# Mathematical Models of Markovian Dephasing

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# Dephasing

**Dephasing**

Decay of coherences in a preferred basis,
usually eigenstates of the system Hamiltonian,
without energy transitions (pure dephasing)

**Decoherence**

Any process that may cause loss of coherence

# Dephasing

**System.** Hilbert space h, Hamiltonian $H_S = \sum_n \epsilon_n P_n$

**State.** Density matrix $\rho$

**Evolution.** $\rho \to \rho_t := \mathcal{T}_{*t}(\rho)$, $\mathcal{T}$ QMS, GKSL generator $\mathcal{L}$

**Dephasing.** Decay of coherences: $P_m\, \rho_t\, P_n \to_{t\to\infty} 0$ for all $m \neq n$

*without energy transitions*: $\mathrm{tr}(\rho_t P_n)$ constant

Example: phase damping. $\mathcal{T}$ QMS on $M_2(\mathbb{C})$ generated by

$$\mathcal{L}(x) = \gamma\,(\sigma_z x \sigma_z - x) \qquad \gamma > 0$$

$$\mathcal{T}_t \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} = \begin{bmatrix} x_{11} & x_{12}\, \mathrm{e}^{-\gamma t} \\ x_{21}\, \mathrm{e}^{-\gamma t} & x_{22} \end{bmatrix}$$

## Definition

$p$ and $q$ two $\mathcal{T}_t$-invariant orthogonal projections mutually orthogonal to each other dephase under a QMS $\mathcal{T}$ if

$$\lim_{t\to\infty} \mathcal{T}_t(p\,x\,q) = 0 \qquad \text{for all} \quad x \in \mathcal{B}(\mathsf{h}).$$

$\mathcal{T}$ is maximally dephasing if $\exists$ *rank-one* orthogonal projections $(p_n)_n$ with $\sum_n p_n = \mathbb{1}$,

- $\mathcal{T}_t(p_n) = p_n$ for all $t, n$,
- $p_n, p_m$ dephasing for all $n \neq m$.

Schrödinger picture (Baumgartner & Narnhofer, J. Phys. A (2008))

$$\lim_{t\to\infty} p_m \mathcal{T}_{*t}(\rho) p_n = 0, \qquad \text{whenever} \quad n \neq m$$

## Maximally dephasing QMSs

1. $\mathcal{T}_t(p_n) = p_n \quad$ for all $t \geq 0$, for all $n$
2. GKSL generator

$$\mathcal{L}(x) = \mathrm{i}[H, x] - \frac{1}{2} \sum_{\ell} \left( L_\ell^* L_\ell x - 2 L_\ell^* x L_\ell + x L_\ell^* L_\ell \right)$$

   $H = H^*$, $\mathbb{1}, L_1, L_2, \ldots$ linearly independent

3. $p_n L_\ell = L_\ell p_n$, $p_n H = H p_n \quad \forall n$
4. basis $(e_n)_n$ s.t. $p_n = |e_n\rangle\langle e_n|$

$$L_\ell = \sum_n \lambda_{\ell,n} |e_n\rangle\langle e_n|, \qquad H = \sum_n \epsilon_n |e_n\rangle\langle e_n|$$

   $\lambda_{\ell,n} \in \mathbb{C}$, $\epsilon_n \in \mathbb{R}$

## Maximally dephasing QMSs

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$$L_\ell = \sum_n \lambda_{\ell,n} |e_n\rangle\langle e_n|, \qquad H = \sum_n \epsilon_n |e_n\rangle\langle e_n|$$

$$\mathcal{T}_t\left(|e_m\rangle\langle e_n|\right) = e^{\left(\sum_\ell\left(-\frac{|\lambda_{\ell,m}|^2}{2} - \frac{|\lambda_{\ell,n}|^2}{2} + \overline{\lambda_{\ell,m}}\lambda_{\ell,n}\right) + \mathrm{i}(\epsilon_m - \epsilon_n)\right)t}|e_m\rangle\langle e_n|$$

$$= e^{\left(-\frac{1}{2}|\lambda_{\bullet,m} - \lambda_{\bullet,n}|^2 + \mathrm{i}\Im\langle\lambda_{\bullet,m},\lambda_{\bullet,n}\rangle + \mathrm{i}(\epsilon_m - \epsilon_n)\right)t}|e_m\rangle\langle e_n|$$

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*decay*

## Maximally dephasing QMSs

$$\mathcal{L}(x) = \mathrm{i}[H,x] - \frac{1}{2}\sum_\ell \left(L_\ell^* L_\ell x - 2L_\ell^* x L_\ell + x L_\ell^* L_\ell\right)$$

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*phase*

# Another definition

J. E. Avron, M. Fraas, G. M. Graf, ... Commun. Math. Phys. (2012)

call $\mathcal{L}$ dephasing iff

$$\ker([\,H,\cdot\,]) \subseteq \ker(\mathcal{L}) \qquad \text{i.e.} \qquad \{\,H\,\}' \subseteq \ker(\mathcal{L})$$

$p$ projection.
$\mathcal{L}(p) = 0$ iff and only if $[\,L_\ell, p\,] = 0 \;\forall \ell$ and $[\,H, p\,] = 0$.

If $H$ has simple spectrum, then $\{\,H\,\}' = \{\,H\,\}''$ and both definitions are equivalent.

Can decay be ascribed to classical noise?

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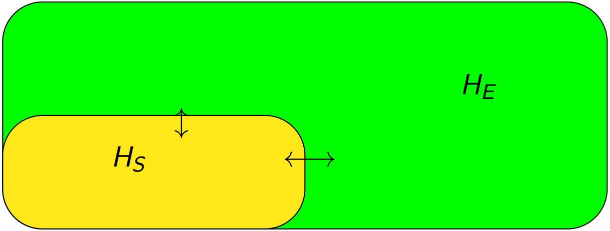
**Aim.** Characterize those QMSs that cannot be described as a unitary dilation using only classical, commutative noise processes and need the full Hudson-Parthasarathy theory.

# Dilations




$$H_{tot} = H_S \otimes \mathbb{1}_E + \mathbb{1}_S \otimes H_E + \text{interaction}, \quad \mathrm{e}^{\mathrm{i}t\,H_{tot}} \rightsquigarrow U_t$$

$$\mathcal{T}_t(x) = \mathrm{Tr}_E\left(U_t^*(x \otimes \mathbb{1}_E)U_t\right)$$

# Essentially commutative (classical) dilation

Maassen & Kümmerer, *Commun. Math. Phys.* (1987)

**Definition**

A dilation is *essentially commutative* if the algebra generated by $U_t^*(x \otimes \mathbb{1}_E)U_t$ with $x \in \mathcal{B}(\mathrm{h})$ is isomorphic to

$$\mathcal{B}(\mathrm{h}) \otimes \mathcal{C}$$

with $\mathcal{C}$ commutative.

i.e. the environment is commutative.

# QMSs with essentially commutative dilations

## Theorem

*(Kümmerer & Maassen, Comm. Math. Phys. 1987) A QMS $\mathcal{T}$ on $M_m(\mathbb{C})$ generated by $\mathcal{L}(x) = G^* x + \sum_\ell L_\ell^* x L_\ell + x\, G$ admits an essentially commutative*† *if and only if*

$$\begin{aligned}\mathcal{L}(x) &= \mathrm{i}[H,x] - \frac{1}{2}\sum_\ell \left(L_\ell^2 x - 2L_\ell x L_\ell + x L_\ell^2\right) \\ &\quad + \sum_j \kappa_j \left(V_j^* x V_j - x\right)\end{aligned}$$

*where $H = H^*$, $L_\ell = L_\ell^*$, $\kappa_j > 0$ and $V_j$ unitaries.*

# QMSs with essentially commutative dilations

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*where $H = H^*$, $L_\ell = L_\ell^*$, $\kappa_j > 0$ and $V_j$ unitaries.*

† HP dilation with Brownian and Poisson noises only

# Hudson-Parthasarathy dilations

$H_E = \Gamma(L^2(\mathbb{R}^d;\mathbb{C}))$ symmetric (Boson) Fock space on $L^2(\mathbb{R}^d;\mathbb{C})$

$$dU_t = \Bigg( \sum_{jk} (S_{jk} - \delta_{jk}\,\mathbb{1}) d\Lambda_{jk}(t)$$

$$+ \sum_j L_j dA_j(t)^\dagger - \sum_{jk} L_j^* S_{jk} dA_k(t) - \Big( \mathrm{i}H + \frac{1}{2} \sum_k L_k^* L_k \Big) dt \Bigg) U_t,$$

$H$, $L_j$ from GKSL, $(S_{jk})_{jk}$ unitary matrix of operators on h

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$H$, $L_j$ from GKSL, $(S_{jk})_{jk}$ unitary matrix of operators on h

$(A_k(t)^\dagger + A_k(t))_{t\geq 0}$, $\mathrm{i}(A_k(t)^\dagger - A_k(t))_{t\geq 0}$ Brownian motions

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$H$, $L_j$ from GKSL, $(S_{jk})_{jk}$ unitary matrix of operators on h

$(\Lambda_{kk}(t) + \xi_k A_k(t)^\dagger + \overline{\xi_k} A_k(t) + |\xi_k|^2 t)_{t\geq 0}$ Poisson processes

# Brownian/Poisson noises? Commutative environment?

$$G := -\left(\mathrm{i}H + \tfrac{1}{2}\sum_k L_k^* L_k\right)$$

$$dU_t = \left(\sum_{jk}(S_{jk} - \delta_{jk}\,\mathbb{1})d\Lambda_{jk} + \sum_j L_j dA_j^\dagger - \sum_{jk} L_j^* S_{jk} dA_k + G dt\right) U_t$$

Example.

$$L_j := \mathrm{i}\sum_n \lambda_{j,n}|e_n\rangle\langle e_n|,\ \lambda_{j,n} \in \mathbb{R} \quad H = \sum_n \epsilon_n |e_n\rangle\langle e_n|, \quad S_{jk} = \delta_{jk}\,\mathbb{1}$$

$$\begin{aligned} dU_t &= \left(\sum_j \mathrm{i}\lambda_{j,\bullet} dA_j^\dagger + \sum_j \mathrm{i}\lambda_{j,\bullet} dA_j + G dt\right) U_t \\ &= \left(\sum_j \mathrm{i}\lambda_{j,\bullet}\, d\left(A_j^\dagger + dA_j\right) + G dt\right) U_t \end{aligned}$$

# Problems

Can maximal dephasing be ascribed only to commutative noises?

i.e.

Does a maximally dephasing QMS admit a dilation with commuting Brownian and Poisson noises?

and related problems

When does it admit a dilation with commuting Brownian noises only?

When does it admit a dilation with commuting Poisson noises only?

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Keep in mind non-uniqueness of $L_\ell$, $H$ in the GKSL representation

$$\mathcal{L}(x) = \mathrm{i}[H, x] - \frac{1}{2}\sum_\ell \left(L_\ell^* L_\ell x - 2L_\ell^* x L_\ell + x L_\ell^* L_\ell\right)$$

# $\mathcal{T}$ both maximally dephasing and essentially commutative

$$\mathcal{L}(x) = \mathrm{i}[H, x] - \frac{1}{2}\sum_{\ell=1}^{d}(L_\ell^* L_\ell x - 2L_\ell^* x L_\ell + x L_\ell^* L_\ell)$$

Maximally dephasing ($((1, .., 1)^t, ..\lambda_{\ell,\bullet}, ..$ linearly independent) $\Rightarrow$

$$L_\ell = \sum_n \lambda_{\ell,n}|e_n\rangle\langle e_n|, \quad \lambda_{\ell,n} \in \mathbb{C} \qquad H = \sum_n \epsilon_n |e_n\rangle\langle e_n|$$

Also essentially commutative if, ... after rotation and translation ...
$L_\ell, H \longrightarrow L'_\ell, H'$ s.t., for all $\ell$,

**either** $\lambda'_{\ell,n} = r_{\ell,n} \in \mathbb{R}$, **or** $\lambda'_{\ell,n} = \nu_j^{1/2} \mathrm{e}^{\mathrm{i}\theta_{\ell,n}} \quad \nu_j > 0, \theta_{\ell,n} \in \mathbb{R}, \ \theta_{\ell,\bullet} \neq 0$

# $\mathcal{T}$ both maximally dephasing and essentially commutative

$$L'_\ell = \sum_k u_{\ell k} L_k + v_\ell, \qquad H' = H + \frac{1}{2\mathrm{i}}\Big(\sum_{\ell,k} \overline{v_\ell} u_{\ell k} L_k - \mathrm{h.c.}\Big) + \mathrm{ct}\,\mathbb{1}$$

**Theorem**

*Max dephasing and essentially commutative if and only if there exists a unitary* $(u_{\ell k})_{1\le \ell,k\le d} \in M_d(\mathbb{C})$, $v \in \mathbb{C}^d$ *s.t., for all* $\ell$

$$L'_\ell = \sum_n \lambda'_{\ell,n} |e_n\rangle\langle e_n|, \qquad H' = \sum_n \epsilon'_n |e_n\rangle\langle e_n|$$

- **either** $\lambda'_{\ell,n} = \sum_k u_{\ell k}\lambda_{k,n} + v_\ell = r_{\ell,n} \in \mathbb{R}$,
- **or** $\lambda'_{\ell,n} = \sum_k \cdots + v_\ell = \nu_j^{1/2} \mathrm{e}^{\mathrm{i}\,\theta_{\ell,n}}, \ \nu_\ell > 0, \ \theta_{\ell,n} \in \mathbb{R}, \ \theta_{\ell,\bullet} \neq 0$

# Dilation by Brownian motions only

$$H = \sum_n \epsilon_n |e_n\rangle\langle e_n|, \quad L_\ell = \sum_n \lambda_{\ell,n} |e_n\rangle\langle e_n|, \ \lambda_{\ell,\bullet} \in \mathbb{C}^m, \ \ell = 1, \dots, d$$

$m = 2 \ \ (\Rightarrow d = 1)$ : can always find a basis s.t. $\lambda_{1,\bullet} \in \mathbb{R}^2$

If $\lambda_{11} = \lambda_{22}$ immediate, if not

$$\phi := \mathrm{Arg}(\lambda_{1,1} - \lambda_{1,2}), \qquad v = -\mathrm{e}^{-i\phi}\lambda_{12}$$

$$\mathrm{e}^{-i\phi} \begin{bmatrix} \lambda_{1,1} \\ \lambda_{1,2} \end{bmatrix} + \begin{bmatrix} v \\ v \end{bmatrix} = \begin{bmatrix} |\lambda_{1,1} - \lambda_{22}| \\ 0 \end{bmatrix}$$

# Obstruction for $m \geq 3$

$$\mathcal{T}_t\left(|e_m\rangle\langle e_n|\right) = e^{\left(-\frac{1}{2}|\lambda_{\bullet,m}-\lambda_{\bullet,n}|^2+i\Im\langle\lambda_{\bullet,m},\lambda_{\bullet,n}\rangle+i(\epsilon_m-\epsilon_n)\right)t}|e_m\rangle\langle e_n|$$

If $\quad \mathfrak{I}_{n,n'} := \Im\left\langle\lambda_{\bullet,n'},\lambda_{\bullet,n}\right\rangle + \left(\epsilon_{n'}-\epsilon_n\right)$

satisfies $\quad \mathfrak{I}_{n,n'} = \omega_{n'} - \omega_n$

then, $\forall\ n, n', n'' \qquad \Delta_{n,n',n''} := \mathfrak{I}_{n,n'} + \mathfrak{I}_{n',n''} + \mathfrak{I}_{n'',n} = 0$

**Theorem**

*$\Delta_{n,n',n''}$ is intrinsic, i.e. independent of the GKSL representation*

Sketch. If $\lambda'_{\ell,n} = \sum_k u_{\ell k}\lambda_{k,n} + v_\ell$ then

$$\Im\left\langle\lambda'_{\bullet,n},\lambda'_{\bullet,n'}\right\rangle = \Im\left\langle\lambda_{\bullet,n},\lambda_{\bullet,n'}\right\rangle + \sum_k \Im\Big(\sum_\ell \overline{v}_\ell u_{\ell k}\Big)\left(\lambda_{k,n}-\lambda_{k,n'}\right)$$

# Obstruction $m = 3,\ d = 1$

$$\mathfrak{I}_{n,n'} := \Im \langle \lambda_{\bullet,n'}, \lambda_{\bullet,n} \rangle + (\epsilon_{n'} - \epsilon_n), \quad \Delta_{n,n',n''} := \mathfrak{I}_{n,n'} + \mathfrak{I}_{n',n''} + \mathfrak{I}_{n'',n}$$

$$\lambda_{1,\bullet} = \begin{bmatrix} 1 \\ \mathrm{i} \\ -1 \end{bmatrix}, \qquad H = 0$$

$$\Delta_{1,2,3} = 2$$

Essentially commutative dilation: NO Brownian, YES Poisson

# Obstruction $m = 3,\ d = 2$

$$\mathfrak{I}_{n,n'} := \Im \left\langle \lambda_{\bullet,n'}, \lambda_{\bullet,n} \right\rangle + (\epsilon_{n'} - \epsilon_n), \quad \Delta_{n,n',n''} := \mathfrak{I}_{n,n'} + \mathfrak{I}_{n',n''} + \mathfrak{I}_{n'',n}$$

$$\lambda_{0,\bullet} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \lambda_{1,\bullet} = \begin{bmatrix} 1 \\ \mathrm{i} \\ 1 \end{bmatrix}, \quad \lambda_{2,\bullet} = \begin{bmatrix} 1 \\ \mathrm{i} \\ -1 \end{bmatrix}, \quad H = 0$$

$$\lambda_{\bullet,1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \lambda_{\bullet,2} = \begin{bmatrix} \mathrm{i} \\ \mathrm{i} \end{bmatrix}, \quad \lambda_{\bullet,3} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\Delta_{1,2,3} = \mathfrak{I}_{1,2} + \mathfrak{I}_{2,3} + \mathfrak{I}_{3,1} = 2 \neq 0$$

no "Brownian only" ess. comm. dilation but
ess. comm. dilation " 2 Poisson" or "1 Poisson + 1 Brownian"

# Max dephasing QMSs with classical Brownian dilations

If $\qquad \mathfrak{I}_{n,n'} := \Im \left\langle \lambda_{\bullet,n'}, \lambda_{\bullet,n} \right\rangle + (\epsilon_{n'} - \epsilon_n) = \omega_{n'} - \omega_n$

then, $\forall\ n, n', n'' \qquad \Delta_{n,n',n''} := \mathfrak{I}_{n,n'} + \mathfrak{I}_{n',n''} + \mathfrak{I}_{n'',n} = 0$

**Theorem**

*A maximally dephasing QMS on $M_m(\mathbb{C})$ admits an essentially commutative dilation only with Brownian motions if and only if*

$$\Delta_{n,n',n''} = 0, \quad \forall n, n', n'' \qquad \textit{(no obstruction)}$$

# Counterexample: no Brownian, no Poisson

There exist maximally dephasing QMSs that do not admit an essentially commutative dilation.

$m = 4,\ d = 1$

$$\lambda_{1,\bullet} = [0, 1, \mathrm{i}, 2]^T, \quad \Delta_{1,2,3} = -1, \quad \Rightarrow \text{NO Brownian}$$

Poisson if and only if, after translation by $v \in \mathbb{C}$

$$|\lambda_{1,j} + v|^2 = |\lambda_{1,1} + v|^2 = |v|^2, \qquad \text{for } j = 2, 3, 4$$

$$\Re(\lambda_{1,j})^2 + 2\Re(v)\Re(\lambda_{1,j}) + \Im(\lambda_{1,j})^2 + 2\Im(v)\Im(\lambda_{1,j}) = 0, \forall j = 2, 3, 4$$

i.e. $\left(\Re(\lambda_{1,j}, \Im(\lambda_{1,j})\right)$, $j = 2, 3, 4$ and $(0, 0)$ lie on the same circle

# Open problems

**Pb. 1.** Obstructions to essentially commutative dilations with Brownian AND Poisson ?

**Pb. 2.** Obstructions to essentially commutative dilations with Poisson noises only?

**Pb. 3.** The infinite dimensional case.

# Open problems

**Pb. 1.** Obstructions to essentially commutative dilations with Brownian AND Poisson ?

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**Pb. 3.** The infinite dimensional case.

**THANK YOU!**

# References


- F. Fagnola, J. E. Gough, H. I. Nurdin, L. Viola, Mathematical Models of Markovian Dephasing, arXiv:1811.11784
- D. Burgarth, P. Facchi,... Can decay be ascribed to classical noise?, *Open Systems & Inf. Dyn.* **24**, 1750001 (2017).
- A Dhahri, F Fagnola, R Rebolledo, The decoherence-free subalgebra of a quantum Markov semigroup with unbounded generator *Inf. Dim. Anal. Quant. Probab. Relat. Top.* **13**, 413-433 (2003).
- J Deschamps, F Fagnola, E Sasso, V Umanità, Structure of uniformly continuous quantum Markov semigroups *Rev. Math. Phys.* **28** (01), 1650003 (2016).