# Relativistic causality and no-signaling

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# Mainly referred to:


- P. H., R. Ramanathan Nat. Comm. **10**, 1701 (2019)
- R. Salazar, M. Kamon, D. Goyeneche, K. Horodecki, D. Saha, R. Ramanathan, P. H., arxiv:1712.01030
- M. Eckstein, P. H. , R. Horodecki and T. Miller, ,,Operational causality in space time" arXiv:1902.05002

# Plan

PART I

1. Bell inequalities.
2. No-signaling boxes – beyond quantum mechanics
3. Digression: v-causal models

4 Can we go beyond no-signaling ? Relativistic causality

5.Relativistically causal boxes.

6. Surprising properties.

PART II

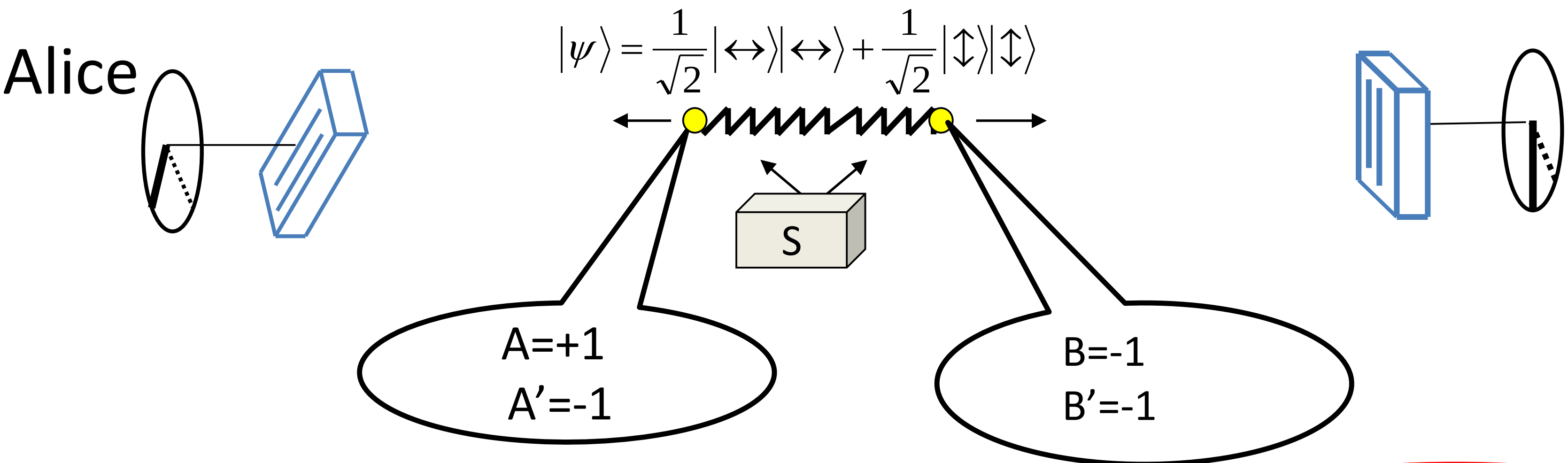
7. Causality of propagating potential statistics – concepts
8. Causality of propagating potential statistics – strong restriction for propagation.

# Quantum Physics and Quantum Entanglement

# The breaktrough discovery of John Bell (1969)

John Bell
(1928-1990)

Assumption the both photons should simultanously carry its preexisting properties A, A' (B,B') equal either ,,I will pass = +1" or ,, I will not pass = - 1" (with respect to each of the two settings of the polariser).




$$A, A', B, B' = \pm 1 \quad \Rightarrow \quad A(B + B') + A'(B - B') = AB + AB' + A'B - A'B' \leq 2$$

The Bell inequality in Clauser-Horne-Shimony-Holt (CHSH) variant

[J. F. Clauser, M. A. Horne, A. Shimony, A., R. A. Holt, PRL (1969)]

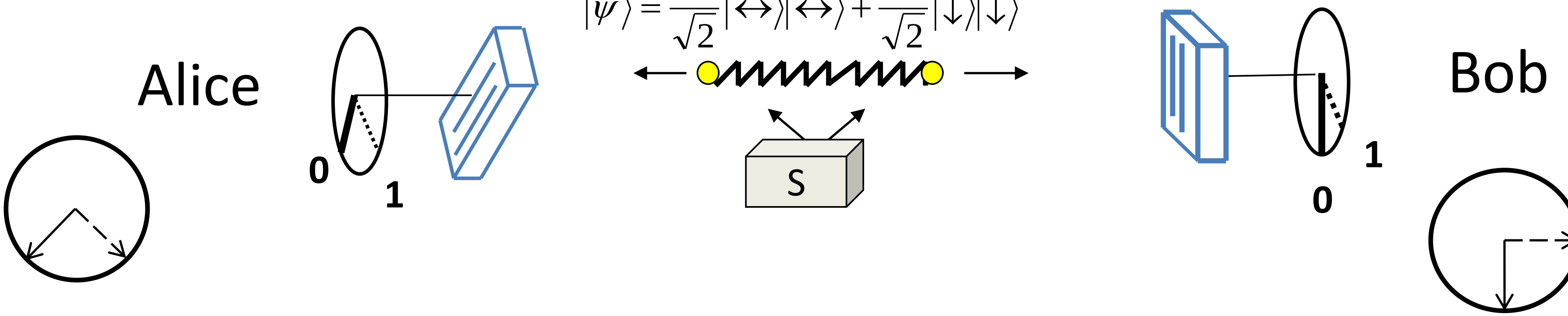
# The breakthrough discovery of John Bell (1969)

John Bell
(1928-1990)

- EPR idea in terms of local hidden variable model (LHV) is refuted on quantum mechanical ground since the Bell inequality is violated

**Settings choosen independently and randomly**

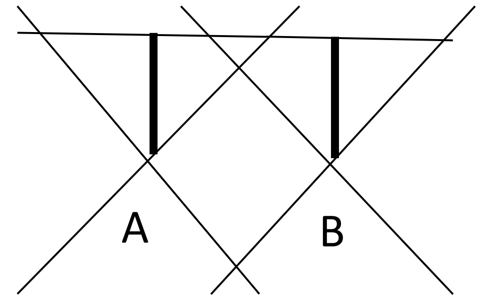
$$|\psi\rangle = \frac{1}{\sqrt{2}}|\leftrightarrow\rangle|\leftrightarrow\rangle + \frac{1}{\sqrt{2}}|\updownarrow\rangle|\updownarrow\rangle$$




$$\mathcal{B}_{\mathbf{Q}} = \langle A_0 B_0 + A_0 B_1 + A_1 B_0 - A_1 B_1 \rangle_{|\psi\rangle} = \color{red}{2\sqrt{2} > } 2 = \mathcal{B}_{\mathbf{LHV}}$$

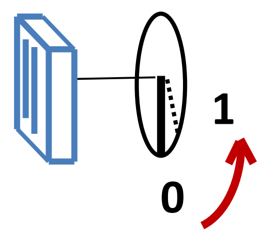
# Bell experiment conditions

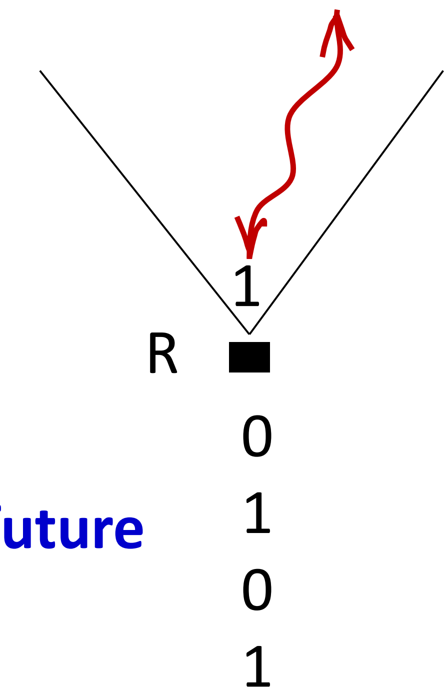
I. Space-like separation
during the whole experiment




II. High enough efficiency detectors
(Bell inequality specific - 83% for CHSH).

III. ,,Free will" assumption
– **local sources of random bits.**




Each of them **correlated only with its future**

*(remember – correlations are reflexive:*
*A is correlated with B ⇔ B is correlated with A !)*

# Bell inequalities - experiments

I. **Space-like separation** √ ***(2013)***
during the whole experiment

II. **High enough efficiency detectors**
(Bell inequality specific).

√ ***(2015)***

III. ,,Free will" assumption
– **local sources of random bits.**

# Bell inequalities - experiments

*NATURE* | LETTER

日本語要約

Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres

**B. Hensen, H. Bernien, A. E. Dréau, A. Reiserer, N. Kalb, M. S. Blok, J. Ruitenberg, R. F. L. Vermeulen, R. N. Schouten, C. Abellán, W. Amaya, V. Pruneri, M. W. Mitchell, M. Markham, D. J. Twitchen, D. Elkouss, S. Wehner, T. H. Taminiau & R. Hanson**

*Nature* **526**, 682–686 (29 October 2015) doi:10.1038/nature15759
Received 19 August 2015 Accepted 28 September 2015 Published online 21 October 2015

More than 50 years ago[1], John Bell proved that no theory of nature that obeys locality and realism[2] can reproduce all the predictions of quantum theory: in any local-realist theory, the correlations between

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***√ (2015)***

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Selected for a Viewpoint in *Physics*

PRL **115,** 250401 (2015) PHYSICAL REVIEW LETTERS week ending 18 DECEMBER 2015

**Significant-Loophole-Free Test of Bell's Theorem with Entangled Photons**

Marissa Giustina,[1,2,*] Marijn A. M. Versteegh,[1,2] Sören Wengerowsky,[1,2] Johannes Handsteiner,[1,2] Armin Hochrainer,[1,2] Kevin Phelan,[1] Fabian Steinlechner,[1] Johannes Kofler,[3] Jan-Åke Larsson,[4] Carlos Abellán,[5] Waldimar Amaya,[5] Valerio Pruneri,[5,6] Morgan W. Mitchell,[5,6] Jörn Beyer,[7] Thomas Gerrits,[8] Adriana E. Lita,[8] Lynden K. Shalm,[8] Sae Woo Nam,[8] Thomas Scheidl,[1,2] Rupert Ursin,[1] Bernhard Wittmann,[1,2] and Anton Zeilinger[1,2,†]

[1]*Institute for Quantum Optics and Quantum Information (IQOQI), Austrian Academy of Sciences, Boltzmanngasse 3, Vienna 1090, Austria*
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[3]*Max-Planck-Institute of Quantum Optics, Hans-Kopfermann-Straße 1, 85748 Garching, Germany*
[4]*Institutionen för Systemteknik, Linköpings Universitet, 581 83 Linköping, Sweden*
[5]*ICFO – Institut de Ciencies Fotoniques, The Barcelona Institute of Science and Technology, 08860 Castelldefels, Barcelona, Spain*
[6]*ICREA – Institució Catalana de Recerca i Estudis Avançats, 08015 Barcelona, Spain*
[7]*Physikalisch-Technische Bundesanstalt, Abbestraße 1, 10587 Berlin, Germany*
[8]*National Institute of Standards and Technology (NIST), 325 Broadway, Boulder, Colorado 80305, USA*

(Received 10 November 2015; published 16 December 2015)

Local realism is the worldview in which physical properties of objects exist independently of measurement and where physical influences cannot travel faster than the speed of light. Bell's theorem

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Selected for a Viewpoint in *Physics*
PRL **115,** 250402 (2015) PHYSICAL REVIEW LETTERS week ending 18 DECEMBER 2015


**Strong Loophole-Free Test of Local Realism***


Lynden K. Shalm,[1,†] Evan Meyer-Scott,[2] Bradley G. Christensen,[3] Peter Bierhorst,[1] Michael A. Wayne,[3,4] Martin J. Stevens,[1] Thomas Gerrits,[1] Scott Glancy,[1] Deny R. Hamel,[5] Michael S. Allman,[1] Kevin J. Coakley,[1] Shellee D. Dyer,[1] Carson Hodge,[1] Adriana E. Lita,[1] Varun B. Verma,[1] Camilla Lambrocco,[1] Edward Tortorici,[1] Alan L. Migdall,[4,6] Yanbao Zhang,[2] Daniel R. Kumor,[3] William H. Farr,[7] Francesco Marsili,[7] Matthew D. Shaw,[7] Jeffrey A. Stern,[7] Carlos Abellán,[8] Waldimar Amaya,[8] Valerio Pruneri,[8,9] Thomas Jennewein,[2,10] Morgan W. Mitchell,[8,9] Paul G. Kwiat,[3] Joshua C. Bienfang,[4,6] Richard P. Mirin,[1] Emanuel Knill,[1] and Sae Woo Nam[1,‡]

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We present a loophole-free violation of local realism using entangled photon pairs. We ensure that all relevant events in our Bell test are spacelike separated by placing the parties far enough apart and by using


III. ,,Free will" assumption
– **local sources of random bits.**

# Bell Bell inequalities - experiments

NATURE | LETTER

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Physics

VIEWPOINT

## Closing the Door on Einstein and Bohr's Quantum Debate

By closing two loopholes at once, three experimental tests of Bell's inequalities remove the last doubts that we should renounce local realism. They also open the door to new quantum information technologies.

by Alain Aspect*

In 1935, Albert Einstein, Boris Podolsky, and Nathan

the Heisenberg uncertainty relations. At the Solvay conference of 1927, however, Bohr successfully refuted all of Ein-

III. ,,Free will" assumption

We present a loophole-free violation of local realism using entangled photon pairs. We ensure that all relevant events in our Bell test are spacelike separated by placing the parties far enough apart and by using

– **local sources of random bits.**

# Bell experiment conditions

**I. Space-like separation** √ ***(2013)***
during the whole experiment

√ ***(2015)***

**II. High enough efficency detectors**
(Bell inequality specific).

III. **,,Free will" assumption**
**– local sources of random bits**

**?**

**perfectly ,,unpredictible" coin**

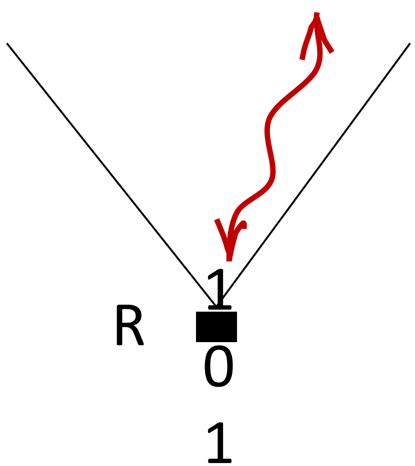
# Randomness and the freedom of an experimentalist

- Is he/she free (i) to make one experiment rather than another ?
  (ii) to make it one way rather then another ?

***Obvious remark:*** *free will is not equivalent to randomness at all*

*Technically speaking*

***,,ontic" randomness = fundamental nonpredictability***
***is needed to perform the Bell test correctly ...***



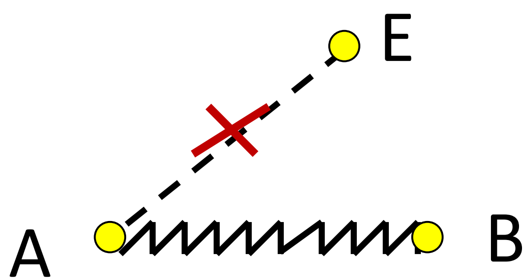

**perfectly ,,unpredictible" coin**

# Quantum  information applications

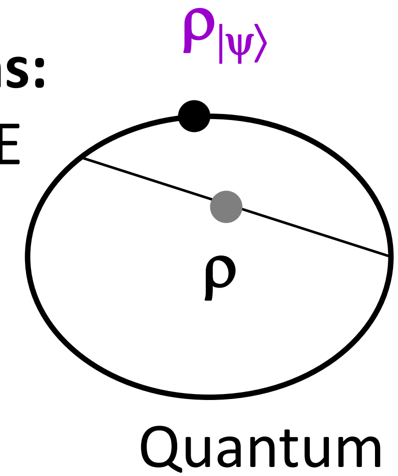
# Quantum Bell value and quantum security

**Purity of** $|\psi\rangle$ gives **monogamy of quantum correlations:**

A **max. entangled with B** $\Leftrightarrow$ **not correlated** with any E




$$|\psi\rangle = \frac{1}{\sqrt{2}}|\leftrightarrow\rangle|\leftrightarrow\rangle + \frac{1}{\sqrt{2}}|\updownarrow\rangle|\updownarrow\rangle$$



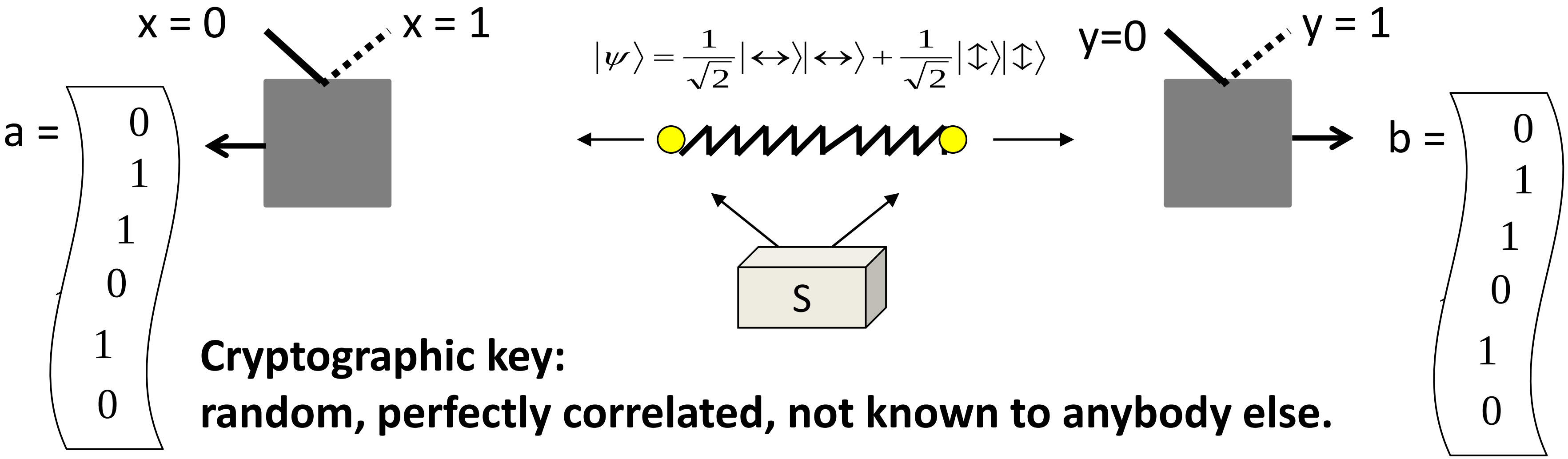

**Violation of Bell inequality:**

- **Strong monogamy witness:**

$$(\mathcal{B}_{AB})^2 + (\mathcal{B}_{AE})^2 \leq (2\sqrt{2})^2$$ [Toner, Verstraete (2006)]

- **Strong certificate of maximal entanglement (physical specification of the experiment devices *not* needed)**

# Device independent quantum security




**Cryptographic key:**
**random, perfectly correlated, not known to anybody else.**

Bell inequality violation guarantees device independent:

- Quantum cryptography [Vidick, & Vaziriani PRL (2012), inspired by Ekert PRL (1991)]
- Q. randomness expansion [Pironio et al. Nature (2009)], [Miller & Shi, J. of ACM (2016)]
- Q. randomness amplification see [ Chung, Shi & Wu, arXiv:1402.4797 , , review Acin & Massanes, Nature (2016)]).

Other applications include:

- Quntum communication complexity reduction

[Brukner, Zukowski, Pan & Zeilinger PRL (2004)], [Brassard et al. PRL (2006)], [Buhrman et al. PNAS (2016)]

# Beyond quantum ?

So far we assumed

(i) quantum mechanics + (ii) Bell inequality violation (experimental assumptions)

However the inequality violation guarantees much more: **the ,,ontic" lack of preexistence** of the properties before the measurements independently on the underlying physics (quantum or not).

$$\cancel{p_{LHV}(AB|xy) = \int p_\lambda(A|x)\, p_\lambda(B|y)\, p(\lambda)\, d\lambda}$$

Is there any chance to exploit that ?

# Bell inequality-based quantum cryptography secure against ,,post-quantum" attack

[ J. Barrett, L. Hardy, A. Kent, Phys. Rev. Lett. (2005) ]

**,,Focus: Thwarting Post-Quantum Spies"**

June 30, 2005• *Phys. Rev. Focus* 15, 2

***"Uncrackable" quantum cryptography can thwart spies even if today's quantum theory is replaced by something better–as long as it remains impossible to send messages faster than light.***

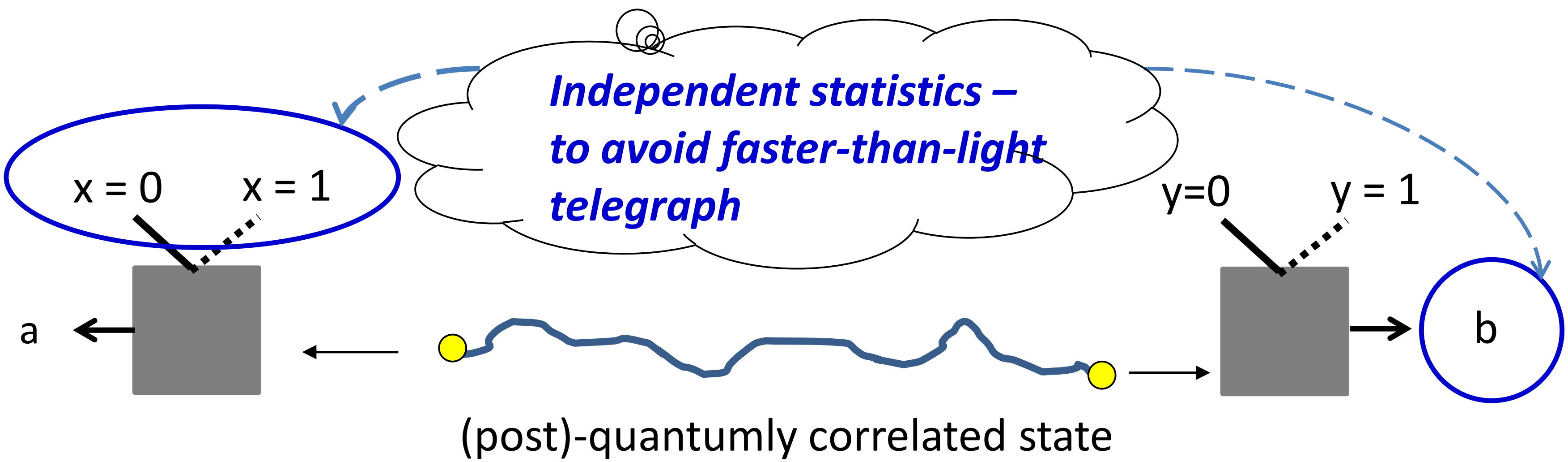
Bell inequality-based quantum cryptography secure against ,,post-quantum" attack

[ J. Barrett, L. Hardy, A. Kent, Phys. Rev. Lett. (2005) ]

**Assumptions leading to the success:**

**1) Specific Bell inequality violation (chain inequality)**

2) **No-signaling condition** for space-like separated labs:




**Quantum mechanics not assumed, only its ,,phenomenology" ie. correlations leading to violation of some Bell inequality.**

# Motivation for studying NS

**Foundations of physics perspective:**

1) Put ultimate limits for information processing in *any* future physical theory.

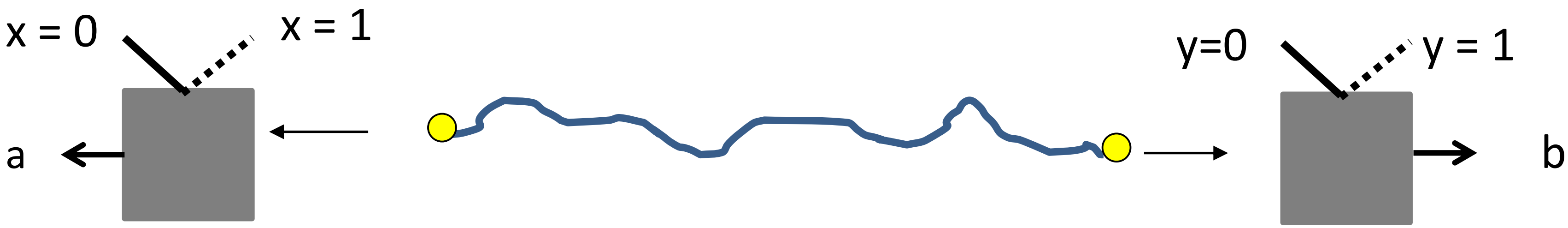
2) Look at quantum mechanics ,,from outside”. (what can be reproduced without referring to the algebraic structure)

**Practical perspective**:

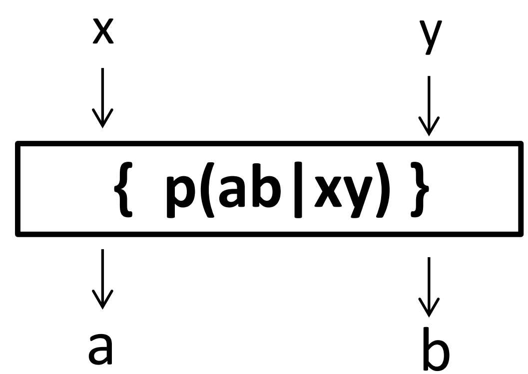
Find (possibly ) new protocols in quantum information processing (sometimes reduction of mathematical formalism may help).

# Theory of ,,no-signaling boxes”

[S. Popescu, D. Rohlich, Found. Phys. 24, 379 (1994)]




**,,Boxes”: statistics of some measurements**

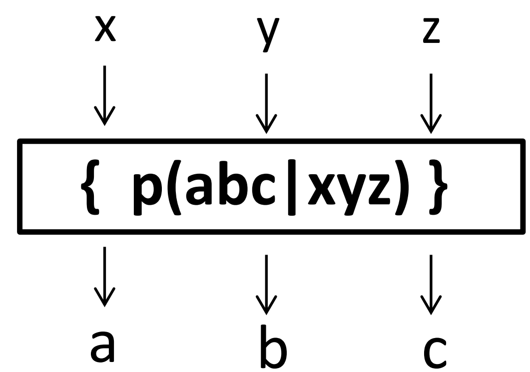



**No-signaling conditions:**

$\Sigma_a p(ab|xy) := p(b|xy) = p(b|y)$ **NS from the right to the left**

(plus the same for a $\rightarrow$b, x $\rightarrow$ y, ,,right” $\leftrightarrow$,,left” )

# No-signaling condition for more parties (natural generalisation)




Three parties:

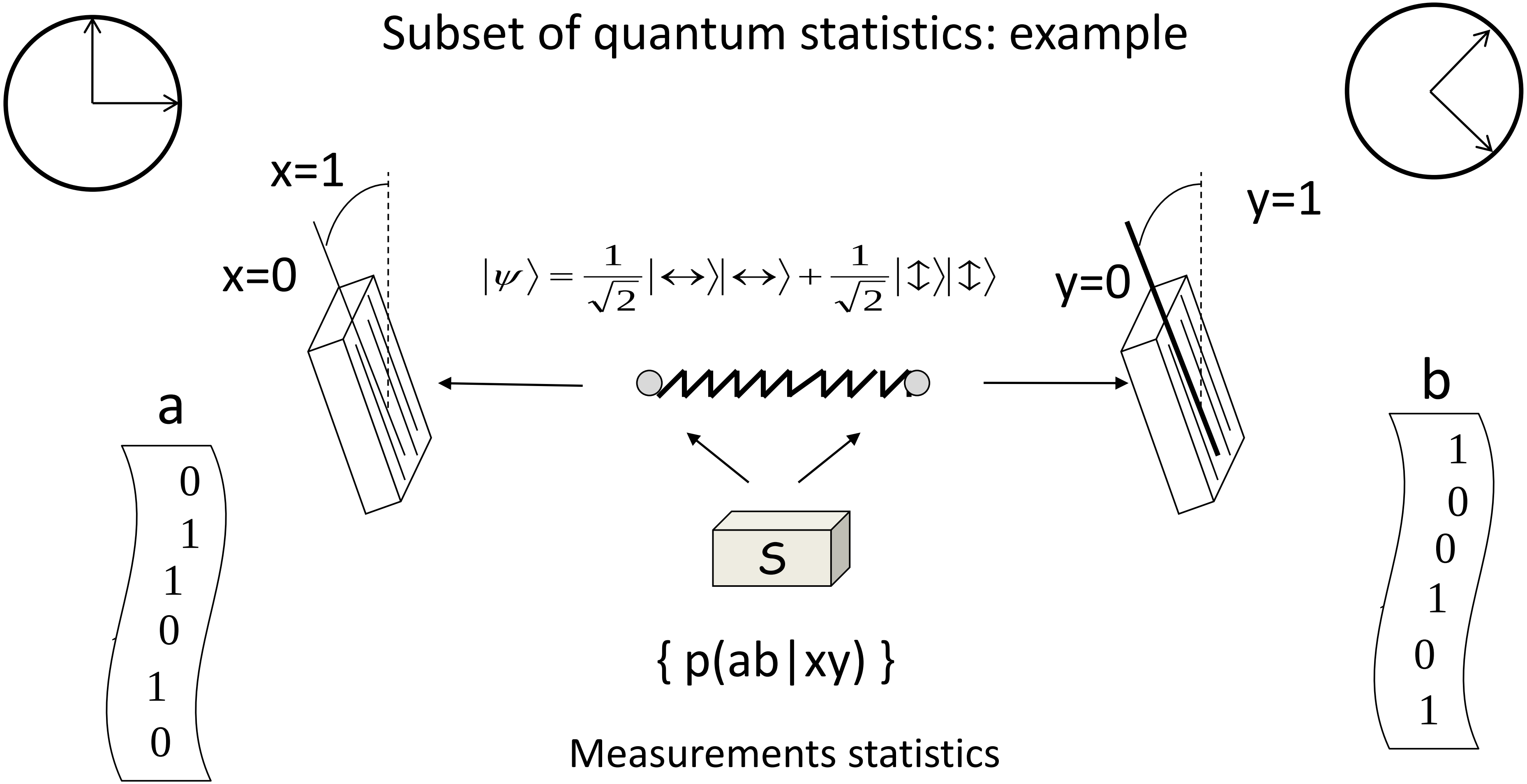
(i) $\Sigma_a p(abc|xyz) := p(bc|xyz) = p(bc|yz)$

(ii) $\Sigma_{ab} p(abc|xyz) := p(c|xyz) = p(c|z)$

+ the same for all permutations of subsystems

Note that from (i) + (ii) the left-right no-signaling of bipartite box { p(bc|yz)} follows automatically.

# Subset of quantum statistics: example




$$\langle A_x\, B_y \rangle_{|\psi\rangle} = \Sigma_{ab=\pm1}\, ab\; p(ab|xy)$$

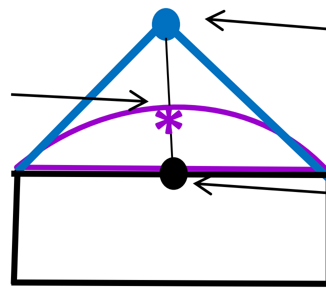
$$\mathcal{B} = \langle A_0\, B_0 \rangle_{|\psi\rangle} + \langle A_0\, B_1 \rangle_{|\psi\rangle} + \langle A_1\, B_0 \rangle_{|\psi\rangle} - \langle A_1\, B_1 \rangle_{|\psi\rangle} = \mathbf{2\sqrt{2}}$$

# Theory of „no-signaling boxes"

**Difficuilty: quantum statistics never extremal**
**- usually some purely deterministic component.**

**Bell-CHSH experiment :**

$$|\psi\rangle = \frac{1}{\sqrt{2}}|\leftrightarrow\rangle|\leftrightarrow\rangle + \frac{1}{\sqrt{2}}|\updownarrow\rangle|\updownarrow\rangle$$




| | y | 0 | | 1 | |
|---|---|---|---|---|---|
| x | a \ b | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | | | 1 |
| | 1 | | | | |
| 1 | 0 | 1 | | | 1 |
| | 1 | | | | |

**2**

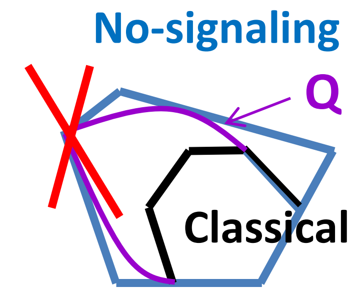
Statistics from $|\psi\rangle$

**$2\sqrt{2}$**

| | y | 0 | | 1 | |
|---|---|---|---|---|---|
| x | a \ b | 0 | 1 | 0 | 1 |
| 0 | 0 | 1/2 | | 1/2 | |
| | 1 | | 1/2 | | 1/2 |
| 1 | 0 | 1/2 | | | 1/2 |
| | 1 | | 1/2 | 1/2 | |

**4 !**

# Theory of „no-signaling boxes"

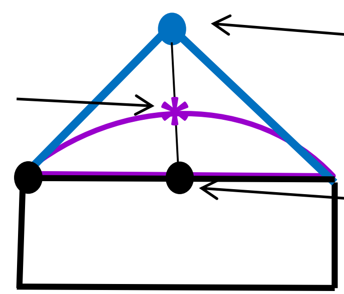



**Difficulty : quantum statistics is never extremal**
**- usually has some purely deterministc component**

Proof in [R. Ramanathan, J. Tuziemski, M. Horodecki, P. H. Phys. Rev. Lett. (2016)]

$$|\psi\rangle = \frac{1}{\sqrt{2}}|\leftrightarrow\rangle|\leftrightarrow\rangle + \frac{1}{\sqrt{2}}|\updownarrow\rangle|\updownarrow\rangle$$

**Bell- CHSH experiment:**




*Quantum statistics is not pure if seen from outside* $\Longrightarrow$ *naive purity-monogamy-based approach to cryptography does not work.*

( Berrett, Hardy & Kent used additional property of chain Bell inequality)

# NC vs QM comparison (I)
# Monogamy relations exist

There are monogamy relations for Bell correlations ([Masanes, Acin, Gisin PRL (2006)],[Toner Proc. R. Soc. A (2009)], universal [Brukner, Pawlowski PRL (2009)] no of Bob labs = no of Bob's settings)

Example: stronger monogamy for Bell function depending on XOR of ourcomes $(a \oplus b)$ :

- Take Bell inequality $\mathcal{B}^{\oplus}{}_{AB} \leq R_{LHV}(\mathbb{B}) < R_{NS}(\mathbb{B})$
- Find how many settings C you need to remove at Bob site to trivialise the inequality to $\mathcal{B}^{\oplus}{}_{AB} \leq R_{LHV}(\mathbb{B}) = R_{NS}(\mathbb{B})$
- The inequality must satisfy the monogamy relation with C+1 Bob's labs

$$\sum_{i=1}^{C+1} \mathcal{B}^{\oplus}{}_{AB^i} \leq (C+1)R_{LHV}(\mathbb{B})$$

# Broken chain and monogamy

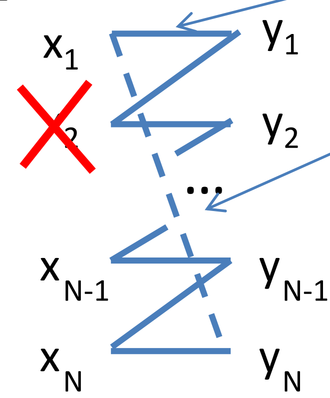
[R. Ramanathan, P.H. PRL (2014)]

correlations (solid)

anticorrrelations (dashed)

$$\mathcal{B}_{AB} \le R_{LHV}(\mathbb{B}) < R_{NS}(\mathbb{B}) = 1$$

graph




$$\mathcal{B}'_{AB} \le R_{LHV}(\mathbb{B}') = R_{NS}(\mathbb{B}') = 1 - 1/2N$$

**Taking one setting out makes inequality trivial (classical = NS)**

$$\mathcal{B}^{(\text{chain, N})}{}_{AB} + \mathcal{B}^{(\text{chain, N})}{}_{AC} \le 2\, R_{LHV}(\mathbb{B}^{(\text{chain, N})})$$

# NC vs QM comparison (II) Purification usually does not exists but complete extension does

In QM any mixed state $\rho_A$ can
(i) be extended to a pure state $\rho_{AB} = |\Psi_{AB}\rangle\langle\Psi_{AB}|$ (purification)
(ii) s. t. all its ansambles of $\rho_A$ can be represented by measurements on B (complete extension)

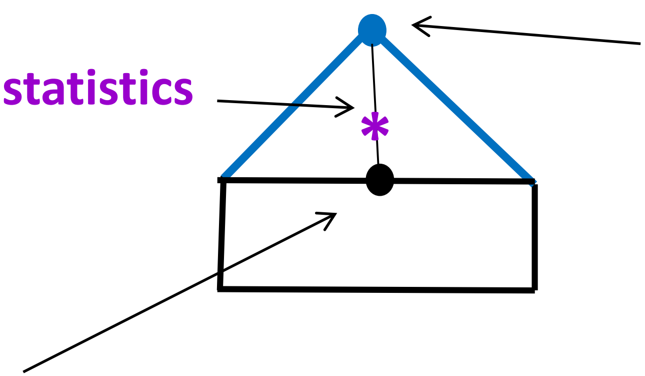
In NC all the above is true except of the purity of extension.

[M. Winczewski, T. Das, K. Horodecki, P. Horodecki, Ł. Pankowski, M. Piani, R. Ramanathan, *No purification in all discrete theories and the power of the complete extension,* arXiv:1810.02222]

# NC vs QM (III)
## Purity = complete statistical independence with ,,environment”

In NC, like in quantum mechanics, if the box is pure (has no nontrivial convex decomposition) then any extension is trivial (product with environment).

$$|\psi\rangle = \frac{1}{\sqrt{2}}|\leftrightarrow\rangle|\leftrightarrow\rangle + \frac{1}{\sqrt{2}}|\updownarrow\rangle|\updownarrow\rangle$$



# NS vs QM (IV)
## Some variants of security agains  NS eavesdropper are possible

**Example.- Partial solution to free-will problem.**
Randomness amplification against NS eavesdropper.

**- R. Renner,  R. Colbeck, Nat.  Phys. (2012),**
**- R. Gallego, L. Masanes,  De La Torre, C. Dhara, L. Aolita, A. AcínNat Comm. (2013)**
**- F. G.S.L. Brandão, R. Ramanathan, A. Grudka, K. Horodecki,**
**M. Horodecki, P. H. , T. Szarek, H. Wojewódka, Nat. Comm. (2016)**
**- F. G.S.L. Brandão, K. Horodecki, M. Horodecki,**
**P. H. , H. Wojewódka, Phys. Rev. Lett. (2016)**
**- H. Wojewodka, F. G. S. L. Brandao, A. Grudka, M. Horodecki, K. Horodecki, P. Horodecki,**
**M.  Pawlowski, R. Ramanathan, M. Stankiewicz, IEEE TIT (2017)**
**-  R. Ramanathan, M. Horodecki, S. Pironio, K. Horodecki, P. H.,**
***Generic randomness amplification schemes using Hardy paradoxes***
**arXiv:1810.11648**

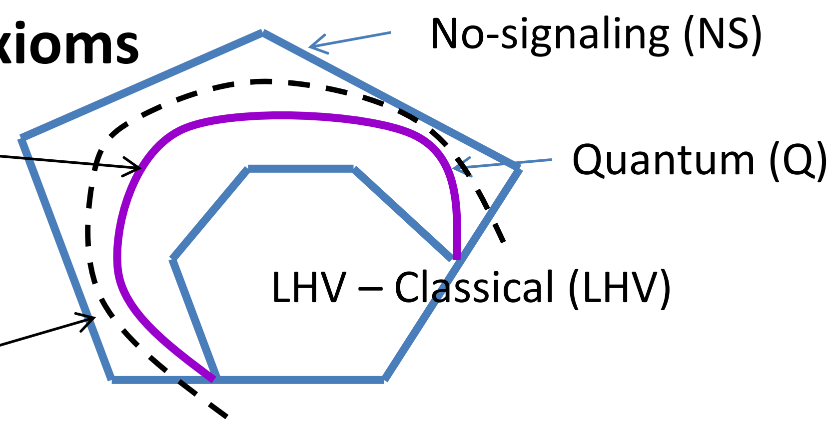
III. **,,Free will” assumption**
**– local sources of random bits**

**(Need of perfectly  unpredictible coin dismissed)**

# Understand/reproduce quantum mechanics from basic principles

**Two approaches:**

## A. Full derivation of Q from LIST of axioms




L. Hardy, quant-ph/0101012 (2001),
G. Chiribella, G. M. D'Ariano, P. Perinotti,
PRA (2010, 2011), arXiv:1506.00398 (2015)

## B. The best outer approximation of Q by a SINGLE information-type (physically motivated) principle:

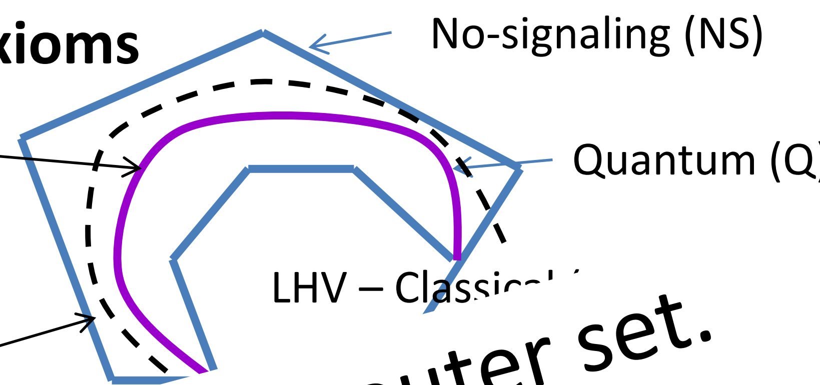
- No-signaling - Rohlich & Popescu PRA 94)
- No trivial communication complexity - Brassard et al. PRL (2006)
- Macroscopic locality - M. Navascues, H. Wunderlich P. R. Soc. (2009)
- Information casuality - Pawlowski et al. Nature (2009).
- Local orthogonality – T. Fritz et al. Nat. Comm. (2013)
- Almost quantum - M. Navascues et al., Nat. Comm. (2014)

*Remark. Second (B) more focused on future physical theories, but the first (A) – harder - also may work in that way (as contains some qualitative principles itself).*

# Understand/reproduce quantum mechanics from basic principles

**Two approaches:**

## A. Full derivation of Q from LIST of axioms

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- In ...ality - Pawlowski et al. Nature (2009).
- Lo ...thogonality – T. Fritz et al. Nat. Comm. (2013)
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NC always a limiting paradigm, an outer set.
Does it need to be so ?

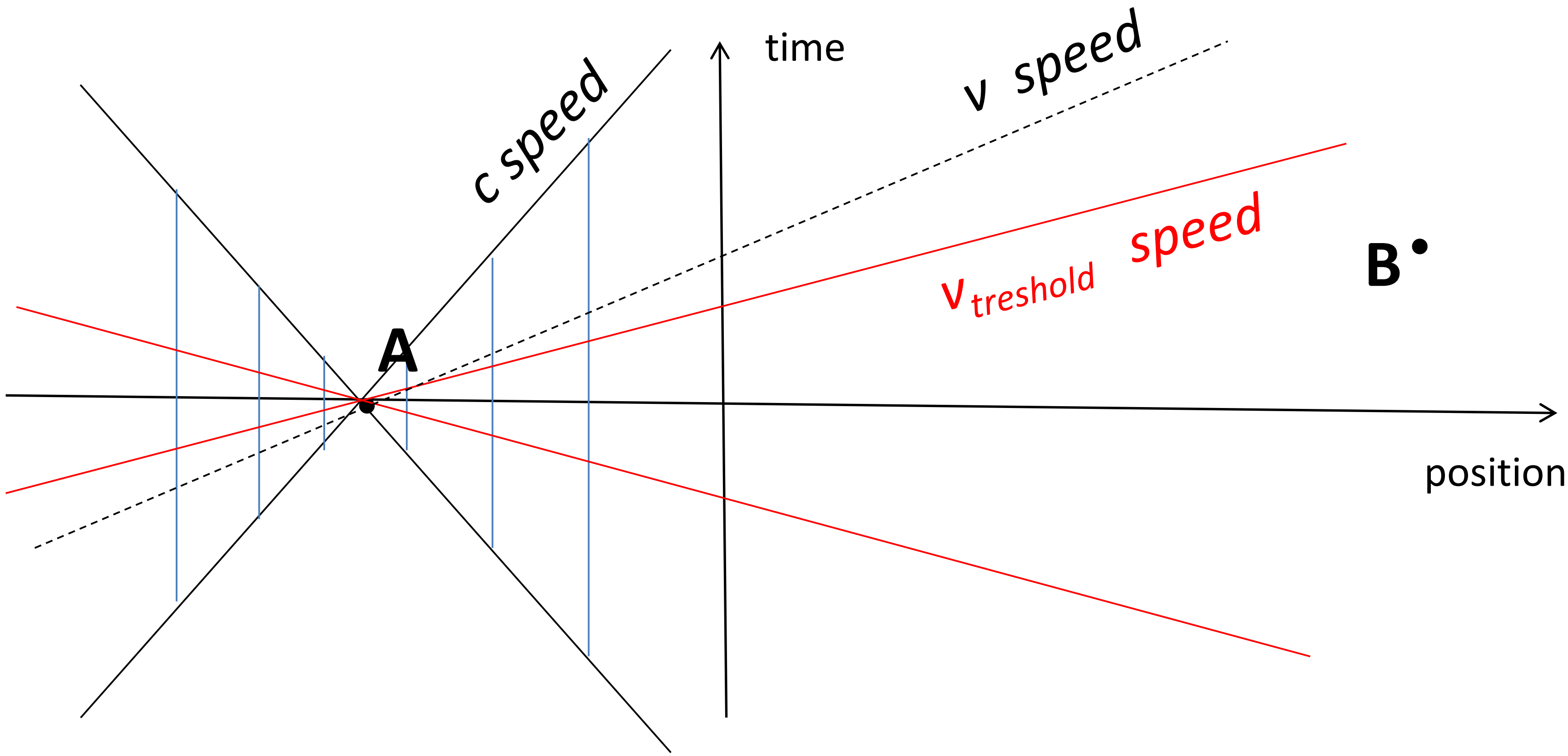
*Remark. Second (B) more focused on future physical theories, but the first (A) – harder - also may work in that way (as contains some qualitative principles itself).*

Digression.

Testing hidden faster than light influences.

Is it possible that Bell inequality violation
is due to hidden v > c influence ?

**In bipartite case** to exclude this for $c < v < v_{treshold}$ requires enough synchronisation (or putting the labs far apart enough)




**Excluding higher v influence requires more and more effort …**

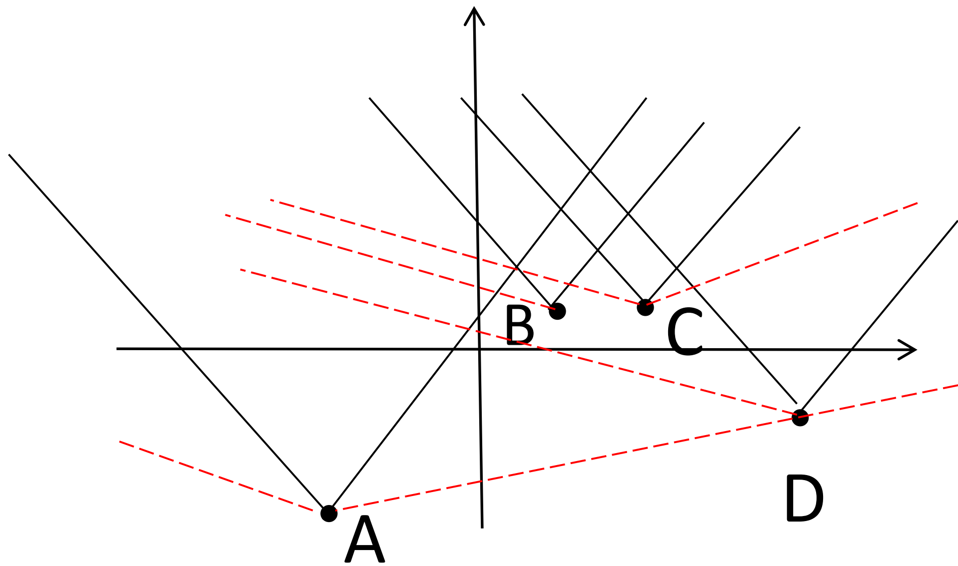
**Can quantum statistics be explained locally via some speed v > $c_{light}$ ?**

(i) **Rohlich-Popescu NS property** ie.

$\Sigma_a$ p(abcd|xyzw)=p(bcd|yzw) etc. …

(ii) **BC correlations locally explained by some signals v > $c_{light}$** coming from A and D

$$p(bc|yz) = \int p(b|y, \lambda)\ \ p(c|z, \lambda)\ \ p(\lambda|AD)\, d\lambda$$




[J. D. Blancal, S. Pironio, A. Acin, Y.-C. Liang, V. Scarani, N. Gisin, Nat. Phys. (2012)]

## Can quantum statistics be explained locally via some speed $v > c_{light}$ ?

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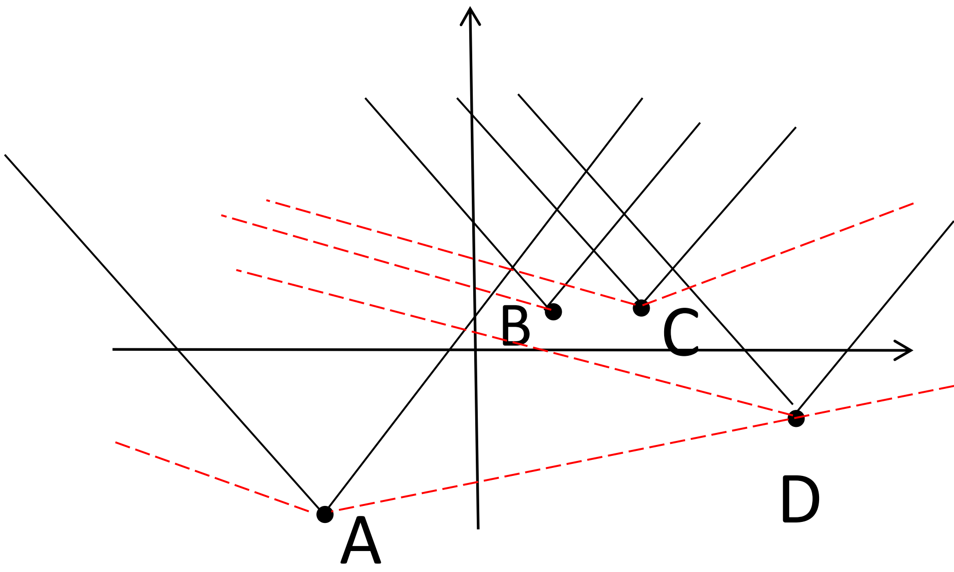
$$p(bc|yz) = \int p(b|y,\lambda)\ p(c|z,\lambda)\ p(\lambda|AD)\, d\lambda$$

**Result.** (Bell-like inequality)

(i) and (ii) $\Rightarrow$ $\mathcal{B} \leq \mathbf{7}$

(made of correlations ACD, ABD)

However quantum mechanics gives $\mathcal{B}_Q \approx \mathbf{7.2}$ **!**




[J. D. Blancal, S. Pironio, A. Acin, Y.-C. Liang, V. Scarani, N. Gisin, Nat. Phys. (2012)]

# Conclusion: refutation of v-causal models

[J. D. Blancal, S. Pironio, A. Acin, Y.-C. Liang, V. Scarani, N. Gisin, Nat. Phys. (2012)]

Any **hidden faster than light** v-influence
would imply **„explicit" signaling faster** than light !

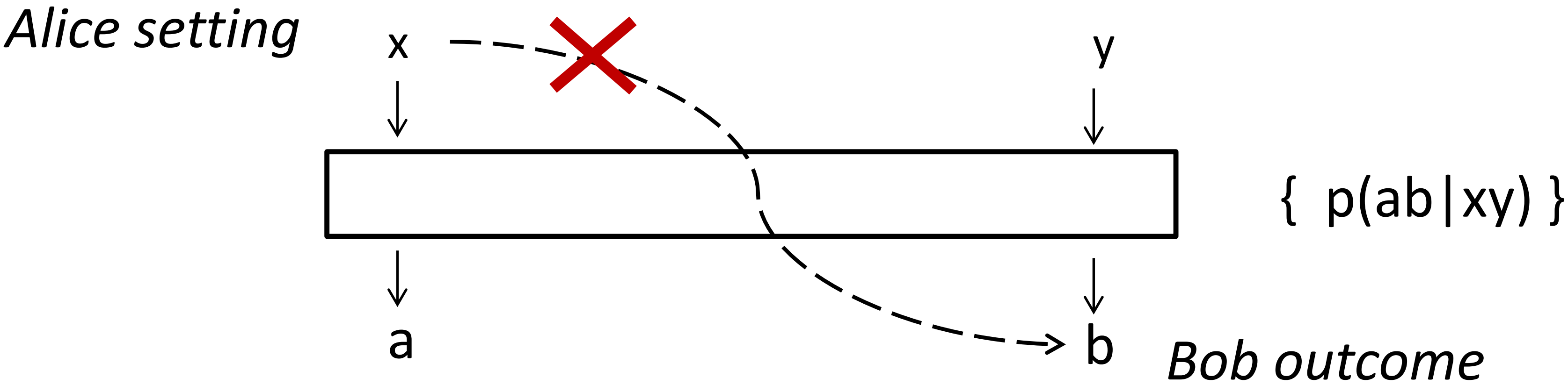
**But we do not observe that $\Rightarrow$ hidden v-influence is ruled out.**

$$
\begin{aligned}
\mathcal{I} \;=\; & -3\langle A_0\rangle - \langle B_0\rangle - \langle B_1\rangle - \langle C_0\rangle - 3\langle D_0\rangle - \langle A_1B_0\rangle - \langle A_1B_1\rangle + \langle A_0C_0\rangle + 2\langle A_1C_0\rangle + \langle A_0D_0\rangle \\
& + \langle B_0D_1\rangle - \langle B_1D_1\rangle - \langle C_0D_0\rangle - 2\langle C_1D_1\rangle + \langle A_0B_0D_0\rangle + \langle A_0B_0D_1\rangle + \langle A_0B_1D_0\rangle - \langle A_0B_1D_1\rangle \\
& - \langle A_1B_0D_0\rangle - \langle A_1B_1D_0\rangle + \langle A_0C_0D_0\rangle + 2\langle A_1C_0D_0\rangle - 2\langle A_0C_1D_1\rangle \le 7,
\end{aligned}
$$

**$\mathcal{B}_{Q} \approx 7.2$**

Can we still go beyond no-signaling condition ?

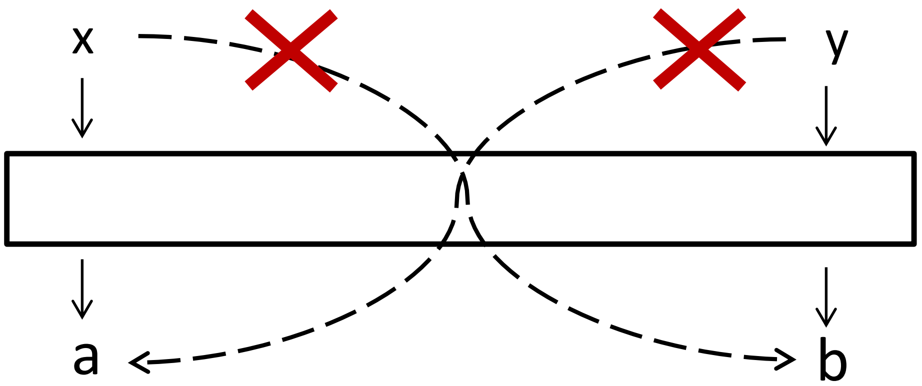
# No-signaling for two observers




$\Sigma_a p(ab|xy) := p(b|xy) = p(b|y)$ no-signaling condition

from the left to the right

# No-signaling for two observers

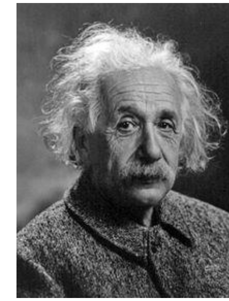


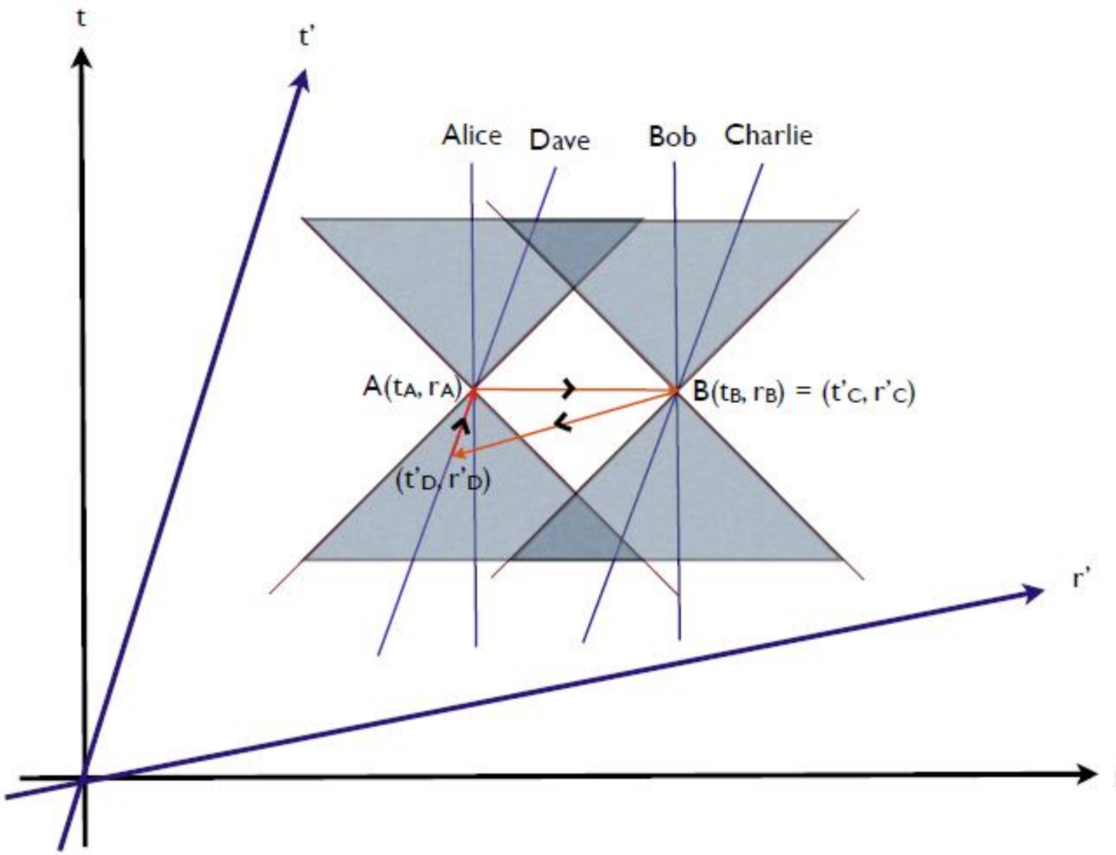

$\{ \ p(ab|xy) \ \}$

$\Sigma_a p(ab|xy) := p(b|xy) = p(b|y)$ no-signaling from the left to the right

$\Sigma_b p(ab|xy) := p(a|xy) = p(a|x)$ no-signaling from the right to the left

# Reason: to avoid causal loop



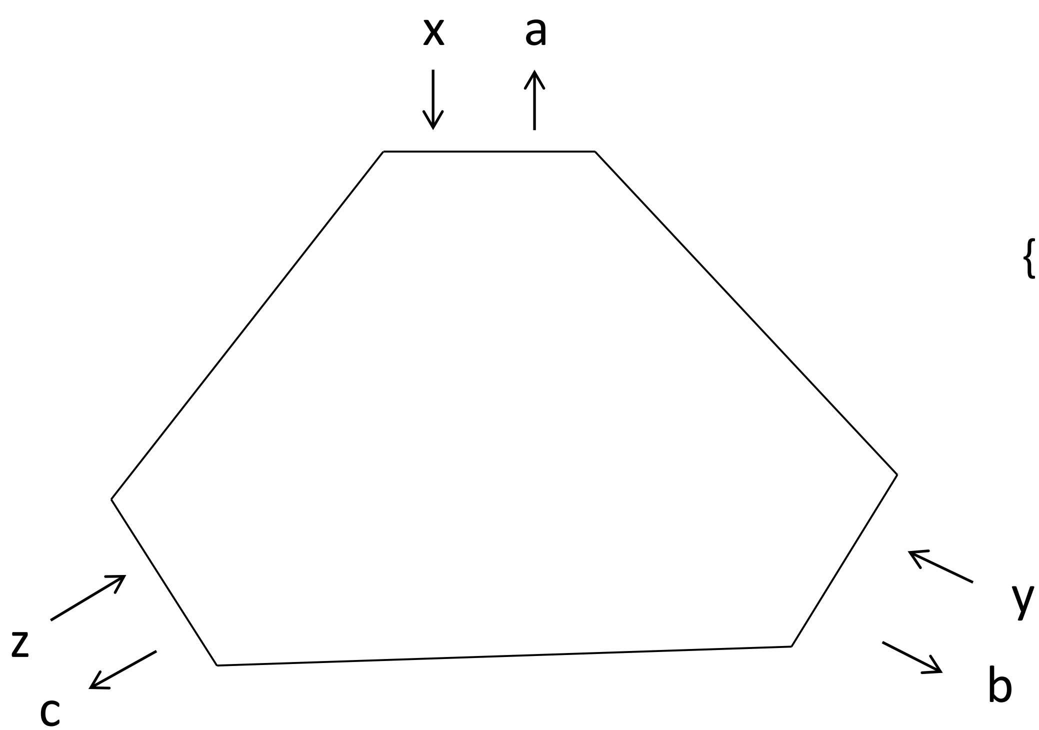



**Special relativity:**

**Superluminal signaling**
**+**
**Relativity of simultaneity**
**=**
**Causal loop**

(grandfather paradoxes etc.)

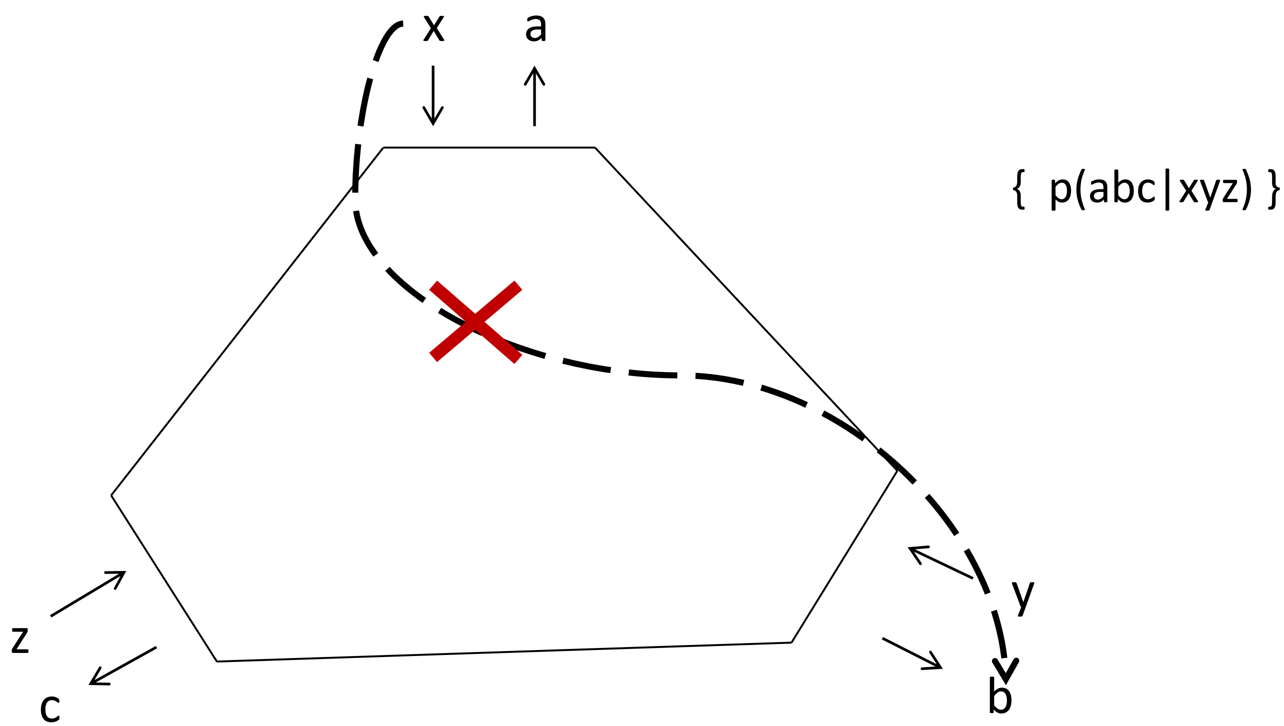
# The case of three observers




$\{ \ p(abc|xyz) \ \}$

# The case of three observers

**The standard NS assumes not only  no point-to point  communication  …**

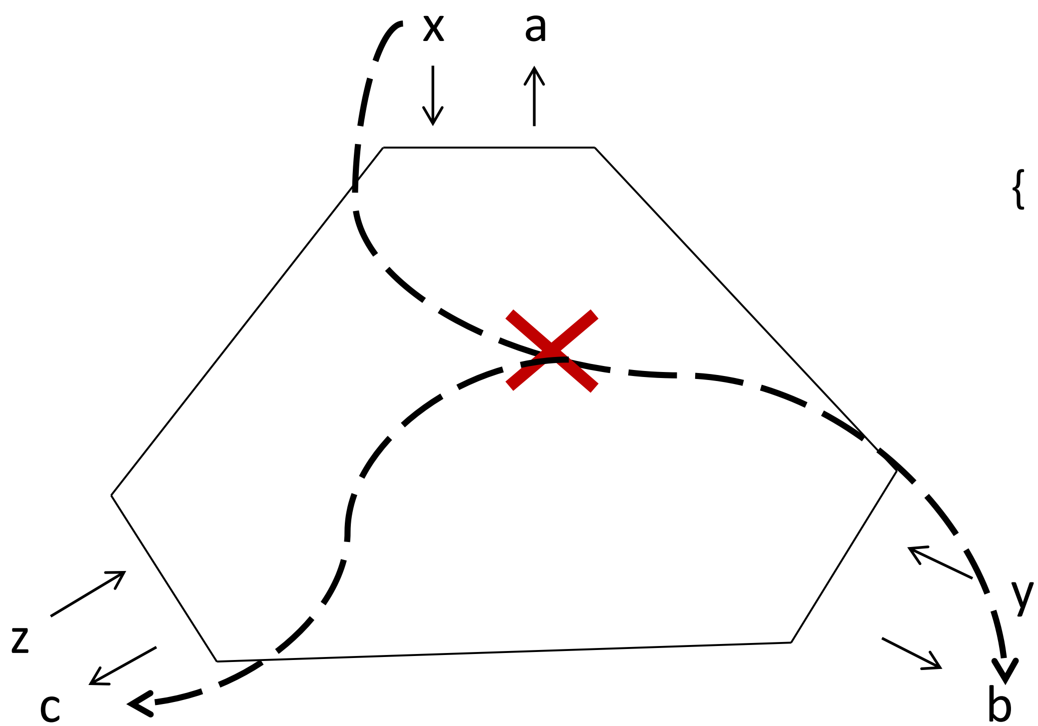



$$\Sigma_a p(ab|xy) := p(b|xy) = p(b|y)$$

…  and the other two but also also an extra one …

# No-signaling for three observers

… **no point-to correlations communication:**

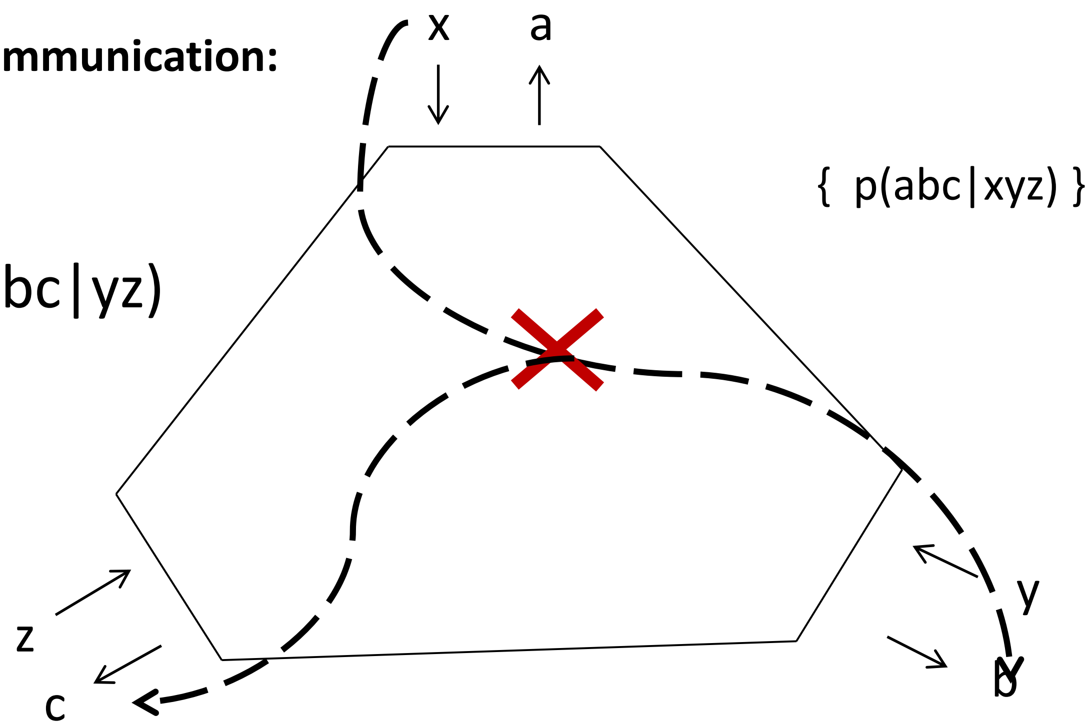



{ p(abc|xyz) }

$$\Sigma_a p(abc|xyz) := p(bc|xyx) = p(bc|yz)$$

# No-signaling for three observers

… **extra no point-to correlations communication:**




$\Sigma_a p(abc|xyz) := p(bc|xyx) = p(bc|yz)$

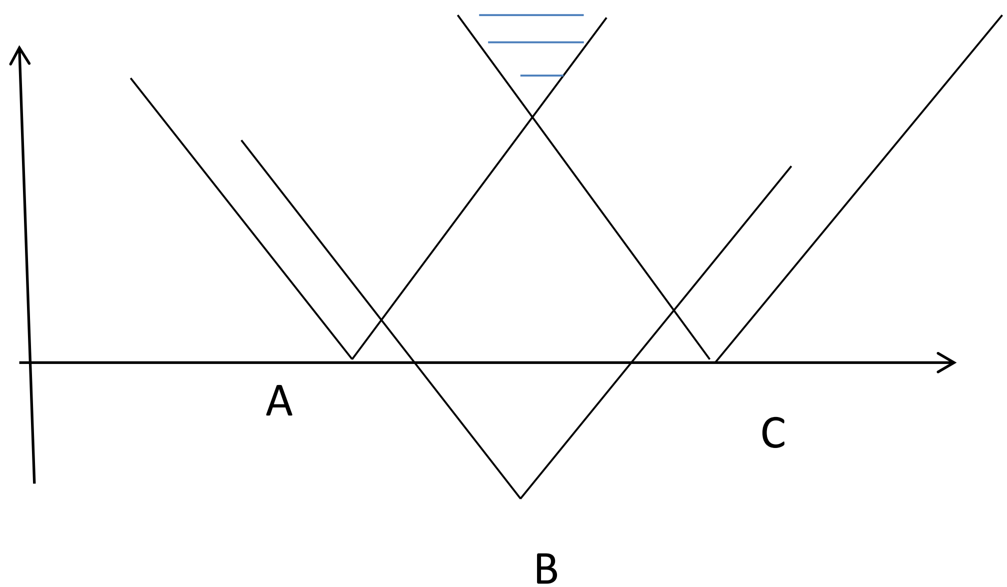
**Does the r**elativistic causality need the above when B and C are „far apart" enough ? **No**.

For infinite speed signaling this crucial observation made in [J. Grunhaus, S. Popescu, D. Rohrlich, „Jamming nonlocal quantum correlations" Phys. Rev. A 53, 3871 (1996)]

# Relativistically Causality and possibility of faster than light influences

) [ P. H. & R. Ramanathan, ,,Relativistic Causality vs. No-Signaling as the limiting paradigm for correlations in physical theories”, Nat. Comm. (2019)]

,,$\sim$” = ,, no mutual influence’’




B

No influences

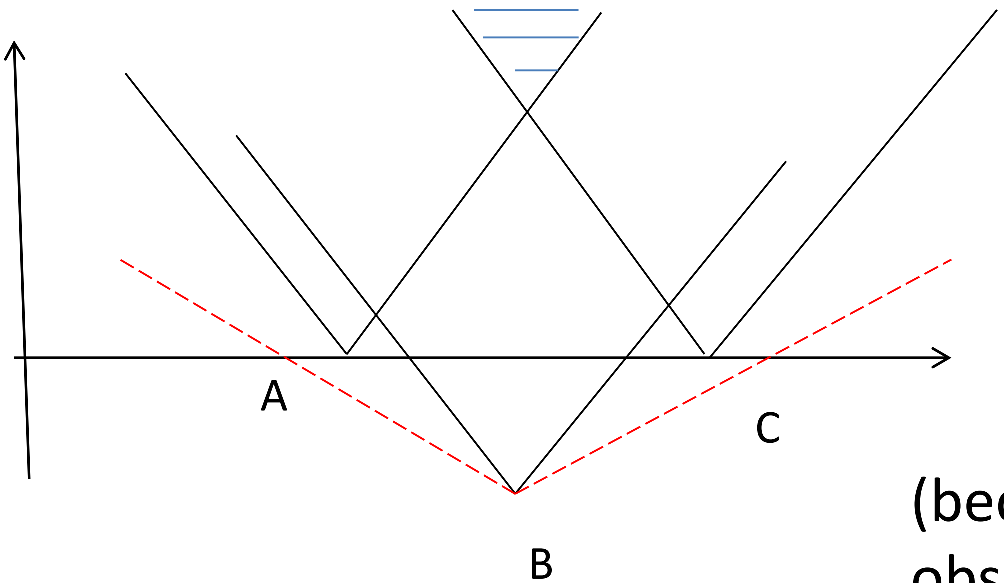
B $\sim$ A

B $\sim$ C

B $\sim$ Corr(A,C)

# Relativistically Causality and possibility of faster than light influences

[ P. H. & R. Ramanathan, „Relativistic Causality vs. No-Signaling as the limiting paradigm for correlations in physical theories", Nat. Comm. (2019)]




No influences

B $\sim$ A

B $\sim$ C

but

B $>_v$ Corr(A,C)

allowed

(because the result could be observed only in the c-future of B)

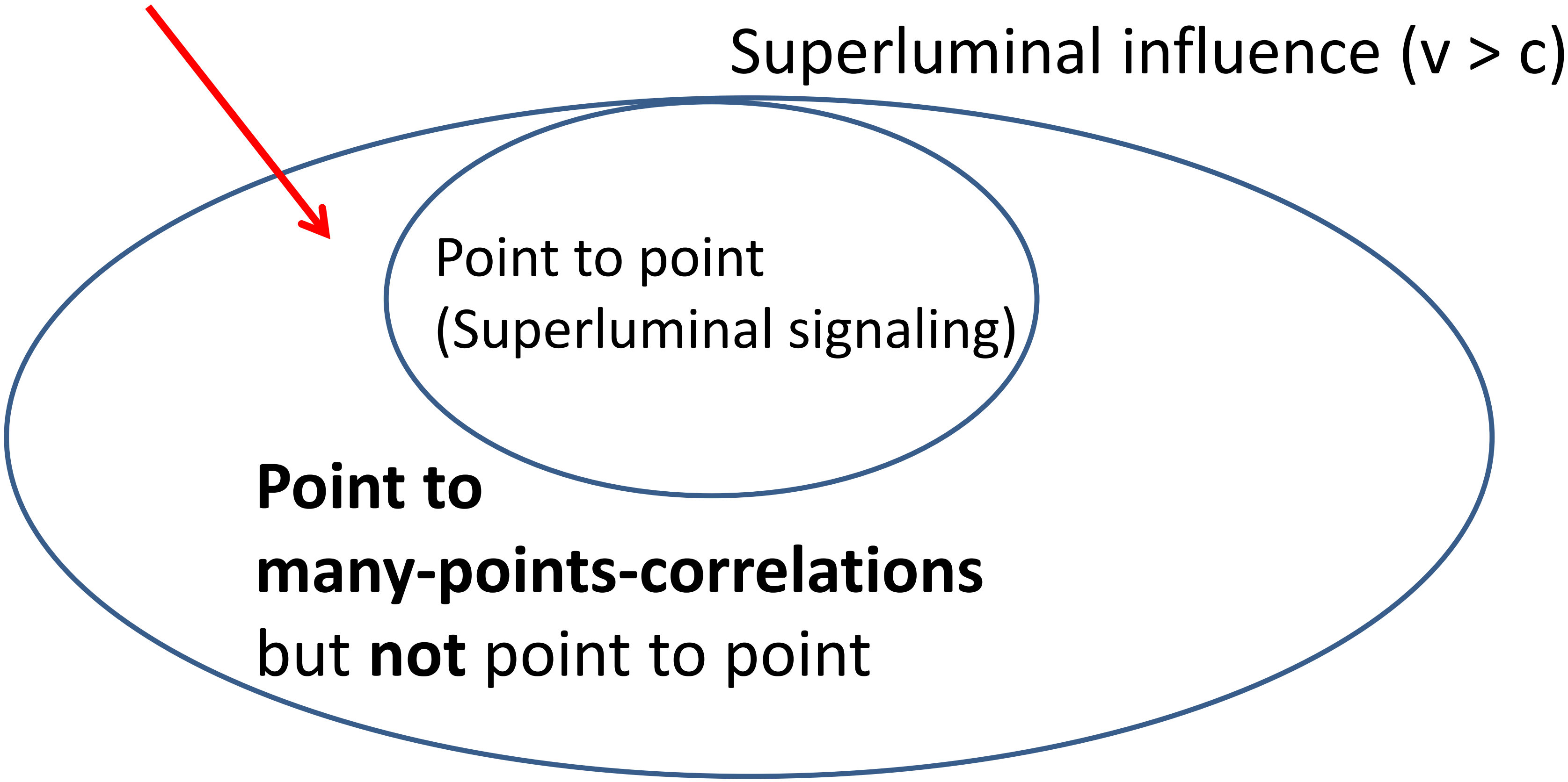
So in flat Minkowski space you may drop the condition

$$\Sigma_b p(abc|xyz) := p(ac|xyz) = p(ac|xz)$$

in ***special*** space-time configurations.

# General summary

Relativistic causality (= no causal loops)
allows for this but for special space-time configurations



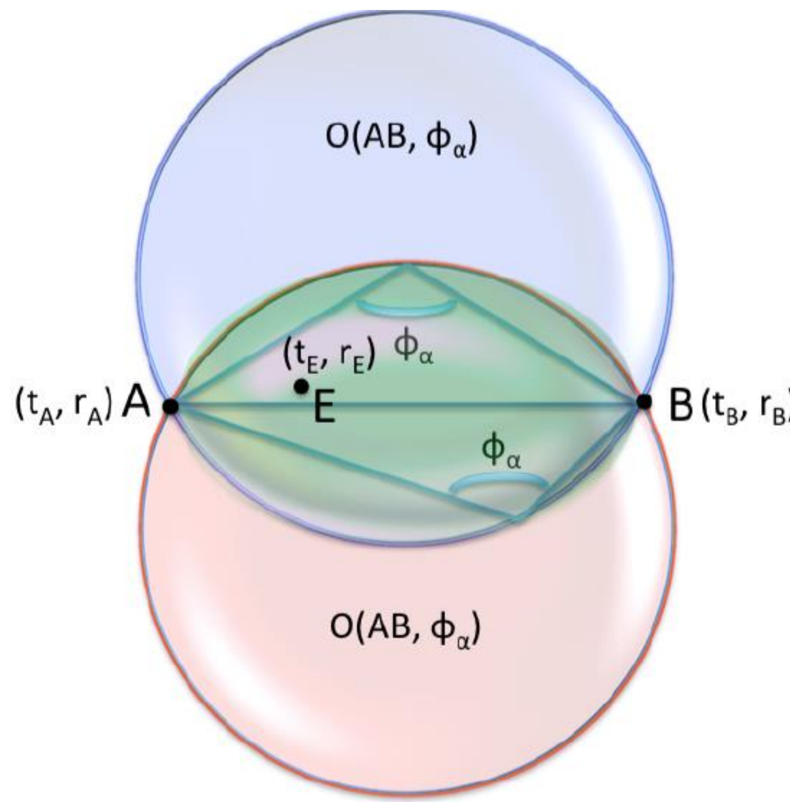

$$NC \not\subseteq RC$$

# Admissible configurations admitting E to influence Corr(A,B) with v > c

[ P. H. & R. Ramanathan, Nature Comm. (2019)]

**Space condition** for $\mathbf{r}_A$ , $\mathbf{r}_B$ , $\mathbf{r}_E$ :

Sum of the segments with
AB cord and the angle
$\phi_\alpha = \pi - 2 \arcsin( c / v)$



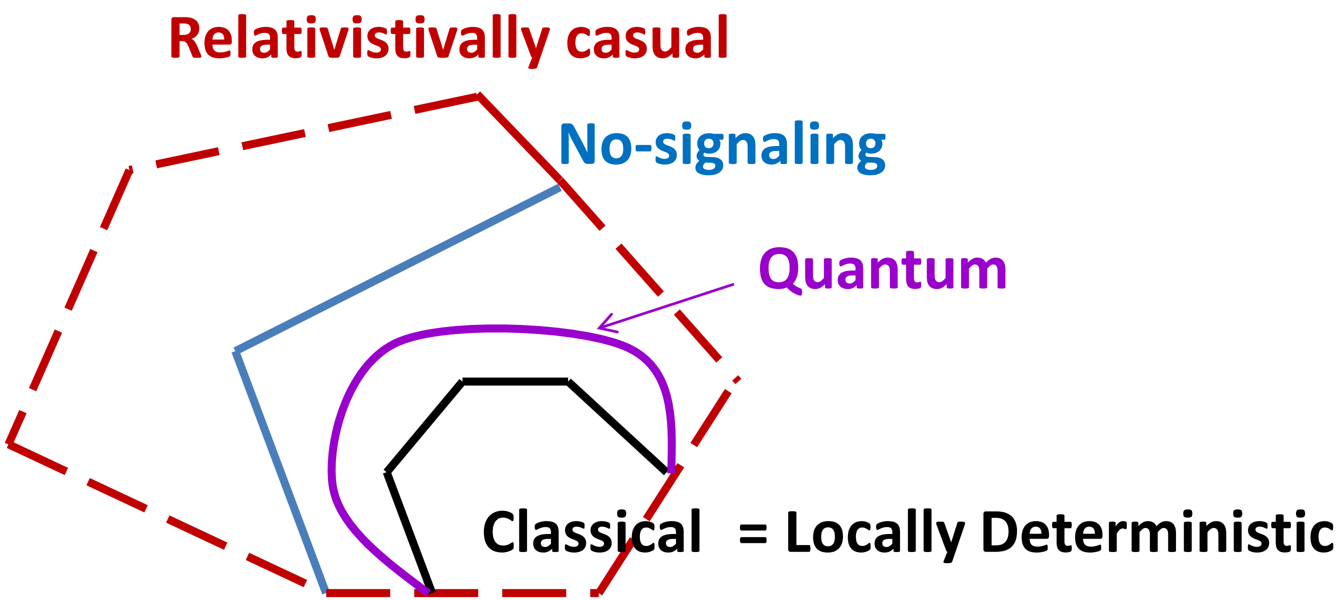

**Time condition** for $t_A$ , $t_B$ , $t_E$ :

$$t_E \leq \min [t_A - | \mathbf{r}_A - \mathbf{r}_E |/ v \ , \ t_B - | \mathbf{r}_B - \mathbf{r}_E |/ v ]$$

# For three and more parties the correlation polytope extended from NS to RC

[ P. H. & R. Ramanathan, Nature Comm. (2019)]

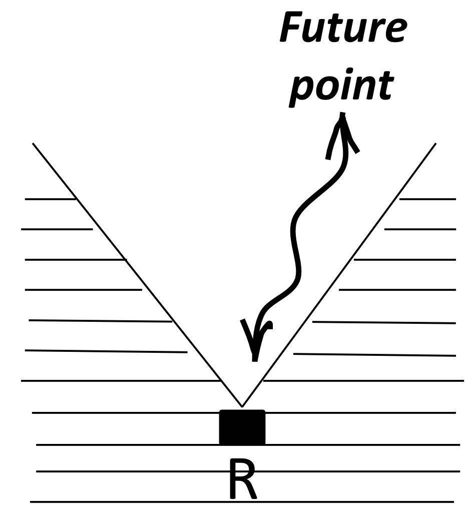
Extend p(abc|xyz) to p(abc|xyz; $t_A\mathbf{r}_A$ ; $t_B\mathbf{r}_B$ ; $t_E\mathbf{r}_E$) then the ,,polytope" extends



# Strange effects in RC beyond NS

## Change of the free will concept

[ P. H. & R. Ramanathan, Nat. Comm. (2019) ]




**Standard:**

Free random bit **only correlated with its Minkowski future**
(= uncorrelated with its complement)

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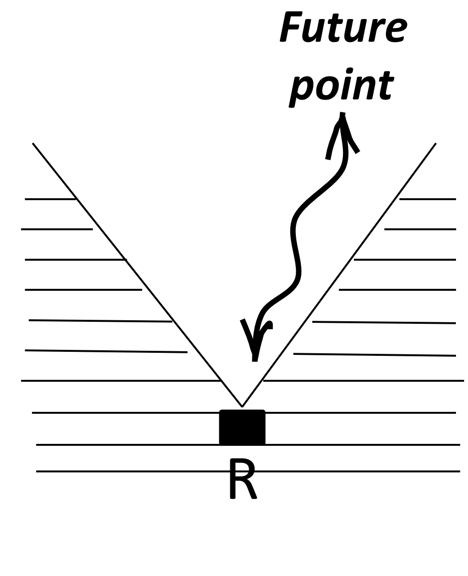
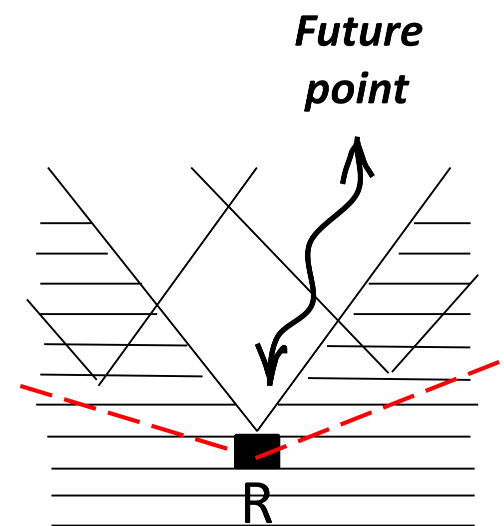



**Present (Relativistic Causality paradigm):**



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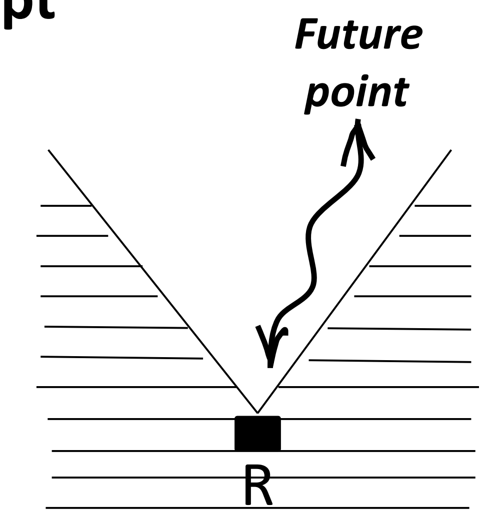
## Modification of the free will concept

[ P. H. & R. Ramanathan, Nat. Comm. (2019)]

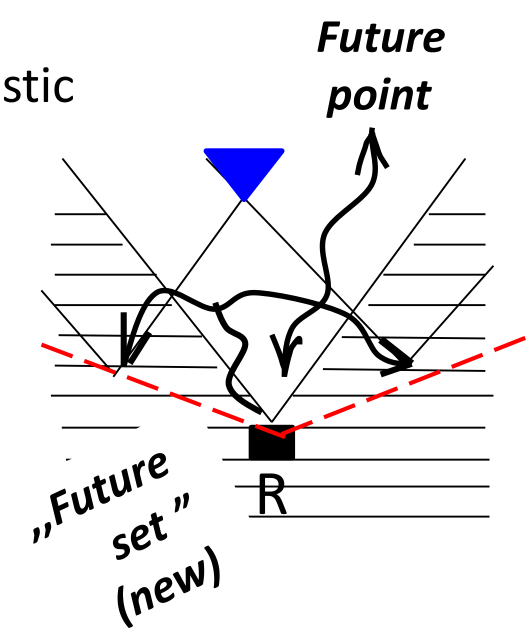



**Standard:**

Free random bit **only correlated with its Minkowski future** (= uncorrelated with its complement)

**Present (Relativistic Causality paradigm):**

Free random bit correlated with (i) its future and (ii) relativistic ,,future-like" sets (in sense of correlations) (= noncorrelated with anything that we can not influence)



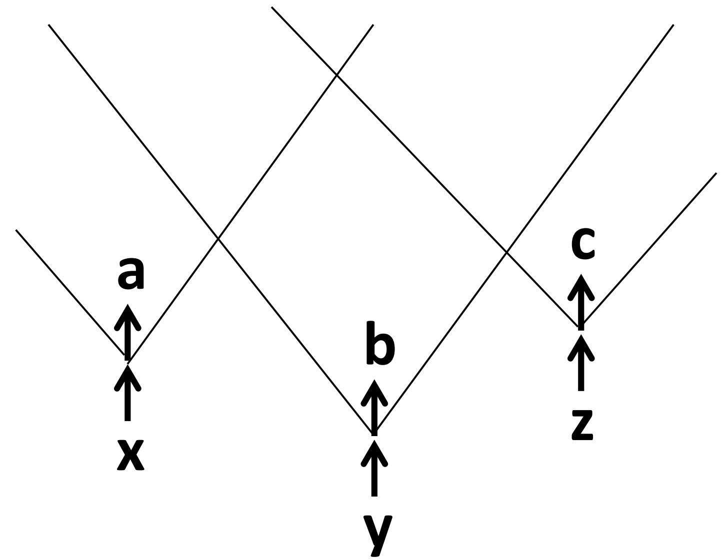
# Strange effects in RC beyond NS

## Modification of the free will concept

[ P. H. & R. Ramanathan,  Nat. Comm. (2019)]

**Free  input bits  (standard NS):**

**P(x|bc,yz) = P(x)**
**P(y|ac,xz) = P(y)**
**P(z|ab,xy) = P(z)**



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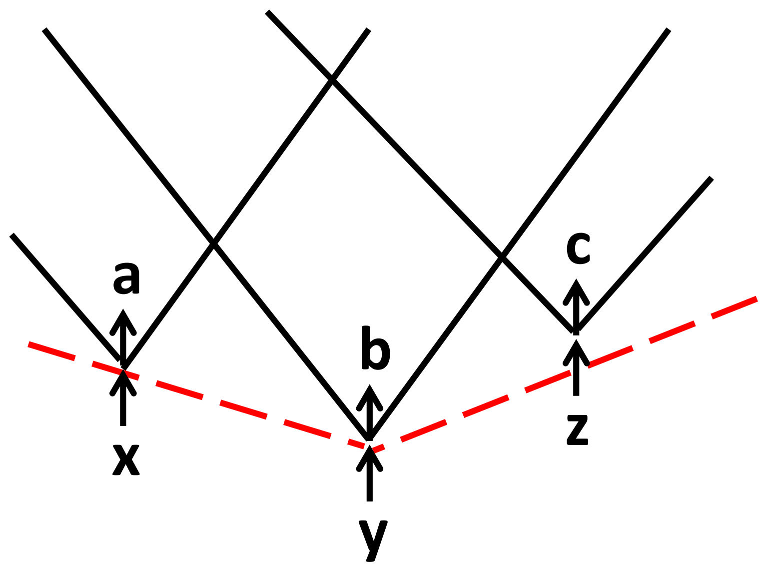



**Free input bits (RC paradigm):**

$$P(x|bc,yz) = P(x)$$
$$P(y|a,x) = P(y)$$
$$P(y|c,z) = P(y)$$
$$P(z|ab,xy) = P(z)$$

(Can be shown to be consistent with the RC conditions on tripartite box)

# Strange effects in RC beyond NS

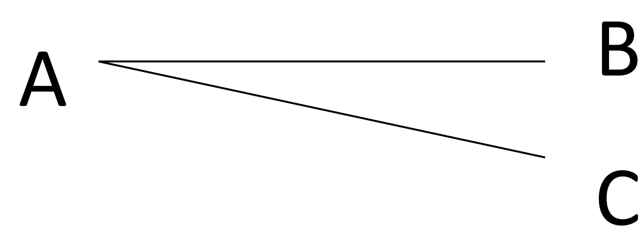
[ P. H. & R. Ramanathan, Nat. Comm. (2019)]

For Bell-CHSH inequality:

$\mathcal{B}_{AB} \leq 2\ (C_{LHV}) < 2\sqrt{2}\ (C_Q) < 4\ (C_{NS}$, also algebraic)

Instead of what NS-type monogamy

$\mathcal{B}_{AB} + \mathcal{B}_{AC} \leq 2\, C_{LHV} = 4$



# Strange effects in RC beyond NS

## Monogamy violation

[ P. H. & R. Ramanathan, Nat. Comm. (2019)]

For Bell-CHSH inequality:

$$\mathcal{B}_{AB} \leq 2 \ (C_{LHV}) < 2\sqrt{2} \ (C_Q) < 4 \ (C_{NS}, \text{algebraic})$$

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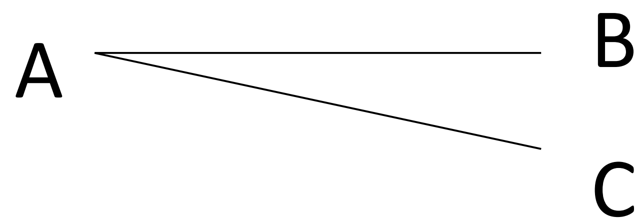



One gets for some boxes extremal monogamy violation:

$$\mathcal{B}_{AB} + \mathcal{B}_{AC} = 2 \, C_{NS} = 8$$

# Strange effects in RC beyond NS

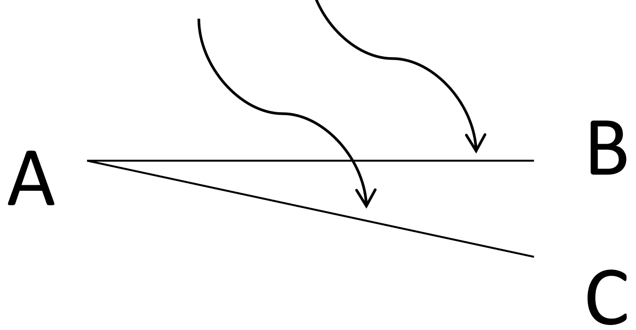
## Monogamy violation and extremal boxes problem

[ P. H. & R. Ramanathan, arXiv:1611.06781, Nat. Comm. accepted ]

$\mathcal{B}_{AB} + \mathcal{B}_{AC} = 2\, C_{NS} = 8$

(extremal violation of monogamy)

PR boxes !



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[ P. H. & R. Ramanathan, Nat. Comm (2019) ]

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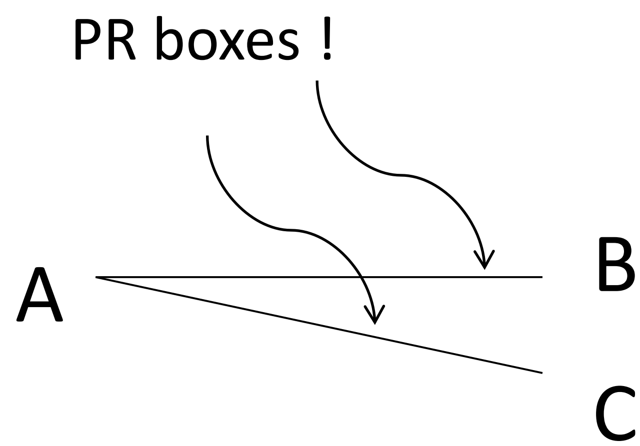
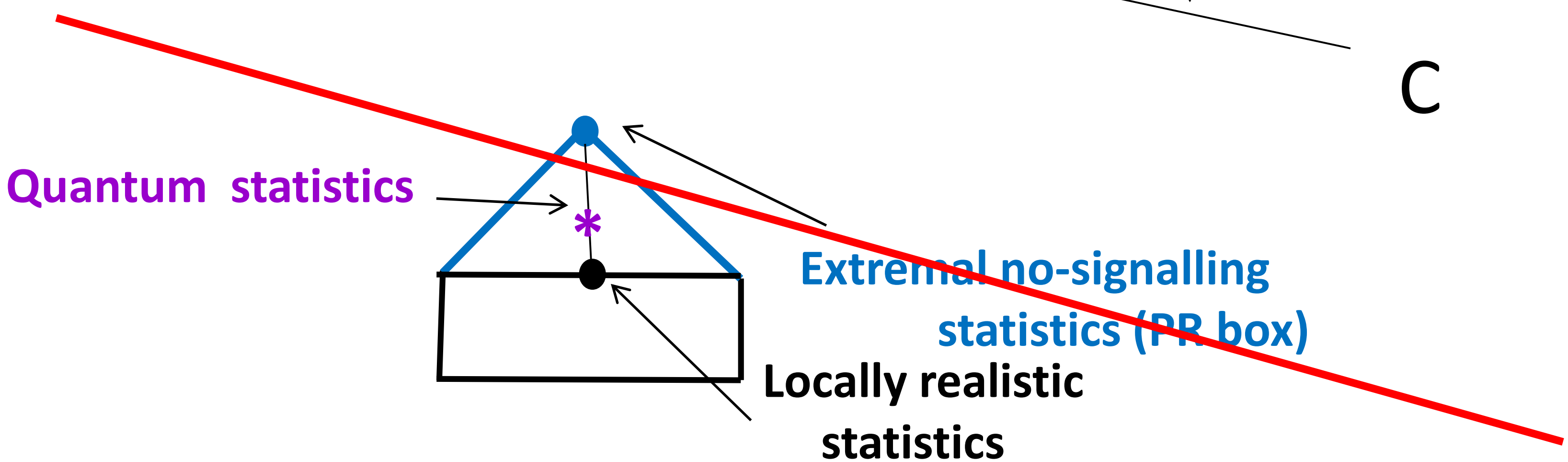
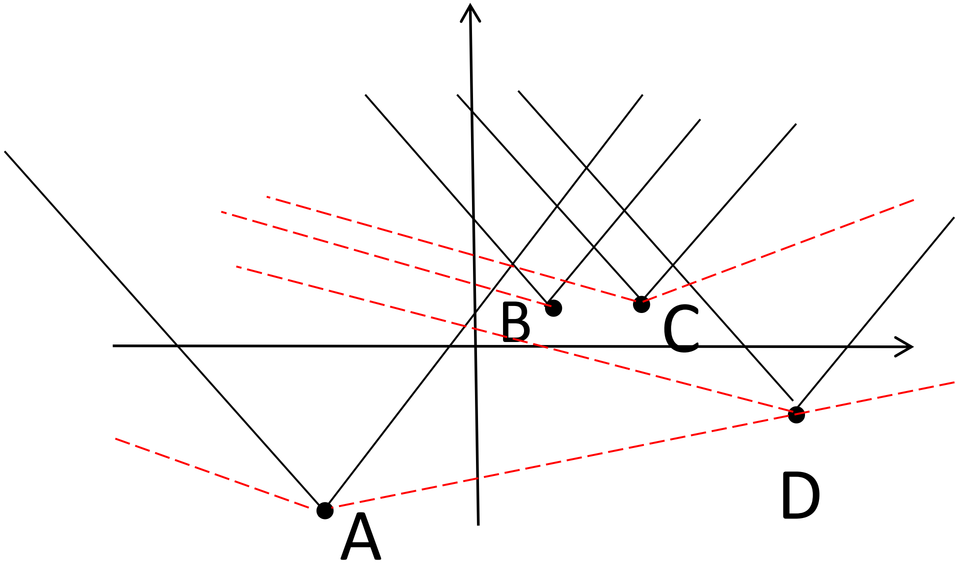



**The concept of extremality loses its power.**
**It no longer means lack of correlations with external world.**

# What about previous (point-to-point) hidden v-causal models ?

Can they be still refuted here ?




[J. D. Blancal, S. Pironio, A. Acin, Y.-C. Liang, V. Scarani, N. Gisin, Nat. Phys. (2012)]

Good news:

Some variant of the the refutation of the hidden v-causal models proven in [J. D. Blancal, Nat. Phys. (2012)] can be shown to survive in RC ( strictly speaking: $v > v_{treshold}$ is not allowed).

# Good news:

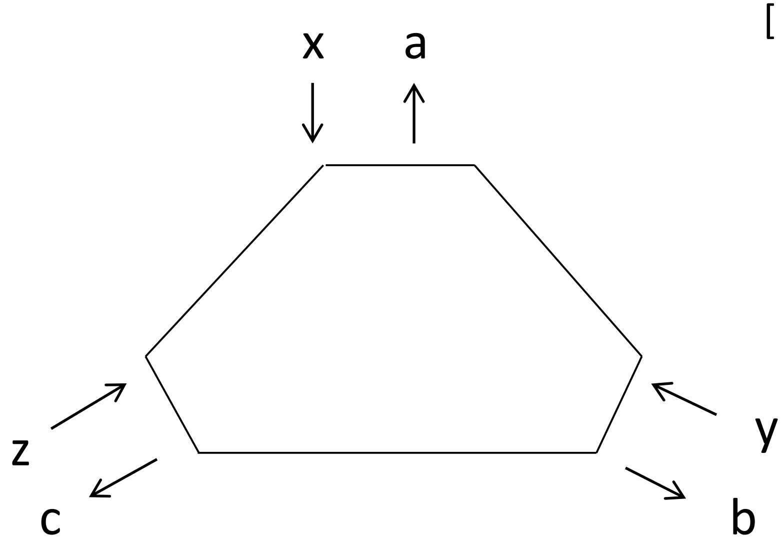
Some variant of the the refutation of the hidden v-causal models proven in [J. D. Blancal, Nat. Phys. (2012)] can be shown to survive in RC ( strictly speaking: $v > v_{treshold}$ is not allowed)

# Questions and facts:

- Randomness amplification ?  For popular Mermin inequality is not possible.  What about  general reandomness and cryptography?  Answers: [R. Salazar et al. (2019), in preparation, tba soon]
- RC does not obey the relativistic independence principle of [A. Carmi, E. Cohen, Sci. Adv. 5, 8370 (2019)].
- It can be rather  viewed as the weakest relativistic principle possible

# Strange effects in RC beyond NC

Complexity comunication problems solved sometimes **much better**




[ Roberto Salazar, Michał Kamon, Dardo Goyeneche, Karol Horodecki, Debashis Saha, Ravishankar Ramanathan, P. H., 1712.01030 ] (submitted)

{ p(abc|xyz) }

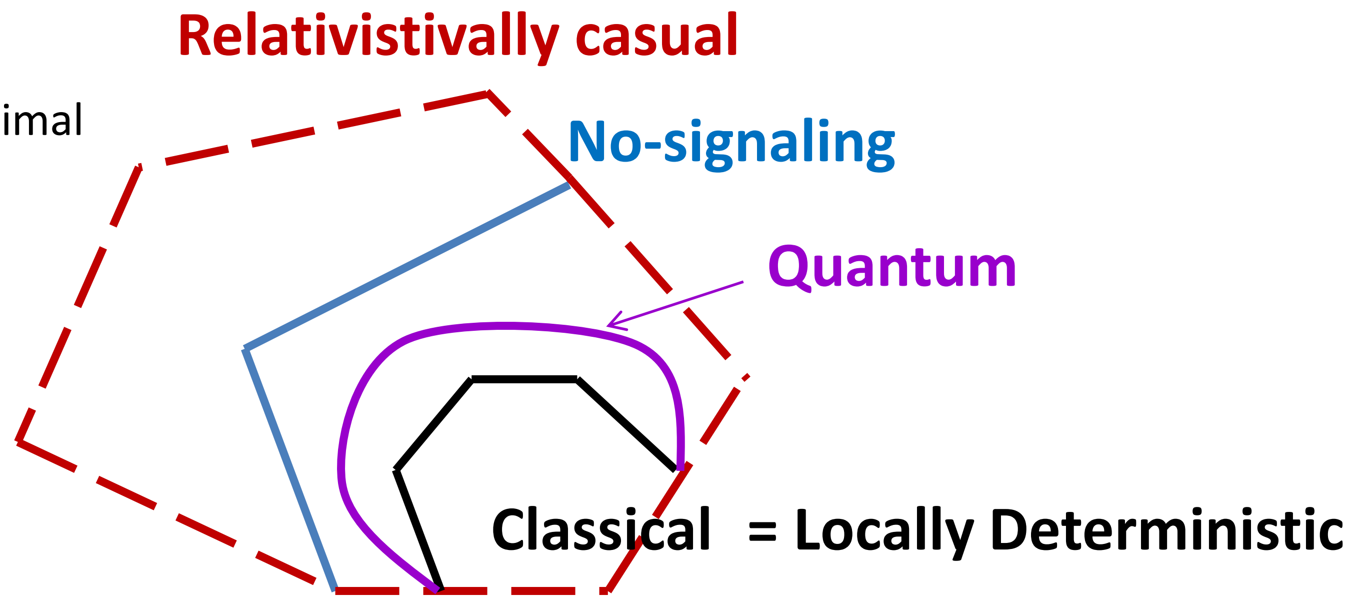
Problem for Alice and Charlie: guess the value function $f(x,y,z) = xy \oplus yz$ exchanging only 1 bit (no communication with Bob).

Probabilities of correct answer: $\mathcal{P}_{LHV} = \mathcal{P}_{Q} = \mathcal{P}_{NS} = 0.75$ , $\mathcal{P}_{RC} = 1$

# Full (3,2,2)-RC-polytope characterisation

[ R. Salazar, M. Kamon, D. Goyeneche, K. Horodecki, D. Saha, R. Ramanathan, P. H., 1712.01030 ]

Other separations in optimal winning probabilites have been found.




APPENDIX F: LIST OF EXTREMAL BOXES *(extremality in a weak sense)*

| Class | Prob. | Condition for RC Extremal Boxes Beyond No-signaling Polytope |
|---|---|---|
| 1 | 1 | $abc(1\oplus x)(1\oplus z) == 1$ |
| | $\frac{1}{2}$ | $b(cx\oplus(a\oplus xy)z) == 1$ |
| 2 | 1 | $abc(1\oplus x)y(1\oplus z) == 1$ |
| | $\frac{1}{2}$ | $a(c\oplus cy\oplus bz)\oplus bx(c\oplus z\oplus yz) == 1$ |
| 3 | $\frac{1}{4}$ | $(1\oplus c)xy\oplus b(c\oplus y\oplus yz\oplus xyz)\oplus a(c\oplus y\oplus z\oplus bz\oplus yz\oplus bcxyz) == 1$ |
| | $\frac{3}{4}$ | $abcxyz == 1$ |
| | $\frac{1}{2}$ | $ab(1\oplus c\oplus y\oplus z\oplus yz\oplus cxyz) == 1$ |

and many more … (190 extremal, only 6 NS of them)

# Conclusions and outlook

- Relativistic Causality: minimal condition to avoid causal loops
- Relativistic Causality is potentially something more than no-signaling: space-time configuration essential
- Randomenss amplification possible or not ?
- Advantages in communication complexity. Other tasks ?
- Point-to-point hidden v-causal models above some treshold value still can be refuted

**General question:**

Is there a physical theory with those properties ?

*,, Dans les champs de l'observation le hasard ne favorise que les esprits préparés (…) ”*

*,,In the fields of observation chance favours only the prepared mind (…).”*

Lecture, University of Lille (7 December 1854)

L. Pasteur

May be it  is good to extended the above also to the theory ground ? (At least while looking for possible future theories)

# PART II
# Propagation of potential statistics in space time

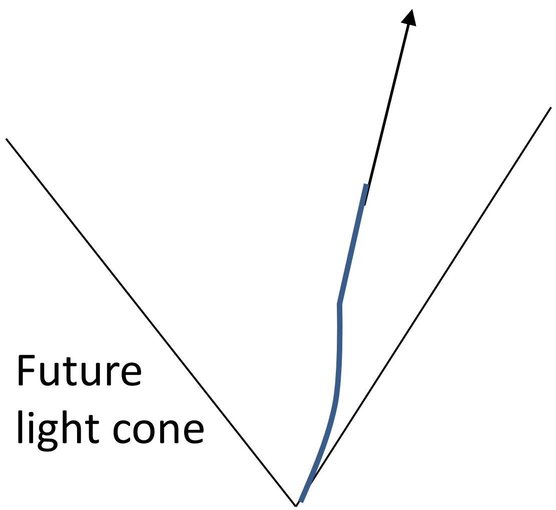
# Motivation

- No-signaling and Relativistic Causality is based on correlation picture – more then one system needed
- No-signaling has no dynamical rules at all, while RC puts space-time constraints on the dynamics of internal degrees of freedom only
- Is it possible to define minimal relativistic causality constraint

    (i) **for single system**

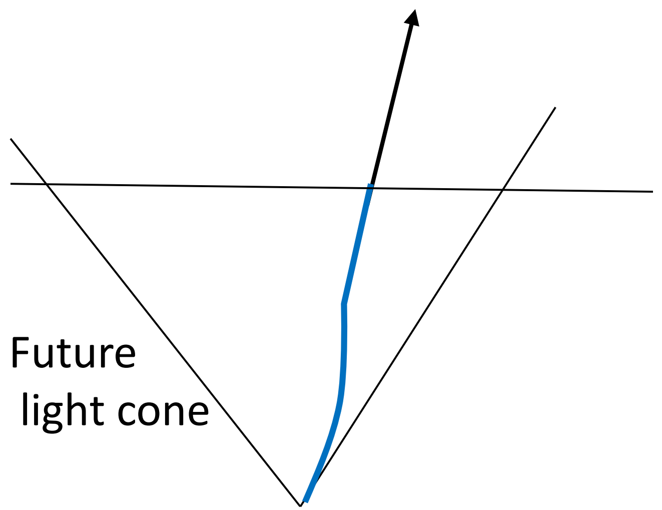
    (ii) **dynamics of arbitrary character** (may be nonlinear) ?

# Propagation of classical particle under causality conditions (I)

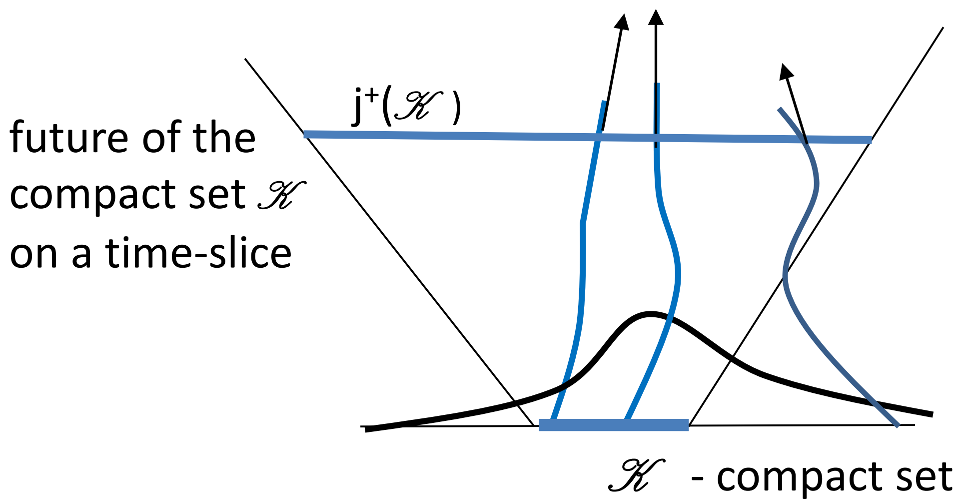



Deterministic case
– position of the particle is known.

# Propagation of classical particle under causality conditions (II)

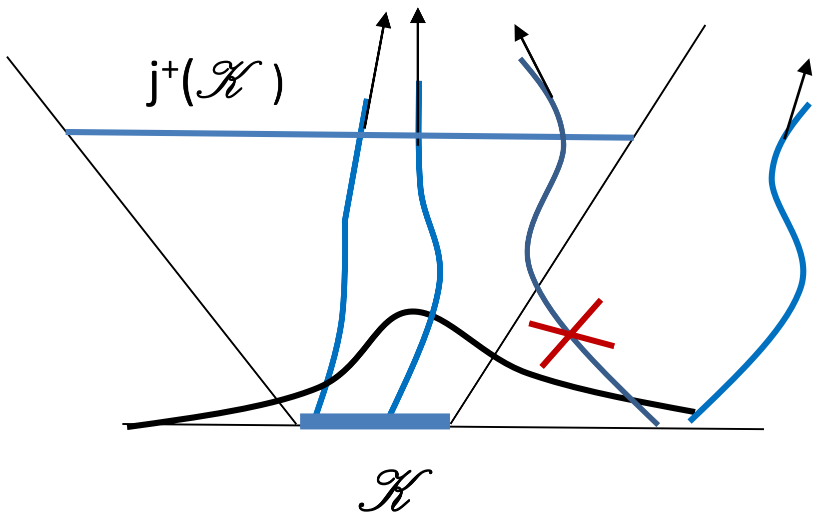



Deterministic case
– position of the particle is known.




Probabilistic case
– position of the particle is unknown.

# Causal propagation of classical distribution (I)

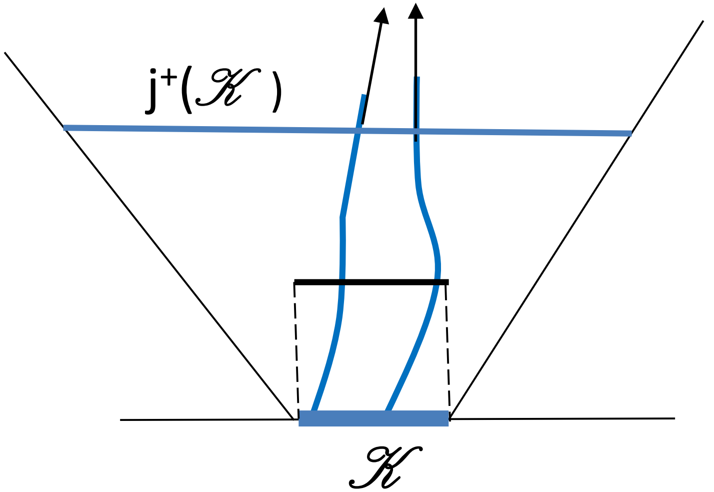



In the case when trajectories do not ,,sneek into" the compact set …

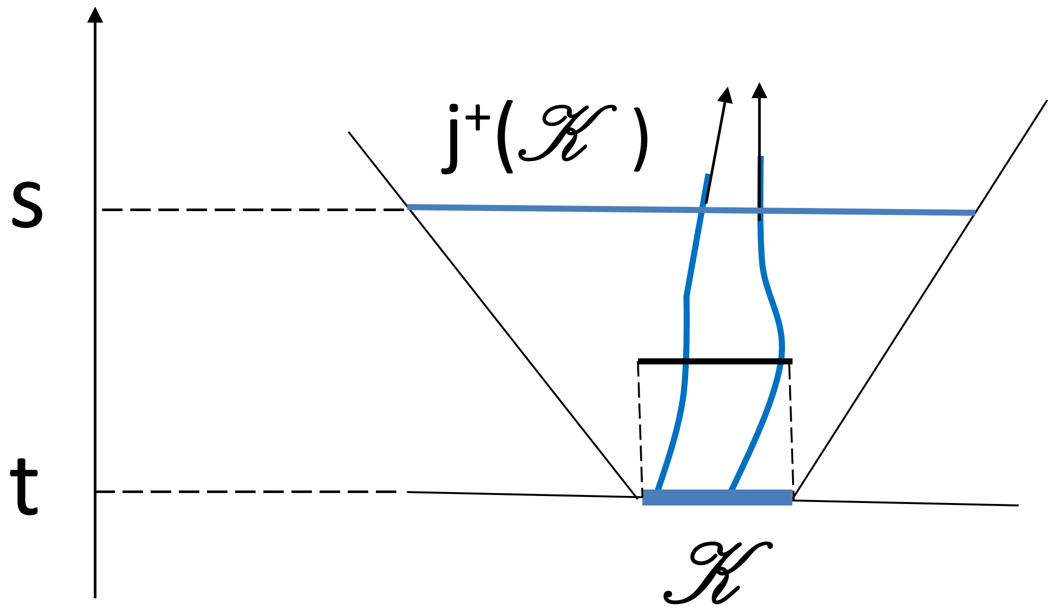
# Causal propagation of classical distribution (II)

… or when we know a priori that the

particle is in region $\mathscr{K}$ …



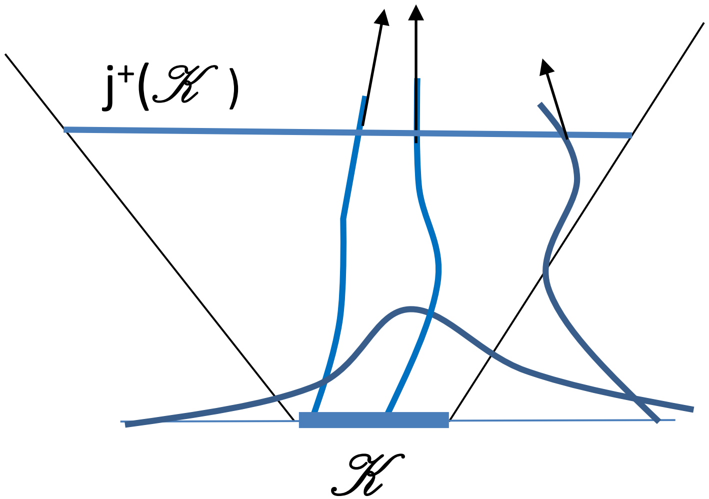
# Causal propagation of classical distribution (II)




… then obviously the measures of the two sets are equal since the particle **is somewhere** in $\mathscr{K}$ and can not leave it due to causality:

$$\mu_t(\mathscr{K}) = \mu_s(\, j^+(\mathscr{K})\,)$$

# Causal propagation of classical distribution (III)

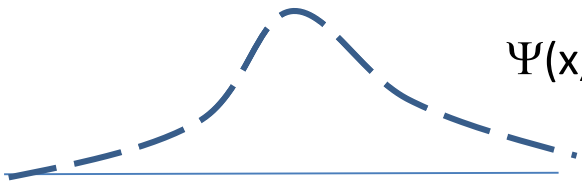



However since in general particle can „sneek into” the region $j^+(\mathscr{K})$ the chances to find it there may only increase:

$$\mu_t(\mathscr{K}) \leq \mu_s(\, j^+(\mathscr{K})\,)$$

# Quantum propagation

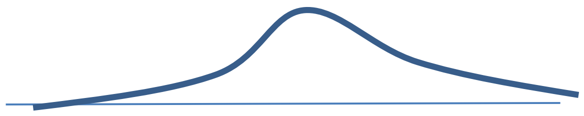
Normalised vector $|\Psi_t\rangle$ form the Hilbert space $H = L^2(R)$ represented by **wavefunction** $\Psi(x,t)$ corresponding to **probability amplitude.**



$\Psi(x,t)$

**The probability density** of spacial distribution of finding a particle **if we perform the measurement is:**

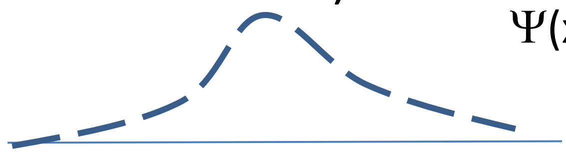
$$\rho(x,t) = |\Psi(x,t)|^2$$



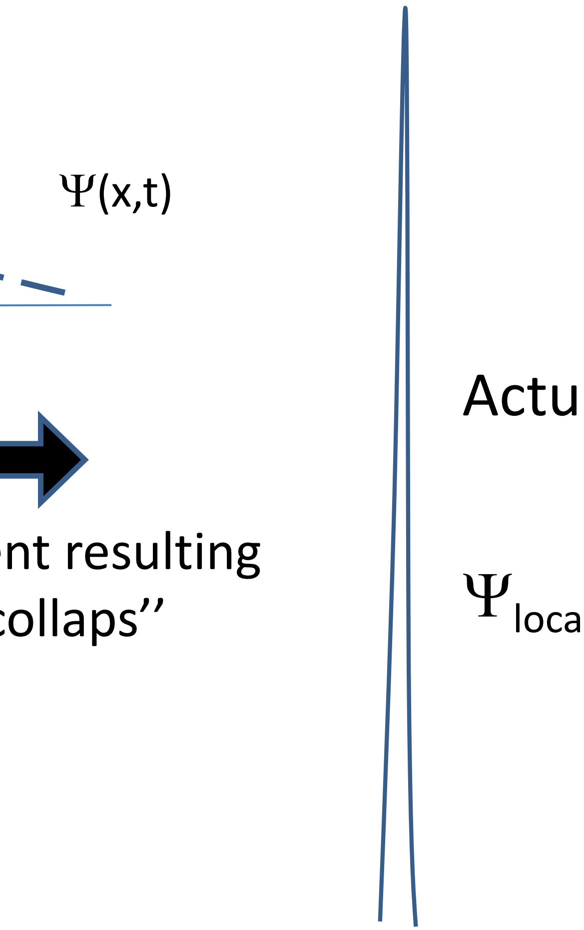
# Quantum collapse

Potentiality - particle is nowhere (wave-like character)

$\Psi(x,t)$



Measurement resulting in a „wave collaps“



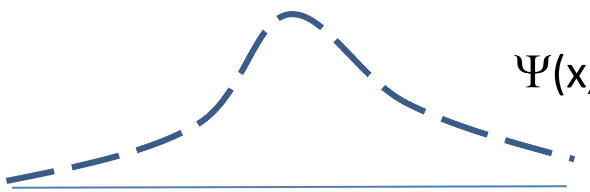
Actualization to

$\Psi_{localised}(x) \approx \delta(x)$

# Quantum propagation

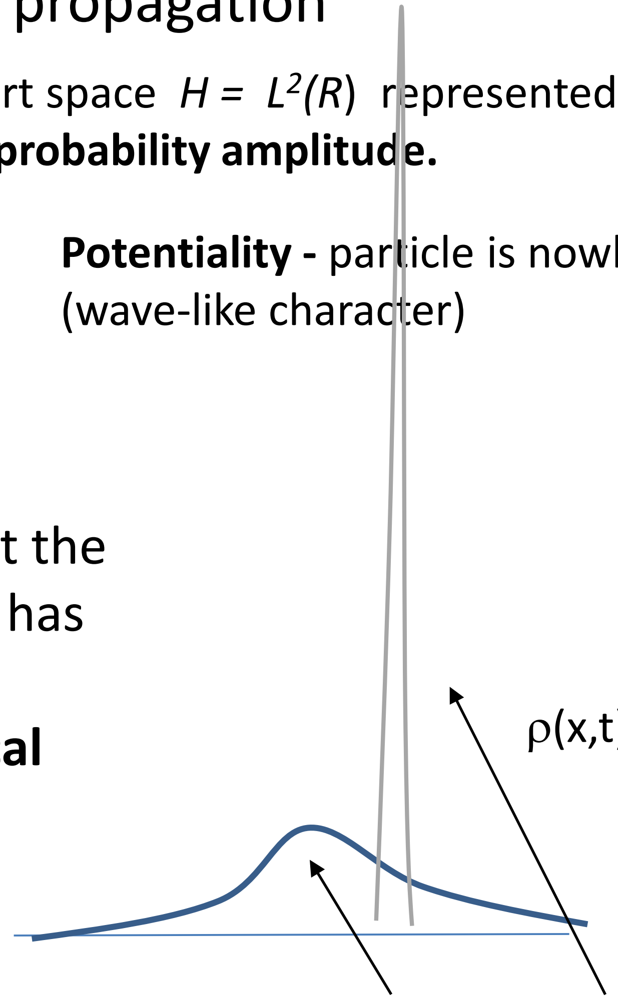
Normalised vector $|\Psi_t\rangle$ form the Hilbert space $H = L^2(R)$ represented by **wavefunction** $\Psi(x,t)$ corresponding to **probability amplitude.**

**Potentiality -** particle is nowhere (wave-like character)



$\Psi(x,t)$

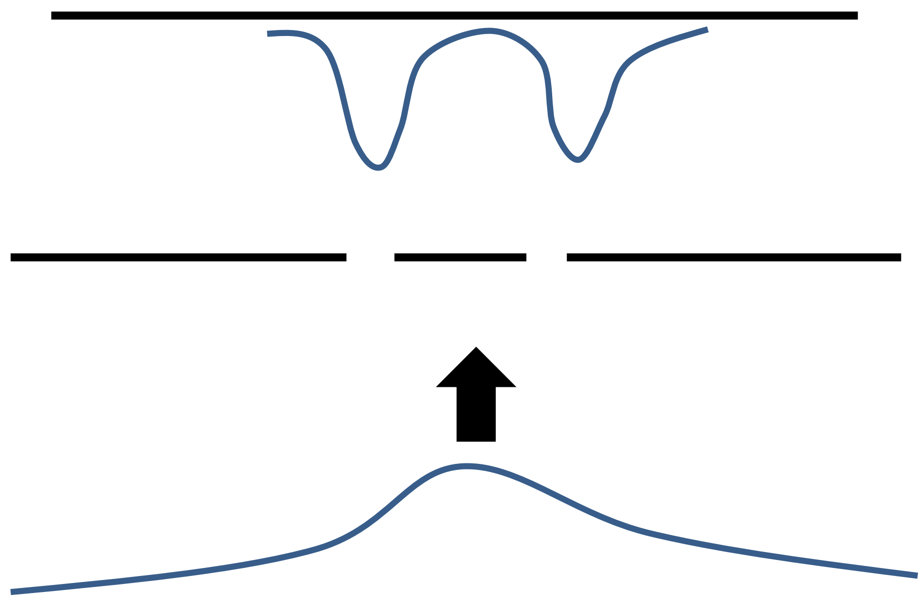
**Actuality –** particle **is somewhere,** but the probability density of getting it there has the form.
Alternatively the density is **our classical lack of knowledge description**



$\rho(x,t) = |\Psi(x,t)|^2$

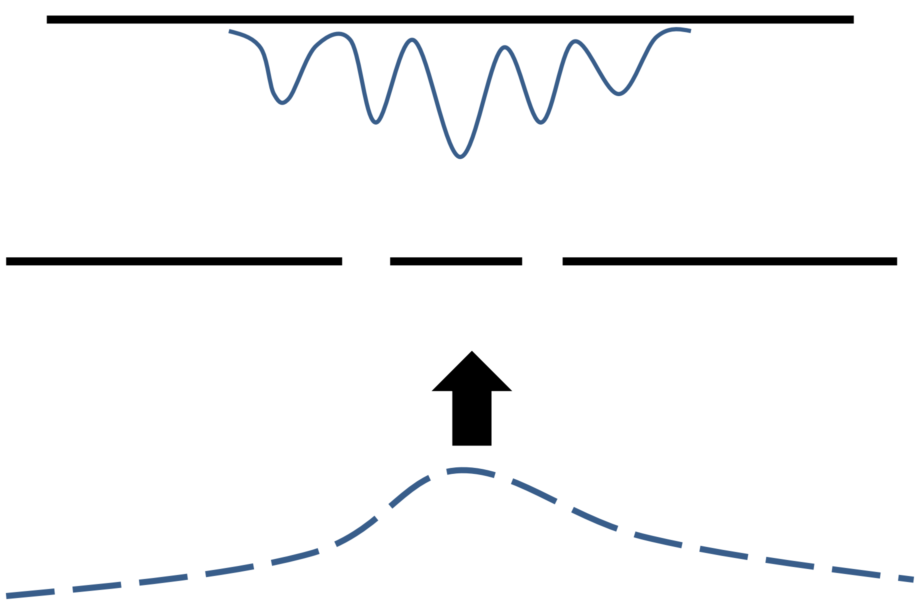
When we forget where the particle is we get a mixture of ,,picks".

# Classical vs quantum interference (I)



Classical propagation

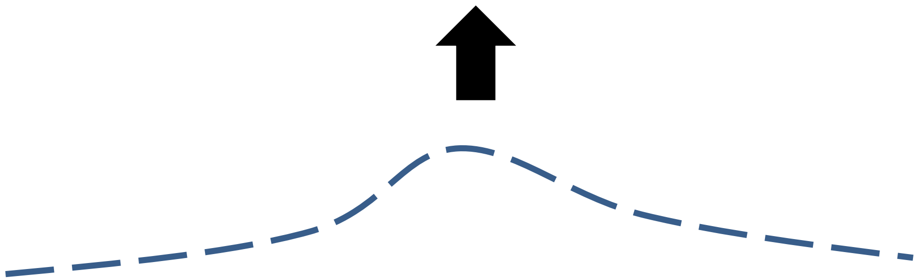
# Classical vs quantum interference (I)



*Interference – nonclassical single particle phenomenon*

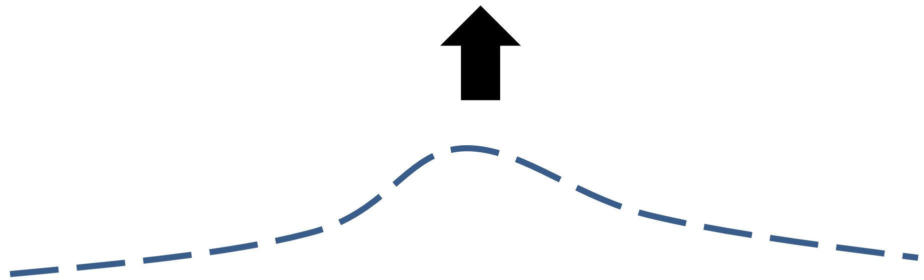
Quantum propagation

Question what is a causal propagation of ,,potential statistics"  ?



- Fully general – (non)linear quantum theory
- Possible regimes:
  1) active (demolition): preparation/absorption of particle is possible
  2) passive (nondemolition) – propagation is given only actualisation (collapse)  is possible (not absorbtion)
  3) both previous ones on demand - 1) +2)

Question what is a causal propagation of ,,potential statistics”  ?



- Fully general – (non)linear quantum theory)
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  1) active (demolition): preparation/absorption of particle is possible
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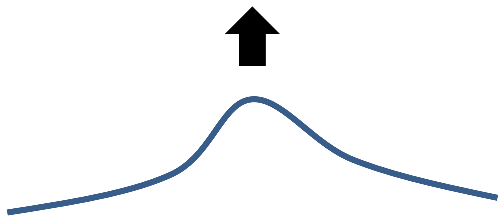
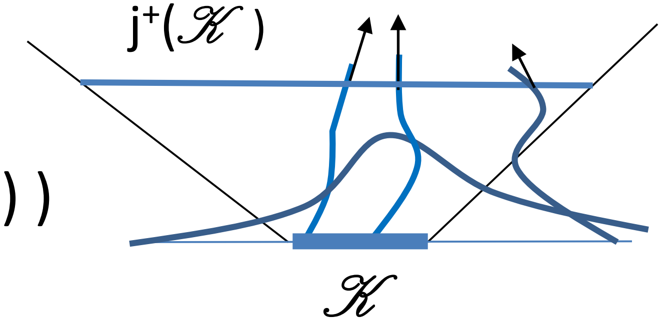
# Classical vs classical-like causal evolution (CE)



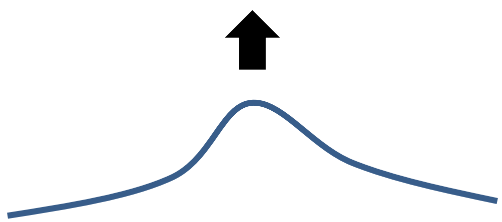
is classical CE iff

$$\mu\,(\mathscr{K}) \leq \mu(\,j^+(\mathscr{K}\,)\,)$$

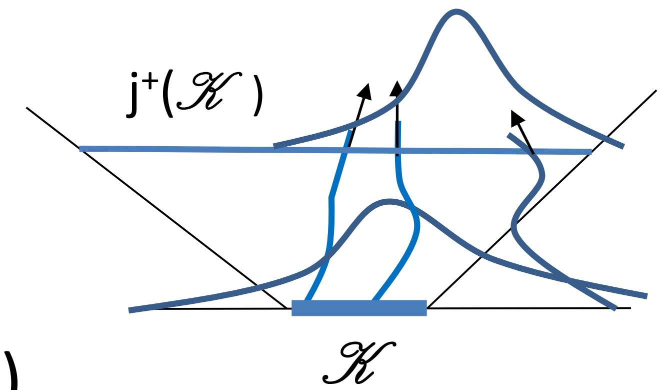


# Classical vs classical-like causal evolution (CE)

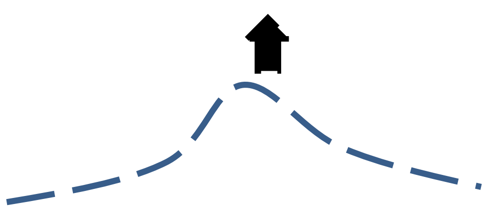
[M. Eckstein and T. Miller PRA, 95 032106 (2016) ]



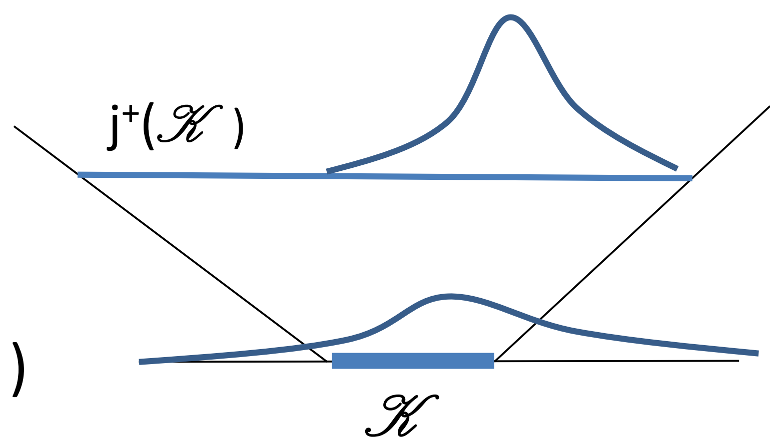
is **classical CE** iff




$$\mu_t(\mathscr{K}) \leq \mu_s(\, j^+(\mathscr{K}\,)\,)$$



**classical-like CE** iff

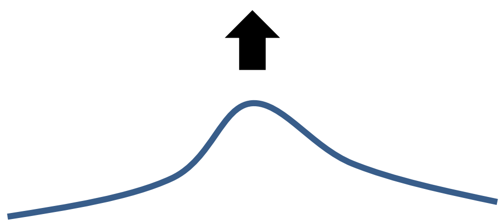


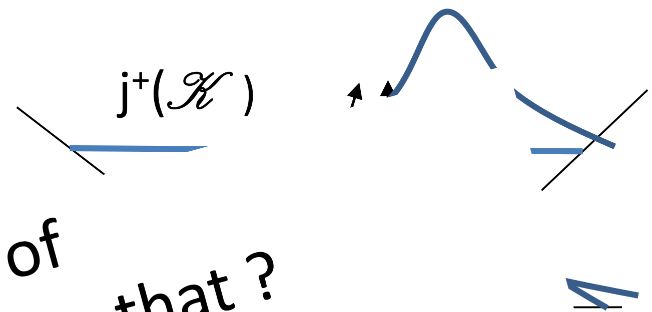

$$\mu'_{\,t}(\mathscr{K}) \leq \mu'_{\,s}(\, j^+(\mathscr{K}\,)\,)$$

with $\mu'(\mathscr{K}) = \int_{\mathscr{K}} |\Psi(x,t)|^2\, dx$

# Classical vs classical-like causal evolution (CE)

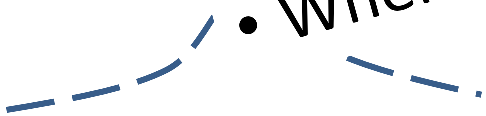
[M. Eckstein and T. Miller PRA, 95 032106 (2016) ]



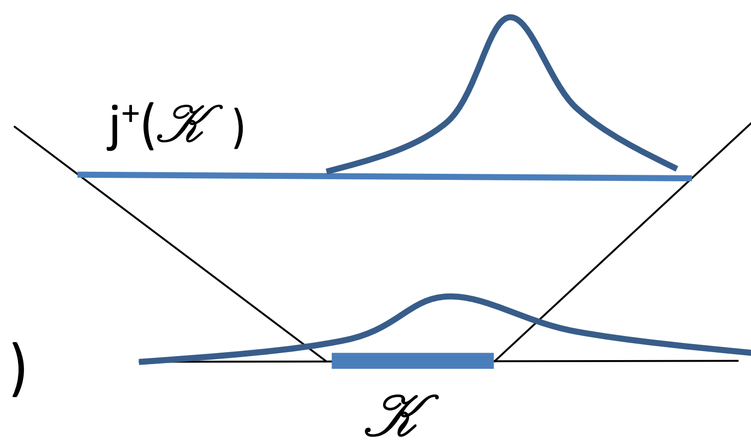



is **classical CE** iff

- Is there any reason for evolution of „potential statistics” to behave like that ?
- When it must behave as if it were classical ?



**classical-like CE** iff

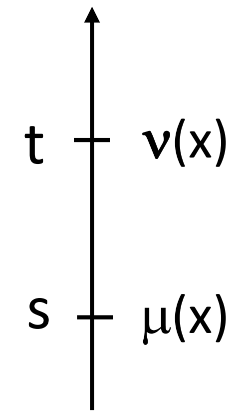



$$\mu'_t(\mathscr{K}) \leq \mu'_s(\, j^+(\mathscr{K})\,)$$

$$\text{with } \mu'(\mathscr{K}) = \int_{\mathscr{K}} |\Psi(x,t)|^2\, dx$$

# The paradigm and consistency conditions (I)

- The potential statistics evolves.
  Had it been fully measured at time **s or (=,,exclusive or") t** (t > s)
  it would have provided the statistics **$\mu$ (x) or $\nu$ (x)** respectively.




- At the second (later) **moment s** a measurement checking the presence (absence) of particle in the region ***K*** is performed with probability **$P(m_K)$, $m_K$=0/1** corresponds to **,,measurement performed/not performed".**

- Possible results are **r =+/-/∅** corresponding to **,,particle detected/not detected/no result"** ( the latter necessary iff **$m_K$=0**).

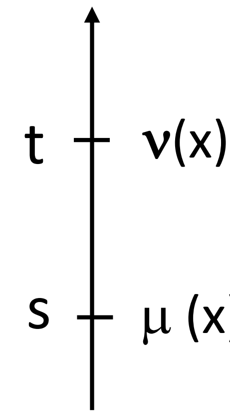
**General problem.-** We ask about behaviour of **$\nu$ (x|$m_K$) in later moment s conditioned upon the measurement in previous moment t.**

Obvious consistency condition $\boldsymbol{\nu(x|0) = \nu(x)}$

# The paradigm and consistency conditions (II)

Particle detected in $K$ if measurement at time s was performed

$$P(+|1) = \mu(\mathcal{K}), \quad P(-|1) = 1 - \mu(\mathcal{K}), \quad P(\emptyset|1) = 0$$
$$P(+|0) = 0, \quad P(-|0) = 0, \quad P(\emptyset|0) = 1$$

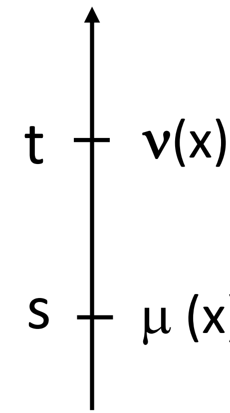



$$\tilde{\nu}(\mathcal{X}, r|m_{\mathcal{K}}) = \nu(\mathcal{X}|r, m_{\mathcal{K}})P(r|m_{\mathcal{K}})$$

$$\sum_r \tilde{\nu}(\mathcal{X}, r|m_{\mathcal{K}}) = \nu(\mathcal{X}|m_{\mathcal{K}})$$

# The paradigm and consistency conditions (III)

$$\tilde{\nu}(\mathcal{X}, r|m_K) = \nu(\mathcal{X}|r, m_K)P(r|m_K)$$




$$\sum_r \tilde{\nu}(\mathcal{X}, r|m_K) = \nu(\mathcal{X}|m_K)$$

Any set. The presence of the particle is checked in this set at later time t conditioned by the fact that its presence in set $K$ had (not) been checked in a previous time s.

$+/-/\varnothing$

corresponds to „found in $K$/ not found in $K$/ no result”

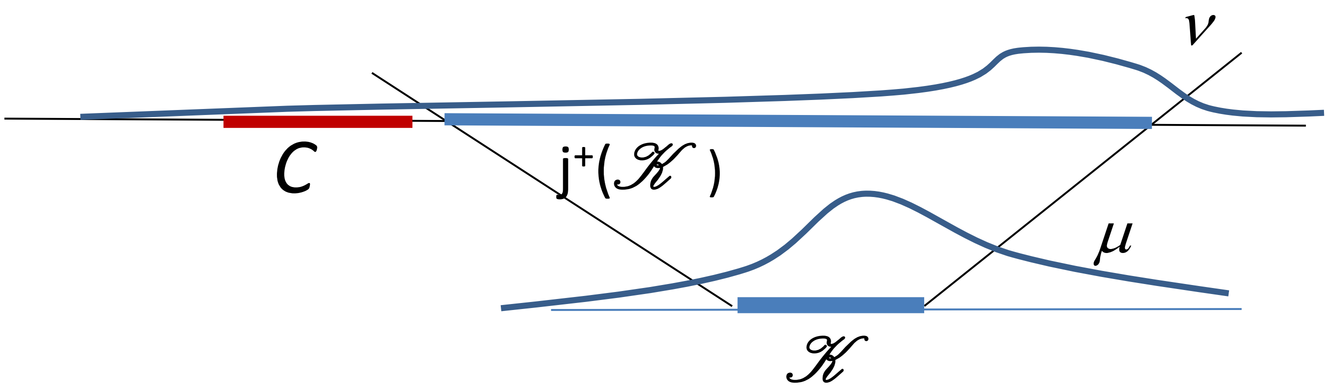
0/1

corresponds to „presence in $K$ checked/ not checked”

# Digression:
# formal NS condition and its operational character

# (Formal) no-signaling (NS) property

[ M. Eckstein, P. H. , R. Horodecki and T. Miller, arXiv:1902.05002 ]




The formal NS property says that the condition

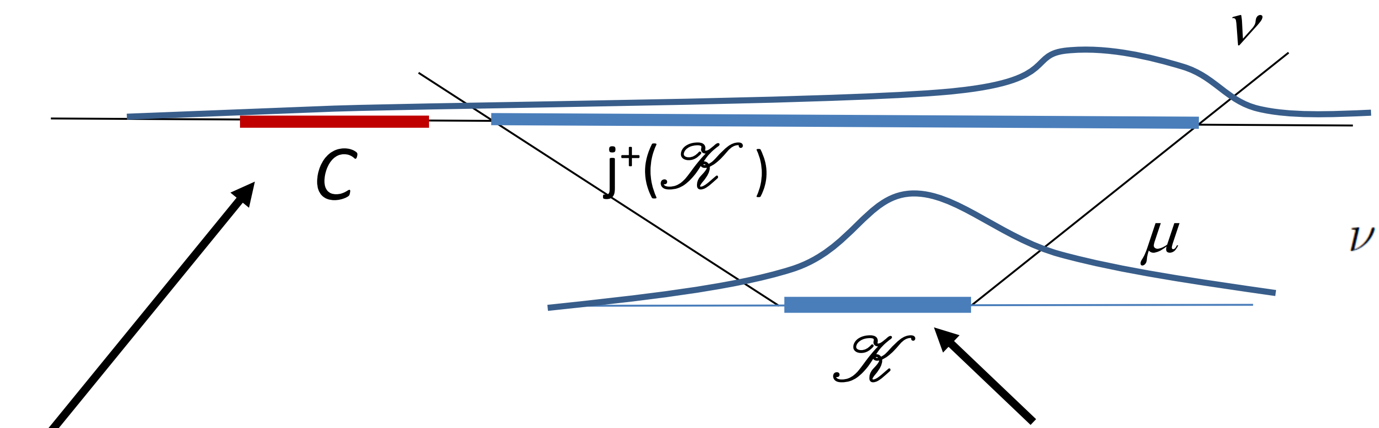
$$\nu(\mathcal{C} \mid 1) = \nu(\mathcal{C} \mid 0)$$

must hold for all compact $\mathscr{K}$ and $C$

when the set C has no intersection with $j^+(\mathscr{K})$ .

# (Formal) no-signaling (NS) property

[ M. Eckstein, P. H. , R. Horodecki and T. Miller, arXiv:1902.05002 ]

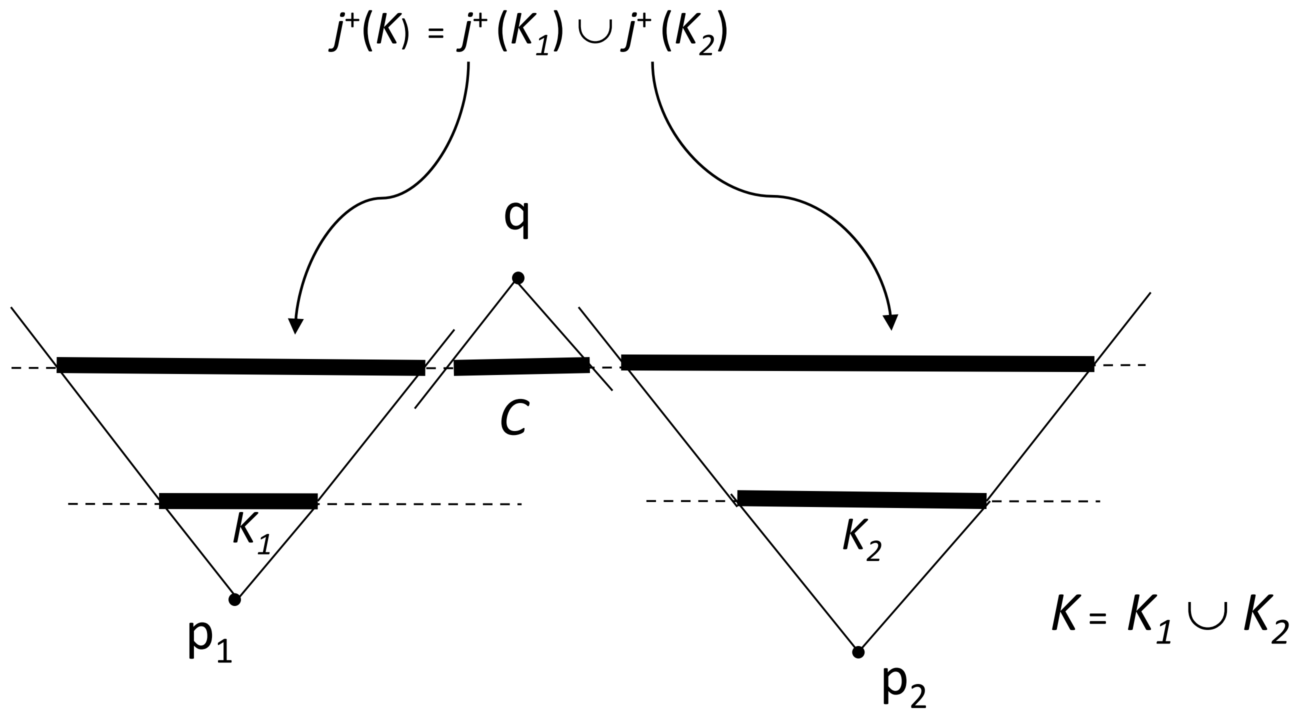



$$\nu(\mathcal{C} \,|\, 1) = \nu(\mathcal{C} \,|\, 0)$$

**Presence/absence of the measurement here** should **not change** statistics **here**

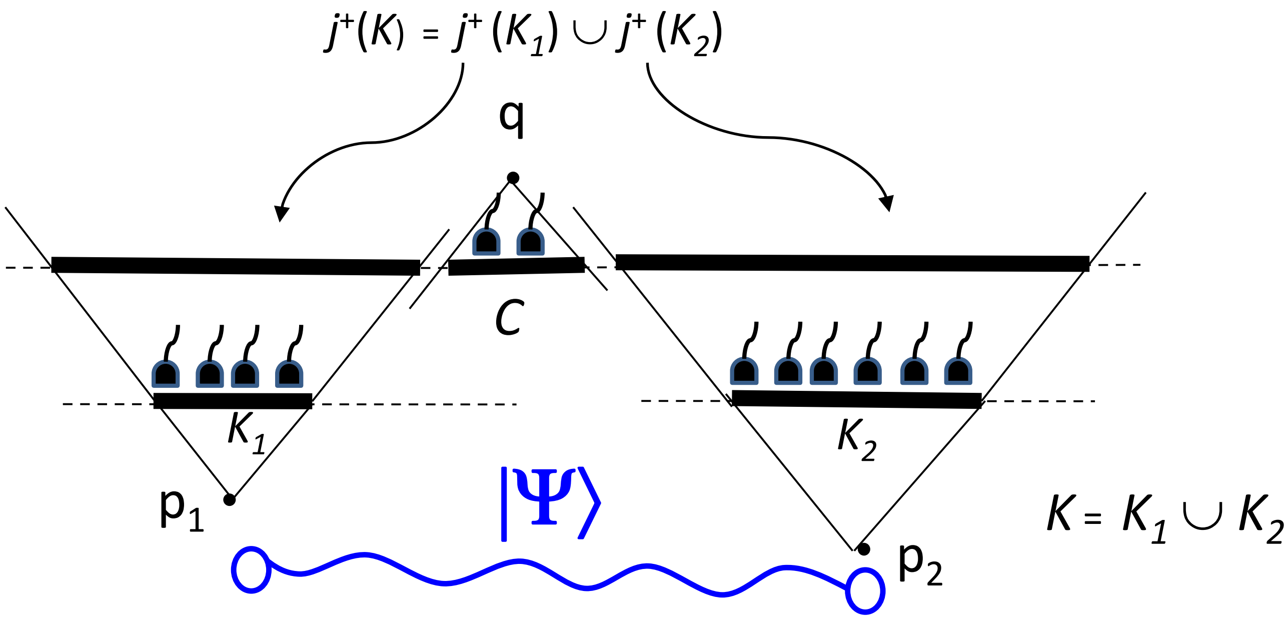
**Problem – it does not mean that we may sent an information when the condition is violated, since the information transfer has a point-to-point character.**
**This is not automatic if K is not convex.**

Suppose that $\nu(C|0) \neq \nu(C|1)$ only if measurement was performed in both regions $K = K_1 \cup K_2$



# Operational theorem

[ M. Eckstein, P. H. , R. Horodecki and T. Miller, arXiv:1902.05002 ]




Measurements in K executed only if the results of $\sigma_z$ measurement on singlet is ,,0". Correlated ,,0"-s results are trasmited outside of the sum of their future cones.

**Theorem 1.** NS is **operational – under a single natural axiom (A1) its violation leads to signaling faster than light.**

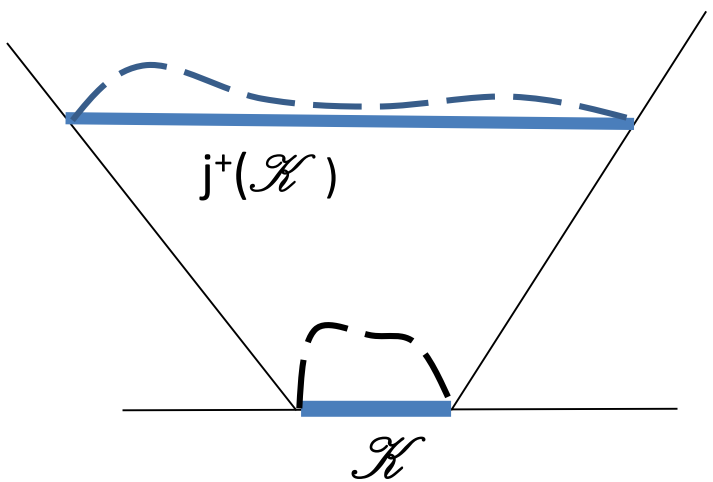
# Natural axiom *A1*

# Natural Axiom *A1*

[ M. Eckstein, P. H. , R. Horodecki and T. Miller,
,,Operational causality in space time” arXiv:1902.05002 ]

*Axiom 1 (A1)* .- If the signal has been detected $(r = +)$ at time $s$ in the region $\mathcal{K}$ $(m_{\mathcal{K}} = 1)$, then then signal must be present with certainty in that region’s future $j^+(\mathcal{K})$ for any later time $t$:

$$\nu(j^+(\mathcal{K})|+, 1) = 1$$




**Reason.** If there were a ,,leackage” then by absorbtion of the particle in *K* we would cancel it signaling outside of the future cone of *K*.

## Classical-like restriction on the dynamics of ,,potential statistics

[ M. Eckstein, P. H. , R. Horodecki and T. Miller,
,,Operational causality in space time” arXiv:1902.05002 ]

**Theorem . –** Assume A1. If ,,potential statistics” violates the formal causality-like evolution condition

$$\mu'_t(\mathscr{K}) \leq \mu'_s( j^+(\mathscr{K}) )$$

then it violates the NS condition which has been shown to be operational.

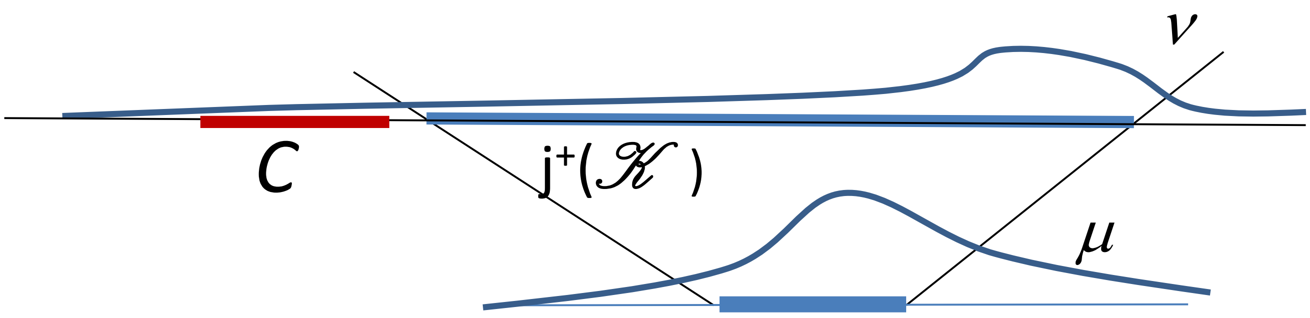
**Conclusion . – Under the minimal assumptions the potential statistics (,,density”) should – in some sense - evolve as if it were classical.**

# Natural axiom *A2* and complete relations

# Natural Axiom *A2* (I)

*Axiom 2 (A2)* .- If the particle has *not* been detected at time $s$ within $\mathcal{K}$, then outside of $J^+(\mathcal{K})$ the evolution of $\mu(\,.\,|\,-,1)$ proceeds with no modification other than re-normalisation:

$$\nu(\mathcal{C}\,|\,-,1) = \frac{\nu(\mathcal{C})}{1-\mu(\mathcal{K})}$$



# Natural Axiom *A2* (II)

[ M. Eckstein, P. H. , R. Horodecki and T. Miller,
,,Operational causality in space time" arXiv:1902.05002 ]

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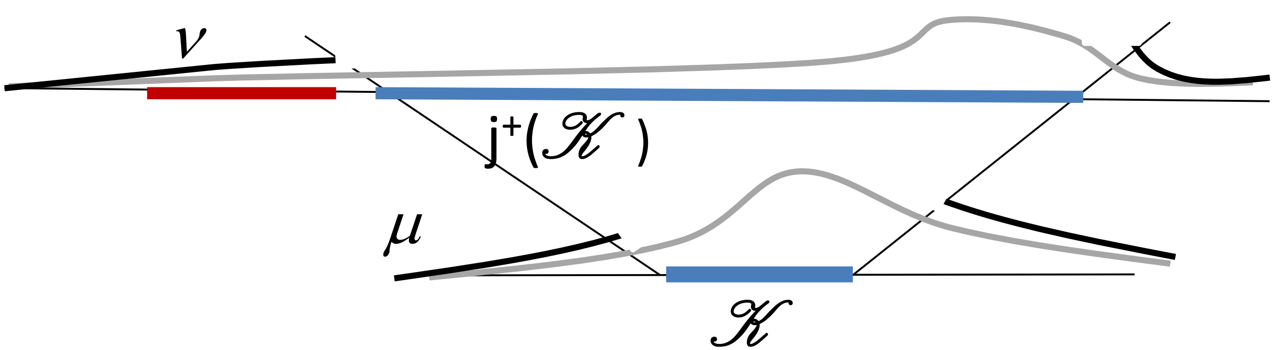
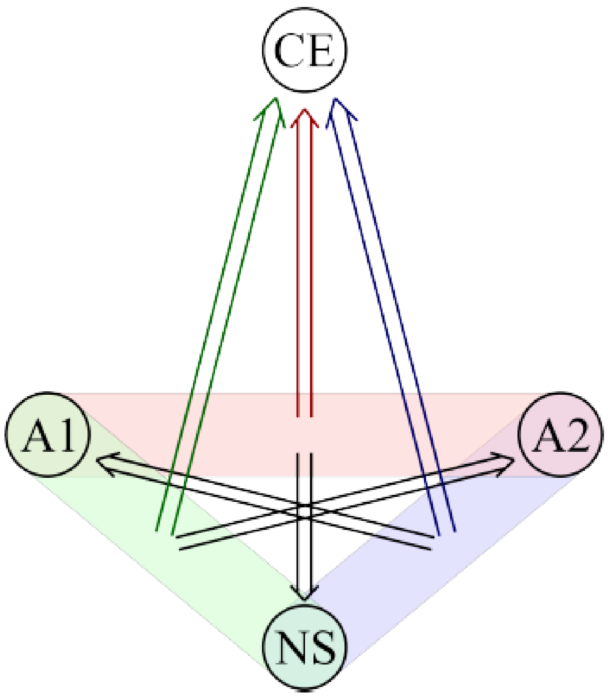



**Reason –** otherwise we could signal outside by executing measurement inside of the region $\mathscr{K}$.

# Complete set of relations

[ M. Eckstein, P. H. , R. Horodecki and T. Miller,
,,Operational causality in space time" arXiv:1902.05002 ]

**Proposition .-**



# Summary of part II

- In the natural scenario of potential statistics (quantum or postquantum, may be nonlinear) from the perspective of causality
  the propagation must behave – in some sense - as if it were
  post-measurement one
- Complete relations of natural conditions necessary for causality
- Single particle Schroedinger equation for some potentials
  provides mechanisms for explicit operational faster than light
  signaling – serious limitations of its validity (can be also quantified).
- Interpretation in terms of single-party box with continuous
  numer of commuting settings (as apposed to
  discrete numer of noncommuting settings for multiparty
  correlation no-signaling/relativiastic causality paradigm)
- Multiparty senario– work in progres

Thank you


MAESTRO
QOLAPS
IDEAS PLUS