Motivation and introduction to Boson Sampling
- Main technical tools and the idea of classical simulation
- Classical simulation of lossy Boson Sampling for:

    (a) Uniform loss model
    (b) Lossy linear optical network
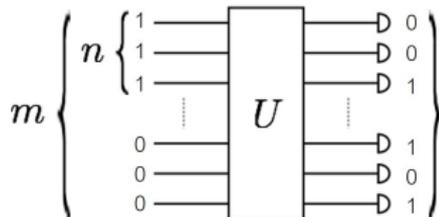
Modes   **Particles**

Modes

Particles

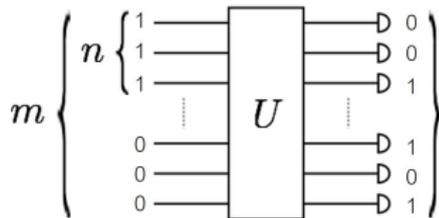Fock space $\mathcal{H} = \mathrm{Fock}_b(\mathbb{C}^m)$

Direct sum of symmetric spaces
$\mathcal{H} = \bigoplus_{l=0}^{\infty} \mathrm{Sym}^l(\mathbb{C}^m)$

Second vs. First Quantization

Modes

Fock space $\mathcal{H} = \mathrm{Fock}_b(\mathbb{C}^m)$

Fock states $|\mathbf{n}\rangle = |n_1, \ldots, n_m\rangle$

Particles

Direct sum of symmetric spaces
$\mathcal{H} = \bigoplus_{l=0}^{\infty} \mathrm{Sym}^l(\mathbb{C}^m)$

$|\mathbf{n}\rangle \propto \mathbb{P}_{\mathrm{sym}} |j_1\rangle \otimes |j_2\rangle \otimes \ldots \otimes |j_n\rangle$,
n_i - # of times $|i\rangle$ appears

Second vs. First Quantization

Modes

Fock space $\mathcal{H} = \mathrm{Fock}_b(\mathbb{C}^m)$

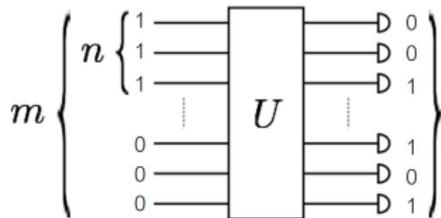Fock states $|\mathbf{n}\rangle = |n_1, \ldots, n_m\rangle$

Mode transformation: $a_i^\dagger \mapsto \sum_j U_{ji} a_j^\dagger$

Particles

Direct sum of symmetric spaces
$\mathcal{H} = \bigoplus_{l=0}^\infty \mathrm{Sym}^l(\mathbb{C}^m)$

$|\mathbf{n}\rangle \propto \mathbb{P}_{\mathrm{sym}} |j_1\rangle \otimes |j_2\rangle \otimes \ldots \otimes |j_n\rangle$,
n_i - # of times $|i\rangle$ appears

Evolution of particles: $\rho \mapsto U^{\otimes n} \rho (U^\dagger)^{\otimes n}$

Particle separable bosonic states

An n particle bosonic state ρ is called **particle separable** ($\rho \in \mathrm{Sep}$) iff

$$\sigma = \sum_\alpha p_\alpha |\phi_\alpha\rangle\langle\phi_\alpha|^{\otimes n} \ , \ \text{where } \{p_\alpha\} \text{ - prob. dist.}$$

Important features:

- **Easy update** of states $|\phi\rangle^{\otimes n}$ under linear optics (acting like $U^{\otimes n}$)

- The particle-number statistics of the state $(U|\phi\rangle)^{\otimes n}$ is **efficiently classically simulable by measuring individual particles**).

- If $\{p_\alpha\}$ - easy to sample from, then sampling from $\tilde{p}_U$ corresponding to boson sampling with input state σ is **efficiently classically simulable**.

Particle separable bosonic states

An n particle bosonic state ρ is called **particle separable** ($\rho \in \mathrm{Sep}$) iff

$$\sigma = \sum_\alpha p_\alpha |\phi_\alpha\rangle\langle\phi_\alpha|^{\otimes n} \ , \ \text{where } \{p_\alpha\} \text{ - prob. dist.}$$

Important features:

- **Easy update** of states $|\phi\rangle^{\otimes n}$ under linear optics (acting like $U^{\otimes n}$)

- The particle-number statistics of the state $(U|\phi\rangle)^{\otimes n}$ is **efficiently classically simulable by measuring individual particles**).

- If $\{p_\alpha\}$ - easy to sample from, then sampling from $\tilde{p}_U$ corresponding to boson sampling with input state σ is **efficiently classically simulable**.
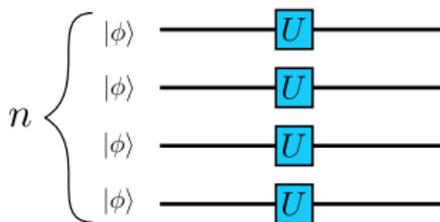
Particle separable bosonic states

An n particle bosonic state ρ is called **particle separable** ($\rho \in \mathrm{Sep}$) iff

$$\sigma = \sum_\alpha p_\alpha |\phi_\alpha\rangle\langle\phi_\alpha|^{\otimes n} \ , \ \text{where } \{p_\alpha\} \text{ - prob. dist.}$$

Important features:

- **Easy update** of states $|\phi\rangle^{\otimes n}$ under linear optics (acting like $U^{\otimes n}$)

- The particle-number statistics of the state $(U|\phi\rangle)^{\otimes n}$ is **efficiently classically simulable by measuring individual particles**).

- If $\{p_\alpha\}$ - easy to sample from, then sampling from $\tilde{p}_U$ corresponding to boson sampling with input state σ is **efficiently classically simulable**.
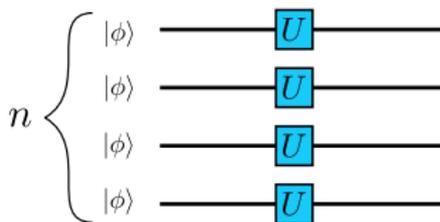
Particle separable bosonic states

An n particle bosonic state ρ is called **particle separable** ($\rho \in \mathrm{Sep}$) iff

$$\sigma = \sum_\alpha p_\alpha |\phi_\alpha\rangle\langle\phi_\alpha|^{\otimes n} \ , \ \text{where } \{p_\alpha\} \text{ - prob. dist.}$$

Important features:

- **Easy update** of states $|\phi\rangle^{\otimes n}$ under linear optics (acting like $U^{\otimes n}$)

- The particle-number statistics of the state $(U|\phi\rangle)^{\otimes n}$ is **efficiently classically simulable by measuring individual particles**).

- If $\{p_\alpha\}$ - easy to sample from, then sampling from $\tilde{p}_U$ corresponding to boson sampling with input state σ is **efficiently classically simulable**.
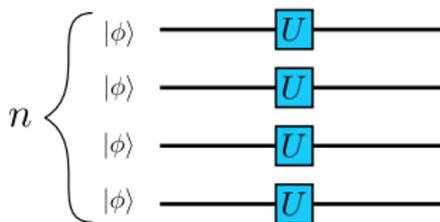
Models of particle losses

- **Model 0**: A fixed number of particles are lost in mode symmetric-manner

$$\rho \mapsto \rho_l = \text{tr}_{n-l}(\rho) \ .$$

- **Model 1**: Every particle is lost with probability $(1 - \eta)$. Equivalently: layer of beamsplitters with trasmitivity η.

$$\rho \mapsto \rho_\eta = \sum_{l=0}^{n} \eta^l (1 - \eta)^{n-l} \binom{n}{l} \rho_l \ .$$

- **Model 2**: **Mode- and location-dependant** losses of photons (**do not commute** with linear optics).

Models of particle losses

- **Model 0**: A fixed number of particles are lost in mode symmetric-manner

$$\rho \mapsto \rho_l = \mathrm{tr}_{n-l}(\rho) \ .$$

- **Model 1**: Every particle is lost with probability $(1 - \eta)$. Equivalently: layer of beamsplitters with trasmitivity η.

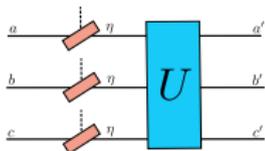$$\rho \mapsto \rho_\eta = \sum_{l=0}^{n} \eta^l (1 - \eta)^{n-l} \binom{n}{l} \rho_l \ .$$

- Model 2: Mode- and location-dependant losses of photons (do not commute with linear optics).

- **Model 0**: A fixed number of particles are lost in mode symmetric-manner

$$\rho \mapsto \rho_l = \operatorname{tr}_{n-l}(\rho) \ .$$

- **Model 1**: Every particle is lost with probability $(1 - \eta)$. Equivalently: layer of beamsplitters with trasmitivity η.

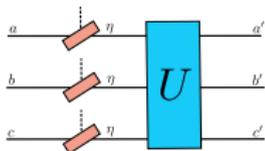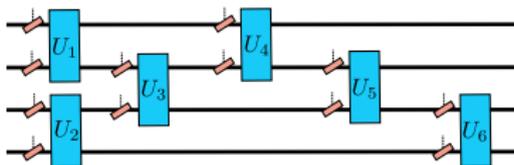$$\rho \mapsto \rho_\eta = \sum_{l=0}^{n} \eta^l (1 - \eta)^{n-l} \binom{n}{l} \rho_l \ .$$

- **Model 2**: **Mode- and location-dependant** losses of photons (**do not commute** with linear optics).

General simulation strategy

- "Pull-out" a lossy channel Λ_{los} before a non-trivial operation is applied. The input state $|\Psi_0\rangle = |1, \ldots, 1\rangle$ is *exchanged* to $\rho_{los} = \Lambda_{los}(\Psi_0)$.
- **Main idea**: Approximate ρ_{los} by **symmetric separable** states in **trace distance**.

$$\Delta = \mathrm{d}_{\mathrm{tr}}(\sigma_*, \rho_{los}) \ .$$

- Finding a suitable σ_* gives the **immediate classical simulation** of Boson Sampling to accuracy Δ in TV (a figure of merit for BS),

$$\mathrm{TV}(\tilde{p}_U, p_U^{\mathrm{BS}}) \leq \mathrm{d}_{\mathrm{tr}}(\sigma_*, \rho_{los}) = \Delta \ .$$

- "Pull-out" a lossy channel Λ_{los} before a non-trivial operation is applied. The input state $|\Psi_0\rangle = |1, \ldots, 1\rangle$ is *exchanged* to $\rho_{los} = \Lambda_{los}(\Psi_0)$.

- **Main idea**: Approximate ρ_{los} by **symmetric separable** states in **trace distance**.
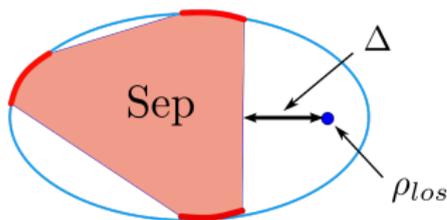

$$\Delta = \mathrm{d_{tr}}(\sigma_*, \rho_{los}) \ .$$

- Finding a suitable σ_* gives the **immediate classical simulation** of Boson Sampling to accuracy Δ in TV (a figure of merit for BS),

$$\mathrm{TV}(\bar{p}_U, p_U^{\mathrm{BS}}) \leq \mathrm{d_{tr}}(\sigma_*, \rho_{los}) = \Delta \ .$$

- "Pull-out" a lossy channel Λ_{los} before a non-trivial operation is applied. The input state $|\Psi_0\rangle = |1, \ldots, 1\rangle$ is *exchanged* to $\rho_{los} = \Lambda_{los}(\Psi_0)$.

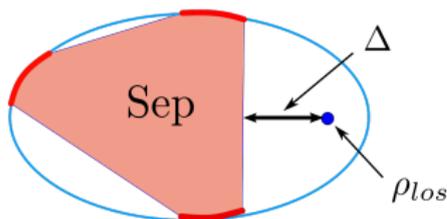- **Main idea**: Approximate ρ_{los} by **symmetric separable** states in **trace distance**.
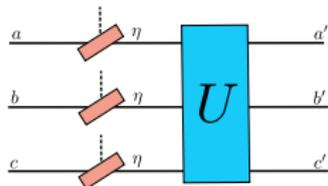

$$\Delta = \mathrm{d_{tr}}(\sigma_*, \rho_{los}) \ .$$

- Finding a suitable σ_* gives the **immediate classical simulation** of Boson Sampling to accuracy Δ in TV (a figure of merit for BS),

$$\mathrm{TV}(\tilde{p}_U, p_U^{\mathrm{BS}}) \leq \mathrm{d_{tr}}(\sigma_*, \rho_{los}) = \Delta \ .$$

Classical simulation for uniform loss model

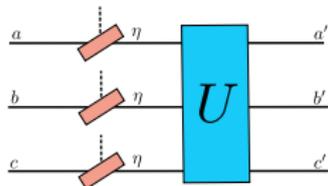The input state for a uniform beamsplitter loss model with transmitivity η,

$$\rho_\eta = \sum_{l=0}^{n} \eta^l (1-\eta)^{n-l} \binom{n}{l} \mathrm{tr}_{n-l}(\Psi_0) \ .$$

We take a probabilistic mixture of "mean-field" states $\sigma_*^{(l)}$ (phase-dephased Fourier transform of $|1\rangle^{\otimes l}$)

$$\sigma_\eta = \sum_{l=0}^{n} \eta^l (1-\eta)^{n-l} \binom{n}{l} \sigma_*^{(l)}.$$

We get $\mathrm{d}_{\mathrm{tr}}(\rho_\eta, \sigma_\eta) \approx \frac{\eta^2 n}{2} = \frac{\langle l \rangle^2}{2n}$, where $\langle l \rangle = \eta n$ is the *average* number of photons left in the network. **Hence** $\langle l \rangle = o(\sqrt{n})$ **implies** $\Delta \to 0$.

The input state for a uniform beamsplitter loss model with transmitivity η,

$$\rho_\eta = \sum_{l=0}^{n} \eta^l (1-\eta)^{n-l} \binom{n}{l} \operatorname{tr}_{n-l}(\Psi_0) \ .$$

We take a probabilistic mixture of "mean-field" states $\sigma_*^{(l)}$ (phase-dephased Fourier transform of $|1\rangle^{\otimes l}$)

$$\sigma_\eta = \sum_{l=0}^{n} \eta^l (1-\eta)^{n-l} \binom{n}{l} \sigma_*^{(l)}.$$

We get $\operatorname{d_{tr}}(\rho_\eta, \sigma_\eta) \approx \frac{\eta^2 n}{2} = \frac{\langle l \rangle^2}{2n}$, where $\langle l \rangle = \eta n$ is the *average* number of photons left in the network. **Hence** $\langle l \rangle = o(\sqrt{n})$ implies $\Delta \to 0$.

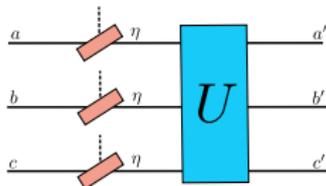The input state for a uniform beamsplitter loss model with transmitivity η,

$$\rho_\eta = \sum_{l=0}^{n} \eta^l (1-\eta)^{n-l} \binom{n}{l} \mathrm{tr}_{n-l}(\Psi_0) \ .$$

We take a probabilistic mixture of "mean-field" states $\sigma_*^{(l)}$ (phase-dephased Fourier transform of $|1\rangle^{\otimes l}$)

$$\sigma_\eta = \sum_{l=0}^{n} \eta^l (1-\eta)^{n-l} \binom{n}{l} \sigma_*^{(l)}.$$

We get $\mathrm{d}_{\mathrm{tr}}(\rho_\eta, \sigma_\eta) \approx \frac{\eta^2 n}{2} = \frac{\langle l \rangle^2}{2n}$, where $\langle l \rangle = \eta n$ is the *average* number of photons left in the network. **Hence** $\langle l \rangle = o(\sqrt{n})$ **implies** $\Delta \to 0$.

Extraction of losses in the realistic loss model

$$\Lambda_{\mathcal{N}}$$

RESULT (Extracting uniform losses from a lossy network)

Let s be a **smallest number of times a photon traverses a beamsplitter as it propagates through the network** $\mathcal{N}$. Let $\Lambda_{\mathcal{N}}$ be the channel associated the network $\mathcal{N}$. Then, **it is possible to "pull-out" uniform losses of transmitivity** $\eta_{\text{eff}} = \eta^s$ from the channel $\Lambda_{\mathcal{N}}$:

$$\Lambda_{\mathcal{N}} = \tilde{\Lambda}_{\mathcal{N}} \circ \Lambda_{\eta_{\text{eff}}} ,$$

where $\Lambda_{\eta_{\text{eff}}}$ - beamspliter loss model, $\tilde{\Lambda}_{\mathcal{N}}$ - linear optics channel.

- **Efficient classical simultion of lossy Boson sampling device** to accuracy $\Delta \approx \frac{n\eta^{2s}}{2}$ in TV- distance.
- **Typically** $s \gtrsim n$. In fact **even if** $s \approx \log(n)$ we can still have $\Delta \to 0$ (for fixed η)!

Extraction of losses in the realistic loss model

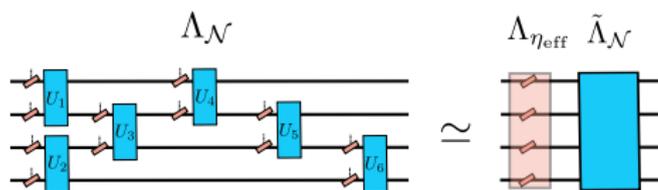### RESULT (Extracting uniform losses from a lossy network)

Let s be a **smallest number of times a photon traverses a beamsplitter as it propagates through the network** $\mathcal{N}$. Let $\Lambda_{\mathcal{N}}$ be the channel associated the network $\mathcal{N}$. Then, **it is possible to "pull-out" uniform losses of transmitivity** $\eta_{\text{eff}} = \eta^s$ from the channel $\Lambda_{\mathcal{N}}$:

$$\Lambda_{\mathcal{N}} = \tilde{\Lambda}_{\mathcal{N}} \circ \Lambda_{\eta_{\text{eff}}} ,$$

where $\Lambda_{\eta_{\text{eff}}}$ - beamspliter loss model, $\tilde{\Lambda}_{\mathcal{N}}$ - linear optics channel.

- **Efficient classical simultion of lossy Boson sampling device** to accuracy $\Delta \approx \frac{n\eta^{2s}}{2}$ in TV- distance.
- **Typically** $s \gtrsim n$. In fact even if $s \approx \log(n)$ we can still have $\Delta \to 0$ (for fixed η)!

Extraction of losses in the realistic loss model

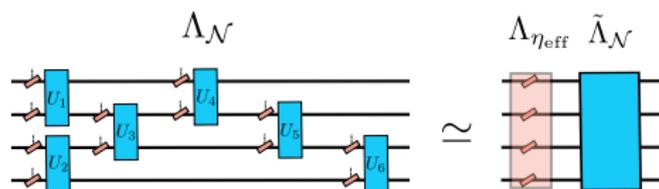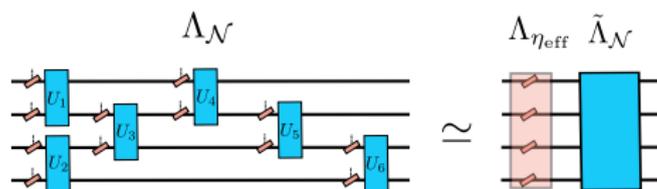## RESULT (Extracting uniform losses from a lossy network)

Let s be a **smallest number of times a photon traverses a beamsplitter as it propagates through the network** $\mathcal{N}$. Let $\Lambda_{\mathcal{N}}$ be the channel associated the network $\mathcal{N}$. Then, **it is possible to "pull-out" uniform losses of transmitivity** $\eta_{\text{eff}} = \eta^s$ from the channel $\Lambda_{\mathcal{N}}$:

$$\Lambda_{\mathcal{N}} = \tilde{\Lambda}_{\mathcal{N}} \circ \Lambda_{\eta_{\text{eff}}} \;,$$

where $\Lambda_{\eta_{\text{eff}}}$ - beamspliter loss model, $\tilde{\Lambda}_{\mathcal{N}}$ - linear optics channel.

- **Efficient classical simultion of lossy Boson sampling device** to accuracy $\Delta \approx \frac{n\eta^{2s}}{2}$ in TV- distance.
- **Typically** $s \gtrsim n$. In fact even if $s \approx \log(n)$ we can still have $\Delta \to 0$ (for fixed η)!

Extraction of losses in the realistic loss model

$$\Lambda_{\mathcal{N}} \qquad \Lambda_{\eta_{\mathrm{eff}}} \ \tilde{\Lambda}_{\mathcal{N}}$$

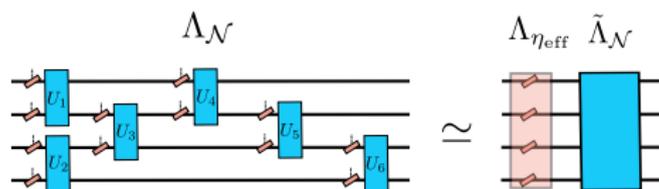### RESULT (Extracting uniform losses from a lossy network)

Let s be a **smallest number of times a photon traverses a beamsplitter as it propagates through the network** $\mathcal{N}$. Let $\Lambda_{\mathcal{N}}$ be the channel associated the network $\mathcal{N}$. Then, **it is possible to "pull-out" uniform losses of transmitivity** $\eta_{\mathrm{eff}} = \eta^s$ from the channel $\Lambda_{\mathcal{N}}$:

$$\Lambda_{\mathcal{N}} = \tilde{\Lambda}_{\mathcal{N}} \circ \Lambda_{\eta_{\mathrm{eff}}} \ ,$$

where $\Lambda_{\eta_{\mathrm{eff}}}$ - beamspliter loss model, $\tilde{\Lambda}_{\mathcal{N}}$ - linear optics channel.

- **Efficient classical simultion of lossy Boson sampling device** to accuracy $\Delta \approx \frac{n\eta^{2s}}{2}$ in TV- distance.
- **Typically** $s \gtrsim n$. In fact even if $s \approx \log(n)$ we can still have $\Delta \to 0$ (for fixed η)!

Extraction of losses in the realistic loss model

$$\Lambda_{\mathcal{N}} \qquad \Lambda_{\eta_{\text{eff}}} \tilde{\Lambda}_{\mathcal{N}}$$

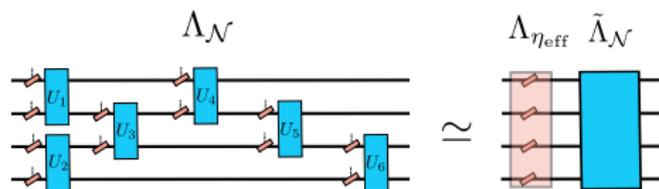### RESULT (Extracting uniform losses from a lossy network)

Let s be a **smallest number of times a photon traverses a beamsplitter as it propagates through the network** $\mathcal{N}$. Let $\Lambda_{\mathcal{N}}$ be the channel associated the network $\mathcal{N}$. Then, **it is possible to "pull-out" uniform losses of transmitivity** $\eta_{\text{eff}} = \eta^s$ from the channel $\Lambda_{\mathcal{N}}$:

$$\Lambda_{\mathcal{N}} = \tilde{\Lambda}_{\mathcal{N}} \circ \Lambda_{\eta_{\text{eff}}} \ ,$$

where $\Lambda_{\eta_{\text{eff}}}$ - beamspliter loss model, $\tilde{\Lambda}_{\mathcal{N}}$ - linear optics channel.

- **Efficient classical simultion of lossy Boson sampling device** to accuracy $\Delta \approx \frac{n\eta^{2s}}{2}$ in TV- distance.
- **Typically** $s \gtrsim n$. In fact **even if** $s \approx \log(n)$ we can still have $\Delta \to 0$ (for fixed η)!

Extraction of losses in the realistic loss model

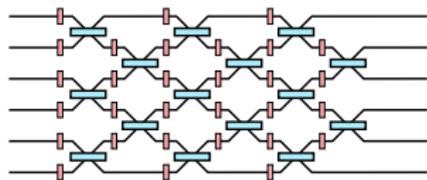## RESULT (Extracting uniform losses from a lossy network)

Let s be a **smallest number of times a photon traverses a beamsplitter as it propagates through the network** $\mathcal{N}$. Let $\Lambda_{\mathcal{N}}$ be the channel associated the network $\mathcal{N}$. Then, **it is possible to "pull-out" uniform losses of transmitivity** $\eta_{\mathrm{eff}} = \eta^s$ from the channel $\Lambda_{\mathcal{N}}$:

$$\Lambda_{\mathcal{N}} = \tilde{\Lambda}_{\mathcal{N}} \circ \Lambda_{\eta_{\mathrm{eff}}} \ ,$$
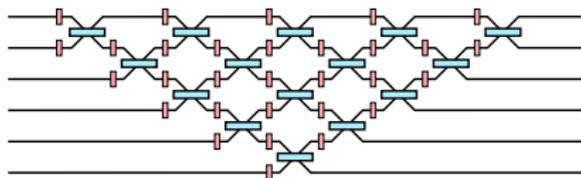
where $\Lambda_{\eta_{\mathrm{eff}}}$ - beamspliter loss model, $\tilde{\Lambda}_{\mathcal{N}}$ - linear optics channel.

- **Efficient classical simultion of lossy Boson sampling device** to accuracy $\Delta \approx \frac{n\eta^{2s}}{2}$ in TV- distance.
- **Typically** $s \gtrsim n$. In fact **even if** $s \approx \log(n)$ we can still have $\Delta \to 0$ (for fixed η)!

Balanced vs. unbalanced networks

(a)

(b)

Example (a): "Balanced" scheme[6] has $s \approx m/2$

Example (b): Traditional triangular scheme[7] has $s = 1$

Not all universal networks are characterized by large shortest paths. **How to classically simulate such situations**?

---

[6]W. Clemens *et al.*, Optica **3**, 12 1460-1465 (2016)
[7]M. Reck, *et al.* Phys. Rev. Lett. **73**, 58 (1994)

Balanced vs. unbalanced networks

(a)   (b)

Example (a): "Balanced" scheme[6] has $s \approx m/2$

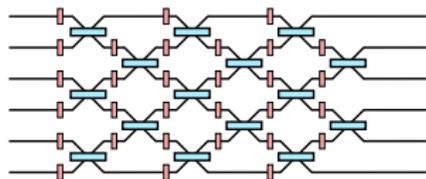**Example (b):** Traditional triangular scheme[7] has $s = 1$

Not all universal networks are characterized by large shortest paths. **How to classically simulate such situations**?
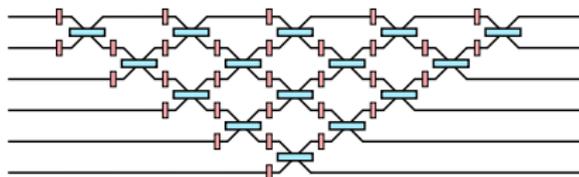
---

[6]W. Clemens *et al.*, Optica **3**, 12 1460-1465 (2016)

[7]M. Reck, *et al.* Phys. Rev. Lett. **73**, 58 (1994)

(a)

(b)

Example (a): "Balanced" scheme[6] has $s \approx m/2$

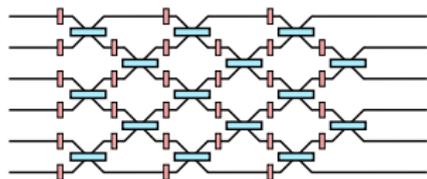**Example (b):** Traditional triangular scheme[7] has $s = 1$

Not all universal networks are characterized by large shortest paths. **How to classically simulate such situations**?
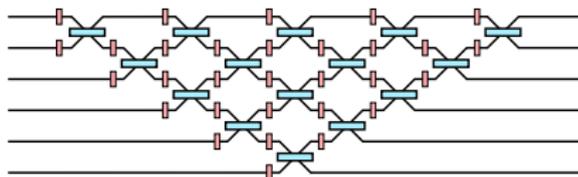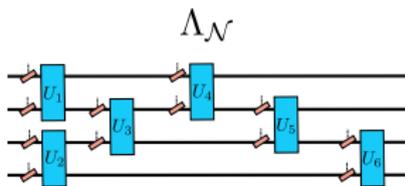
---

[6]W. Clemens *et al.*, Optica **3**, 12 1460-1465 (2016)

[7]M. Reck, *et al.* Phys. Rev. Lett. **73**, 58 (1994)

$\Lambda_{\mathcal{N}}$

RESULT (Extracting **nonuniform** losses from a lossy network)

Let s_i be a **smallest number of times a photon traverses a beamsplitter as it propagates through the network** $\mathcal{N}$ **starting form mode** i. Let $\Lambda_{\mathcal{N}}$ be the channel associated the network $\mathcal{N}$. Then, **it is possible to "pull-out" nonuniform losses characterized by transmitivities** $\eta_{i,\text{eff}} = \eta^{s_i}$ **from the** channel $\Lambda_{\mathcal{N}}$:

$$\Lambda_{\mathcal{N}} = \bar{\Lambda}_{\mathcal{N}} \circ \Lambda_{\vec{\eta}_{\text{eff}}} \ ,$$

where $\Lambda_{\vec{\eta}_{\text{eff}}}$ - nonuniform beamspliter loss model, $\bar{\Lambda}_{\mathcal{N}}$ -linear optics channel.

Quantitative characterization of unbalanced losses via network's geometry

---

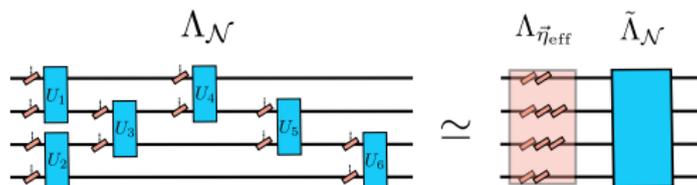### RESULT (Extracting **nonuniform** losses from a lossy network)

Let s_i be a **smallest number of times a photon traverses a beamsplitter as it propagates through the network $\mathcal{N}$ starting form mode** i. Let $\Lambda_{\mathcal{N}}$ be the channel associated the network $\mathcal{N}$. Then, **it is possible to "pull-out" nonuniform losses characterized by transmitivities** $\eta_{i,\text{eff}} = \eta^{s_i}$ from the channel $\Lambda_{\mathcal{N}}$:

$$\Lambda_{\mathcal{N}} = \bar{\Lambda}_{\mathcal{N}} \circ \Lambda_{\vec{\eta}_{\text{eff}}} \ ,$$

where $\Lambda_{\vec{\eta}_{\text{eff}}}$ - nonuniform beamspliter loss model, $\bar{\Lambda}_{\mathcal{N}}$ -linear optics channel.

Quantitative characterization of unbalanced losses via network's geometry

Extraction of losses in the realistic loss model (II)

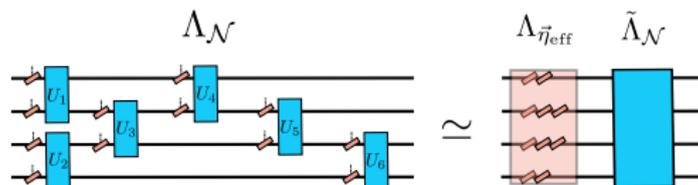## RESULT (Extracting **nonuniform** losses from a lossy network)

Let s_i be a **smallest number of times a photon traverses a beamsplitter as it propagates through the network $\mathcal{N}$ starting form mode i.** Let $\Lambda_{\mathcal{N}}$ be the channel associated the network $\mathcal{N}$. Then, **it is possible to "pull-out" nonuniform losses characterized by transmitivities $\eta_{i,\mathrm{eff}} = \eta^{s_i}$ from the** channel $\Lambda_{\mathcal{N}}$:

$$\Lambda_{\mathcal{N}} = \bar{\Lambda}_{\mathcal{N}} \circ \Lambda_{\vec{\eta}_{\mathrm{eff}}} \; ,$$

where $\Lambda_{\vec{\eta}_{\mathrm{eff}}}$ - nonuniform beamsplitter loss model, $\bar{\Lambda}_{\mathcal{N}}$ -linear optics channel.

Quantitative characterization of unbalanced losses via network's geometry

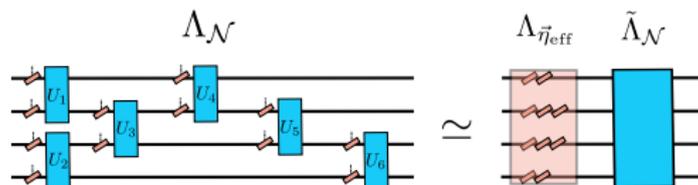RESULT (Extracting **nonuniform** losses from a lossy network)

Let s_i be a **smallest number of times a photon traverses a beamsplitter as it propagates through the network $\mathcal{N}$ starting form mode i**. Let $\Lambda_{\mathcal{N}}$ be the channel associated the network $\mathcal{N}$. Then, **it is possible to "pull-out" nonuniform losses characterized by transmitivities $\eta_{i,\text{eff}} = \eta^{s_i}$ from the** channel $\Lambda_{\mathcal{N}}$:

$$\Lambda_{\mathcal{N}} = \tilde{\Lambda}_{\mathcal{N}} \circ \Lambda_{\vec{\eta}_{\text{eff}}} \ ,$$

where $\Lambda_{\vec{\eta}_{\text{eff}}}$ - nonuniform beamspliter loss model, $\tilde{\Lambda}_{\mathcal{N}}$ -linear optics channel.

Quantitative characterization of unbalanced losses via network's geometry

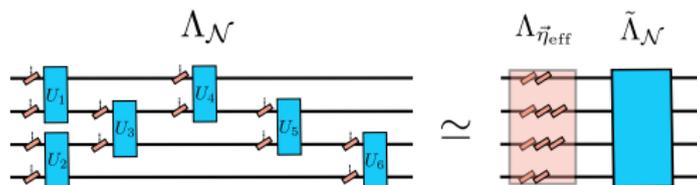### RESULT (Extracting **nonuniform** losses from a lossy network)

Let s_i be a **smallest number of times a photon traverses a beamsplitter as it propagates through the network $\mathcal{N}$ starting form mode i.** Let $\Lambda_\mathcal{N}$ be the channel associated the network $\mathcal{N}$. Then, **it is possible to "pull-out" nonuniform losses characterized by transmitivities $\eta_{i,\mathrm{eff}} = \eta^{s_i}$ from the** channel $\Lambda_\mathcal{N}$:

$$\Lambda_\mathcal{N} = \tilde{\Lambda}_\mathcal{N} \circ \Lambda_{\vec{\eta}_{\mathrm{eff}}} \ ,$$

where $\Lambda_{\vec{\eta}_{\mathrm{eff}}}$ - nonuniform beamsplitter loss model, $\tilde{\Lambda}_\mathcal{N}$ -linear optics channel.

Quantitative characterization of unbalanced losses via network's geometry

(a)　　　　　　　　　　(b)

How to leverage this to get efficient classical simulation?

(a)   (b)

How to leverage this to get efficient classical simulation?
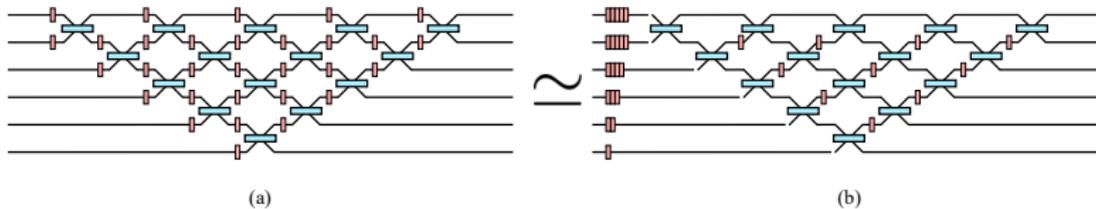
New easy instances of Boson Sampling

Standard input · Type A input · Type B input

$m \left\{ n \left\{ \begin{matrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ \vdots \\ 0 \end{matrix} \right. \right.$

$k \left\{ \begin{matrix} n_1 \\ \vdots \\ n_k \\ 0 \\ \vdots \\ 0 \\ 0 \end{matrix} \right.$

$k = O(1)$

$k \left\{ \begin{matrix} n_1 \\ 1 \\ 1 \\ 0 \\ \vdots \\ 0 \end{matrix} \right.$

$k = O(\log n)$
$n_1 = n - (k-1)$
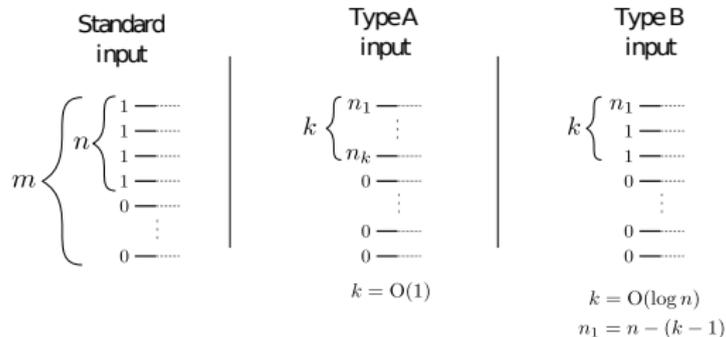
RESULT (Classical simulation of boson sampling for unbalanced inputs)

Consider the following types of input states:

A A Fock state with photons distributed arbitrarily in $k = O(1)$ different input modes (or bins).

B A Fock state with photons distributed in $k = O(\log n)$ bins such that all but $k - 1$ photons are in the kth mode.

Then, **there exists an efficient classical algorithm** simulating the corresponding instances of boson sampling.

	Standard input	Type A input	Type B input

$$m \left\{ n \left\{ \begin{matrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ \vdots \\ 0 \end{matrix} \right. \right.$$

$$k \left\{ \begin{matrix} n_1 \\ \vdots \\ n_k \\ 0 \\ \vdots \\ 0 \\ 0 \end{matrix} \right.$$
$$k = O(1)$$

$$k \left\{ \begin{matrix} n_1 \\ 1 \\ 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{matrix} \right.$$
$$k = O(\log n)$$
$$n_1 = n - (k-1)$$

Proof sketch[8]

- Measurements of occupation numbers $\longleftrightarrow$ measurement of subsequent particles the standard basis $\mathbf{r} = (r_n, r_{n-1}, \ldots, r_1)$.

- Sample one particle at the time using the identity

$$p(r_n, r_{n-1}, \ldots, r_1) = p(r_n | r_{n-1}, \ldots, r_1) \cdot p(r_{n-1} | r_{n-2}, \ldots, r_1) \cdot \ldots \cdot p(r_1) \, .$$

- Not too many "simple" marginal probabilities ($\approx$permanents) have to be computed.

[8]Generalization of P. Clifford and R. Clifford, *The Classical Complexity of Boson Sampling*, SODA (2018)

Standard input — Type A input — Type B input

$k = O(1)$

$k = O(\log n)$
$n_1 = n - (k-1)$

Proof sketch[8]

- Measurements of occupation numbers $\longleftrightarrow$ measurement of subsequent particles the standard basis $\mathbf{r} = (r_n, r_{n-1}, \ldots, r_1)$.
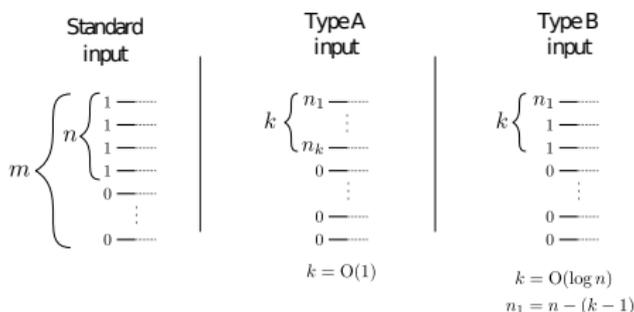
- Sample one particle at the time using the identity

$$p(r_n, r_{n-1}, \ldots, r_1) = p(r_n | r_{n-1}, \ldots, r_1) \cdot p(r_{n-1} | r_{n-2}, \ldots, r_1) \cdot \ldots \cdot p(r_1) \,.$$

- Not too many "simple" marginal probabilities ($\approx$permanents) have to be computed.

[8]Generalization of P. Clifford and R. Clifford, *The Classical Complexity of Boson Sampling*, SODA (2018)

New easy instances for Boson Sampling (II)

Proof sketch[8]

- Measurements of occupation numbers $\longleftrightarrow$ measurement of subsequent particles the standard basis $\mathbf{r} = (r_n, r_{n-1}, \ldots, r_1)$.
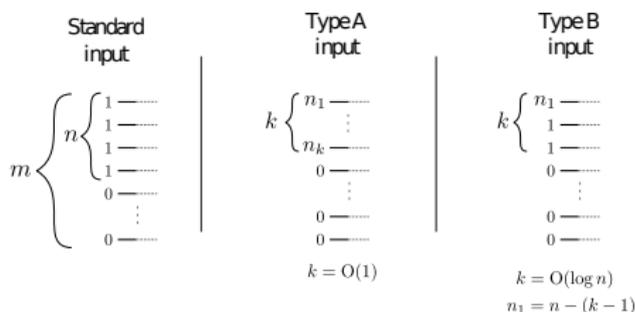
- Sample one particle at the time using the identity

$$p(r_n, r_{n-1}, \ldots, r_1) = p(r_n | r_{n-1}, \ldots, r_1) \cdot p(r_{n-1} | r_{n-2}, \ldots, r_1) \cdot \ldots \cdot p(r_1) .$$

- Not too many "simple" marginal probabilities ($\approx$permanents) have to be computed.

[8]Generalization of P. Clifford and R. Clifford, *The Classical Complexity of Boson Sampling*, SODA (2018)

Proof sketch[8]

- Measurements of occupation numbers $\longleftrightarrow$ measurement of subsequent particles the standard basis $\mathbf{r} = (r_n, r_{n-1}, \ldots, r_1)$.
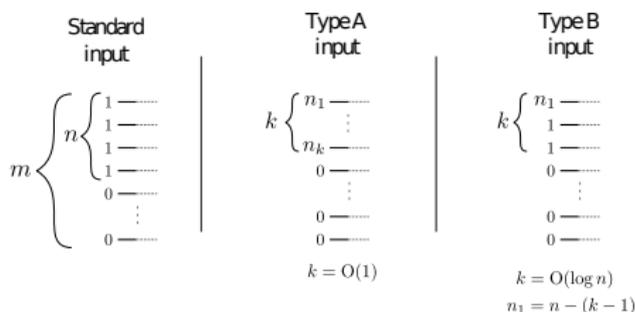
- Sample one particle at the time using the identity

$$p(r_n, r_{n-1}, \ldots, r_1) = p(r_n | r_{n-1}, \ldots, r_1) \cdot p(r_{n-1} | r_{n-2}, \ldots, r_1) \cdot \ldots \cdot p(r_1) .$$

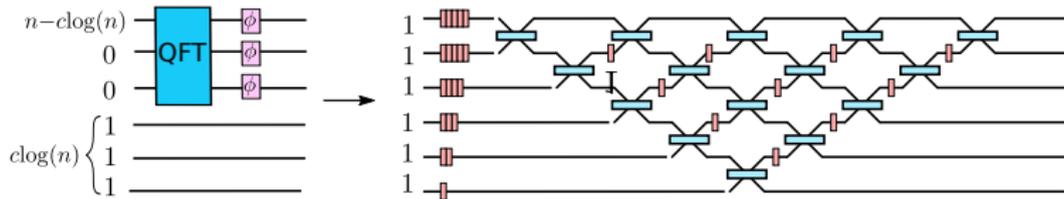- Not too many "simple" marginal probabilities ($\approx$permanents) have to be computed.

[8]Generalization of P. Clifford and R. Clifford, *The Classical Complexity of Boson Sampling*, SODA (2018)

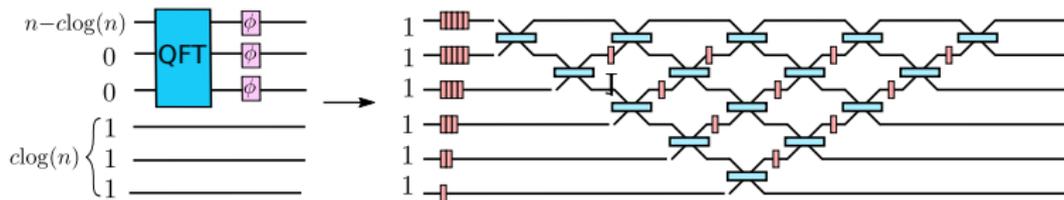**Idea:** Ignore modes that are not too lossy and put there exact inputs. For the remaining lossy modes use the same strategy as before. **States obtained in this way can be obtained as mixtures of sates of Type B**.

Idea: Ignore modes that are not too lossy and put there exact inputs. For the remaining lossy modes use the same strategy as before. **States obtained in this way can be obtained as mixtures of sates of Type B**.

Informal formulation of the result: Consider arbitrary lossy photonic network $\mathcal{N}$. Assume that there are at most $\mathrm{O}(\log(n))$ input modes characterized by depth smaller than s. Then, for standard BS input state $|\Psi_0\rangle$ it is possible to efficiently simulate output probability distribution to accuracy $\Delta \approx \frac{\eta^{2s}n}{2}$.

- Linear-optical networks **are heavily affected** by photon losses.

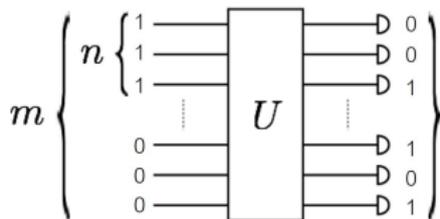- **Consequence for lossy Boson Sampling devices**: classical simulation of output statistics to precision Δ in TV-distance:

  (a) Uniform losses: average number of photons that are left $\langle l \rangle = o(\sqrt{n})$, then $\Delta \approx \frac{\langle l \rangle^2}{2n}$.

  (b) Lossy optical networks: $\Delta \approx \frac{\eta^{2s}n}{2}$, if less then $O(\log(n))$ input modes have depth smaller that s.

- **New technical results**: extraction of nonuniform losses from lossy networks and new classes of classically simulable inputs for BS

Conclusions

- Linear-optical networks **are heavily affected** by photon losses.
- **Consequence for lossy Boson Sampling devices**: classical simulation of output statistics to precision Δ in TV-distance:
  - (a) Uniform losses: average number of photons that are left $\langle l \rangle = \mathrm{o}(\sqrt{n})$, then $\Delta \approx \frac{\langle l \rangle^2}{2n}$.
  - (b) Lossy optical networks: $\Delta \approx \frac{\eta^{2s}n}{2}$, if less then $O(\log(n))$ input modes have depth smaller that s.
- New technical results: extraction of nonuniform losses from lossy networks and new classes of classically simulable inputs for BS
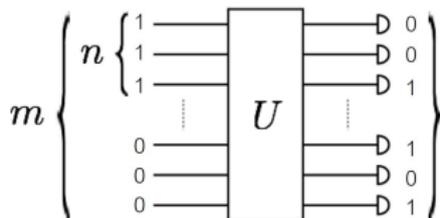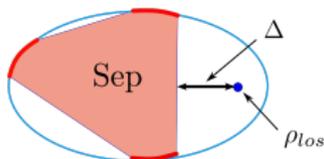
Conclusions

- Linear-optical networks **are heavily affected** by photon losses.
- **Consequence for lossy Boson Sampling devices**: classical simulation of output statistics to precision Δ in TV-distance:
  - (a) Uniform losses: average number of photons that are left $\langle l \rangle = \mathrm{o}(\sqrt{n})$, then $\Delta \approx \frac{\langle l \rangle^2}{2n}$.
  - (b) Lossy optical networks: $\Delta \approx \frac{\eta^{2s} n}{2}$, if less then $O(\log(n))$ input modes have depth smaller that s.
- **New technical results**: extraction of nonuniform losses from lossy networks and new classes of classically simulable inputs for BS

- Using Total-variation distance instead of trace distance.
- Boson Sampling with shallow optical circuits
- *Easy* error-mitigation schemes?
- Similar techniques to other quantum advantage proposals?
- Relation to Quantum de-Finetti theorems?
- Relation between *weight structure* and *hardness of sampling* for irreps of compact Lie groups?

Thank you for your attention!

Check out[9] NJP paper and a fresh follow-up `arXiv:1906.XXXX`

---

[9] Also: `arXiv:1712.10037` for independent work by R. Garcia-Parton *et al.*

Soon: Postdoc and PhD positions @ Quantum Computing Group in Center for Theoretical Physics (founded by Team-Net project)

$$\rho_{l,n} = \frac{1}{\binom{n}{l}} \sum_{\sum_i x_i = l, \ 0 \le x_i \le 1} |x_1, \ldots, x_n\rangle\langle x_1, \ldots, x_n|$$

RESULT (Closest seperable state to a lossy Fock state)

Trace distance of $\rho_{l,n}$ to the set of symmetric separable l-particle states is

$$\Delta_l = 1 - \frac{n!}{n^l(n-l)!} \ .$$

Moreover, an optimal separable state σ_* attaining Δ_l can be chosen to be

$$\sigma_* = \frac{1}{(2\pi)^n} \int_0^{2\pi} d\varphi_1 \ldots \int_0^{2\pi} d\varphi_n \left(V_{\varphi_1,\ldots,\varphi_n} |\phi_0\rangle\langle\phi_0| V_{\varphi_1,\ldots,\varphi_n}^\dagger \right)^{\otimes l},$$

where $|\phi_0\rangle = (1/\sqrt{n}) \sum_{i=1}^n |i\rangle$ and $V_{\varphi_1,\ldots,\varphi_n} = \exp\left(-i\sum_{i=1}^n \varphi_i |i\rangle\langle i|\right)$.

Consequence: Lossy Boson-Sampling can be **efficiently approximated** to accuracy Δ_l in TV-distance. Moreover,

$$l = o(\sqrt{n}) \Rightarrow \Delta_l \approx \frac{l^2}{2n} \ , \quad l = \omega(\sqrt{n}) \Rightarrow \Delta_l \to 1 \ .$$

Classical simulation for $n - l$ particles lost

$$\rho_{l,n} = \frac{1}{\binom{n}{l}} \sum_{\sum_i x_i = l, \ 0 \leq x_i \leq 1} |x_1, \ldots, x_n\rangle\langle x_1, \ldots, x_n|$$

RESULT (Closest seperable state to a lossy Fock state)

Trace distance of $\rho_{l,n}$ to the set of symmetric separable l-particle states is

$$\Delta_l = 1 - \frac{n!}{n^l (n-l)!} \ .$$

Moreover, an optimal separable state σ_* attaining Δ_l can be chosen to be

$$\sigma_* = \frac{1}{(2\pi)^n} \int_0^{2\pi} d\varphi_1 \ldots \int_0^{2\pi} d\varphi_n \left(V_{\varphi_1,\ldots,\varphi_n} |\phi_0\rangle\langle\phi_0| V_{\varphi_1,\ldots,\varphi_n}^\dagger \right)^{\otimes l} \ ,$$

where $|\phi_0\rangle = (1/\sqrt{n}) \sum_{i=1}^n |i\rangle$ and $V_{\varphi_1,\ldots,\varphi_n} = \exp\left(-i \sum_{i=1}^n \varphi_i |i\rangle\langle i|\right)$.

Consequence: Lossy Boson-Sampling can be **efficiently approximated** to accuracy Δ_l in TV-distance. Moreover,

$$l = o(\sqrt{n}) \Rightarrow \Delta_l \approx \frac{l^2}{2n} \ , \quad l = \omega(\sqrt{n}) \Rightarrow \Delta_l \to 1 \ .$$

Classical simulation for $n - l$ particles lost

$$\rho_{l,n} = \frac{1}{\binom{n}{l}} \sum_{\sum_i x_i = l, \ 0 \leq x_i \leq 1} |x_1, \ldots, x_n\rangle\langle x_1, \ldots, x_n|$$

RESULT (Closest seperable state to a lossy Fock state)

Trace distance of $\rho_{l,n}$ to the set of symmetric separable l-particle states is

$$\Delta_l = 1 - \frac{n!}{n^l(n-l)!} \ .$$

Moreover, an optimal separable state σ_* attaining Δ_l can be chosen to be

$$\sigma_* = \frac{1}{(2\pi)^n} \int_0^{2\pi} d\varphi_1 \ldots \int_0^{2\pi} d\varphi_n \left(V_{\varphi_1, \ldots, \varphi_n} |\phi_0\rangle\langle\phi_0| V_{\varphi_1, \ldots, \varphi_n}^\dagger \right)^{\otimes l} \ ,$$

where $|\phi_0\rangle = (1/\sqrt{n}) \sum_{i=1}^n |i\rangle$ and $V_{\varphi_1, \ldots, \varphi_n} = \exp\left(-i \sum_{i=1}^n \varphi_i |i\rangle\langle i|\right)$.

Consequence: Lossy Boson-Sampling can be **efficiently approximated** to accuracy Δ_l in TV-distance. Moreover,

$$l = o(\sqrt{n}) \Rightarrow \Delta_l \approx \frac{l^2}{2n} \ , \ \ l = \omega(\sqrt{n}) \Rightarrow \Delta_l \to 1 \ .$$