49 Symposium on Mathematical Physics

June 17–18, 2017, Toruń, Poland

BOOK OF ABSTRACTS

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Topological classification of the set of all expansive dynamical systems on totally disconnected compact metric spaces

The concepts of various kinds of entropy have been playing important role in studying the topological structures of dynamical systems. For example, topological entropy, which can become a homeomorphism invariant enabling us to know whether any two topological dynamical systems are homeomorphic to each other or not, was introduced by Adler, Konheim and McAndrew. In the first half of this talk, we will apply the concept of Gödel number to the numbering problem of all expansive dynamical systems on totally disconnected compact metric spaces, more concretely speaking, we will show that the set of such dynamical systems is countable. Since topological entropy is not a complete topological invariant on the set of all topological dynamical systems, it is quite reasonable that there exists a pair of topological dynamical systems whose entropy values are equal to each other. Therefore, in the second half of this talk, we will present a way of discriminating more finely one topological dynamical system of such a pair from the other.

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An operational approach to Bell inequalities: application to qutrits

Bell inequalities were a revolutionary discovery to better understand the intrinsic non-local behavior of quantum mechanics. In general, they have been studied in relation to well-known entangled states. We focus instead on a not so well known operational approach, trying to grasp the fundamental cause of their violation in an analytical way. We apply this formalism to qutrits, where we find new inequalities, and also propose a method to link maximally entangled states with inequalities.

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Energy as a detector of nonlocality in many-body systems

We present a method to show that low-energy states of quantum many-body interacting systems in one spatial dimension are nonlocal. We assign a Bell inequality to the Hamiltonian of the system in a natural way and we efficiently find its classical bound using dynamic programming. The Bell inequality is such that its quantum value for a given state, and for appropriate observables, corresponds to the energy of the state. Thus, the presence of nonlocal correlations can be certified for states of low enough energy. Our work opens the possibility for the use of low-energy states of commonly studied Hamiltonians as multipartite resources for quantum information protocols that require nonlocality.

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Bounds on bipartite entanglement from marginal measures

We discuss the problem of analyzing and characterizing all possible bipartite correlations, quantum and classical, between two quantum systems in a situation of restricted access, i.e. when only the reduced density matrices (marginals) are known. Such a problem is in strong connection with the Quantum Marginal Problem (QMP), that is the problem of finding the necessary and sufficient constraints on the joint spectrum ensuring the compatibility with the given marginals. However, its solution introduces restrictions only on the purity of the joint state. Therefore, we search for restrictions on entanglement and other correlation measures stemming from the assigned marginals. In particular we address this problem for the negativity of two qubits and then expand it to higher dimensions.

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Spectral properties of quantum operations — algebraic methods

Spectral properties of quantum operations are important for investigation of the iterative dynamics of an open quantum system. A key role is played by the peripheral spectrum, i.e. the set of unimodular eigenvalues of the operator. Its structure is related to the algebra generated by Kraus operators: if Kraus operators generate the full matrix algebra, i.e. the operation is irreducible, then the peripheral spectrum is a cyclic subgroup of U(1). We present a way of generalizing this theorem to channels that are not necessary irreducible. Analysis of the structure of the algebra associated to exemplary quantum channels based on Shemesh and Amitsur-Levitzki theorem is presented.

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Applications of the Fedorov theorem to quantum tomography (poster)

It is common knowledge that the Gorini-Kossakowski-Sudarshan-Lindblad equation has a simple solution in the exponential form. For a quantum system with evolution given by such an equation one can easily determine the minimal number of distinct observables required for quantum tomography (the ability to reconstruct the initial density matrix from mean values of observables measured at different time instants). Such an approach to quantum tomography that one can measure each observable at a set of discrete time instants is called the stroboscopic tomography. The question which still has not been answered is whether one can apply the stroboscopic approach to quantum tomography to systems with evolution given by equations with time-dependent generators. There is a number of theorems that give the conditions under which one can find the solutions of such equations.The Fedorov theorem enables one to write the solution of an evolution equation with a time-dependent generator in closed form. This theorem will be applied to a specific example of quantum system evolution along with the motivation behind the research and possible further implications.

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Quantum filtering for environment in single photon state

We consider a discrete model of quantum trajectories for a system interacting with the environment defined as a series of qubits prepared in the entangled state being an analogue of a continuous mode single-photon state. The qubits interact in turn one by one with the quantum system and they are subsequently measured. We show that the stochastic evolution of the quantum system is given then by the set of coupled difference equations. We describe their interpretation and present their limit for the continuous in time observation.

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Uncertainty relations based on conditional majorization

We briefly review uncertainty relations based on entropy and majorization. Next we introduce the concept of conditional majorization. We use this concept to derive conditional uncertainty relations. We consider tripartite and bipartite scenario. We find lower bound on uncertainty that two or one party have about outcomes of two measurements performed by the remaining party.

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Quantum state independent contextuality can improve 1-way communication

State independent contextuality is a very strong property of some special sets of quantum measurements that can not be jointly performed due to quantum concommutativity. Given such set of measurements and completely arbitrary quantum state it is impossible to design the hypothetical preexisting values of all the measurements on that state in a consistent way. We shall describe the 1-way communication protocol [1] in which the above quantum phenomenon can improve an efficiency of the so called oblivious transfer protocol. The protocol is fully operational, in particular it outperforms the one using classical system with no extra assumptions on the dimensionality of the latter.

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The existence of decoherence-free subspaces and an effective criterion

The existence of decoherence-free subspaces in open quantum systems is a beneficial property for applications. However, it is usually difficult to recognise if a given system has such a subspace. In this presentation, we will establish a criterion for the existence of decoherence-free subspaces for quantum channels and some open systems. Indeed, the criterion we will study is "effective", i.e. the existence of a decoherence-free subspace can be tested by a step-by-step procedure in a finite number of steps.

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Games and monogamy in the relativistically causal correlations

Recently a new paradigm for the physically realizable correlations among observables was introduced in ref. [1]. The corresponding set of correlations includes the convex set of no-signaling correlations and extends it to include some signaling correlations when the number of parties is equal or more than three. This signaling correlations are called Relativistic Causal (RC) because they cannot be used to send information from an event to other causally preceding event in a given spacetime. Here [2] we characterize this new resource by computing its advantage on a set of probabilistic games. First we show that for Guest your neighbor's input (GYNI) game [3] RC boxes provide a maximum success probability which is higher than for the best no-signaling box under several different promises. Furthermore, we show that for all unique games [4] the monogamy relation induced by that game is violated by RC correlations. On the other hand, we show that some inequalities still preserve their monogamy for RC correlations, thus showing that there exists a strict hierarchy among monogamy

relations. We also compute the extremal boxes for the RC correlation polytope in the three-partite, two input, two output scenario [6], [7].

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Measurement uncertainty from no-signaling and non-locality (poster)

One of the formulations of Heisenberg uncertainty principle, concerning so-called measurement uncertainty, states that the measurement of one observable modifies the statistics of the other. Here, we derive such a measurement uncertainty principle from two comprehensible assumptions: the no-signaling principle, and violation of Bell inequalities (non-locality). The uncertainty is established for a pair of observables of one of two spatially separated systems that exhibit non-local correlations. To this end, we introduce a gentle form of measurement acquiring only partial information about one of the observables. We then bound disturbance of the remaining observables by the amount of information gained from the gentle measurement, minus a correction depending on the degree of non-locality. The obtained quantitative expression resembles the quantum mechanical formulations, yet it is derived without the quantum formalism and complements the known qualitative effect of disturbance implied by non-locality and no-signaling.

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Long time behavior of nonlocal quantum gates

We investigate the long time behaviour of unitary quantum gate acting on two subsystems of dimension N. We have derived a formula for the dynamics of a typical nonlocal two-qubit gate, underlining the connection with the dynamics of a free particle in a billiard. We show that the time trajectory of such a gate is equivalent to an ensemble of random gates uniformly distributed according to a suitable measure. In case of arbitrary dimensions we show that entropy averaged either over time or over an ensemble of unitary gates gives asymptotically the same results.

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Mutual entropy on quantum encoding

In this talk, we review the algebraic and operational approach to quantum entanglement as generalized encoding scheme given by Belavkin and Ohya. In their scheme c-q correspondences such as quantum encodings can be treated as d(diagonal)-entanglement leading to a special class of separable compound states. The mutual information for the d-compound states and q(quantum)-compound (or entangled) states leads to two types of entropies for given quantum states. We will discuss the difference of such two types of entropies from the viewpoint of quantum correlations. On the basis of the above, the understanding of mutual information via entanglement will be discussed.

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Quantum dynamics of two coupled spins under controllable and fluctuating magnetic fields (poster)

The quantum dynamics of two spins $\hat{\mathbf{j}}_1$ and $\hat{\mathbf{j}}_2$ (with values $j_1 \geq j_2$), subjected to external and controllable time-dependent magnetic fields and under a $\hat{\mathbf{J}}^2 = (\hat{\mathbf{j}}_1 + \hat{\mathbf{j}}_2)^2$-conserving bilinear coupling is investigated. Each eigenspace of $\hat{\mathbf{J}}^2$ is dynamically invariant and the Hamiltonian of the total system restricted to any one of such $(2j_2 + 1)$ eigenspaces possesses the SU(2) structure of the Hamiltonian of a single fictitious spin acted upon by the given controllable magnetic field. We show that such a reducibility holds regardless of the time-dependence of the externally applied field as well as of the statistical properties of the Overhauser noise, here represented as a classical fluctuating magnetic field. Exploiting such a remarkable result, the time evolution of the joint transition probabilities of the two spins $\hat{\mathbf{j}}_1$ and $\hat{\mathbf{j}}_2$ between two prefixed factorized states is examined, bringing to light peculiar dynamical properties of the system under scrutiny. In particular, when the noise-induced nonunitary dynamics of the two coupled spins is properly taken into account, the paper reports explicit analytical expressions for the joint Landau-Zener transition probabilities. The possibility of taking advantage of these exact results to envision a feedback-test on the reliability of the modelling adopted for unavoidable environmental effects in a given set-up is finally briefly discussed.

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Quaestiones disputatae de observatione

Understanding quantum measurements has been one of the central problems of quantum theory since its beginning. It is of central interest in quantum mechanics as it provides the link between the quantum world and the world of everyday experience. One of the features of the latter is its robust, objective character, contrasting the delicate nature of quantum systems. In our work, we use the quantum state discrimination in a central system model to show how its evolution leads to the broadcasting of information, and how orthogonalization and decoherence factors allow to monitor the distance of the state in question to the one perfectly broadcasting information, in any moment of time. We analyze in a model-independent way the celebrated von Neumann measurement process, using recent techniques of information flow, studied in open quantum systems. We show that objective results typically appear in quantum measurements, provided we macroscopically coarse-grain the measuring apparatus and wait long enough. Our results are manifestly universal and are a generic property of von Neumann measurements.

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Structure of irreducibly covariant quantum channels for finite groups

We obtain an explicit characterization of linear maps, in particular, quantum channels, which are covariant with respect to an irreducible representation U of a finite group G, whenever $U \otimes U_c$ is simply reducible (with U_c being the contragradient representation). Using the theory of group representations, we obtain the spectral decomposition of any such linear map. The eigenvalues and orthogonal projections arising in this decomposition are expressed entirely in terms of representation characteristics of the group G. This in turn yields necessary and sufficient conditions on the eigenvalues of any such linear map for it to be a quantum channel. We also obtain a wide class of quantum channels which are irreducibly covariant by construction. For two-dimensional irreducible representations of the symmetric group $S(3)$, and the quaternion group Q, we also characterize quantum channels which are both irreducibly covariant and entanglement breaking.

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Recent progress in reducible QED

Reducible QED was investigated by Marek Czachor and coworkers. Recently, the theory was simplified and formulated in a rigorous manner. Two topics are emphasized: Possibly, the Coulomb forces can be treated as emergent forces; The S-matrix can be formulated in such a way that all Feynman diagrams are finite without the need of renormalization.

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Spontaneous moralization problem in quantum stochastic walk (poster)

The spontaneous moralization of a quantum stochastic walk is the effect of propagation on the moral graph instead of the original graph. We argue that this effect is undesired, especially in the case of directed graphs. We also propose a procedure to correct for this effect. We analyse the corrected model using the scaling exponent. We show that the fast propagation of walk results from the model itself and not from the existence of additional amplitude transitions. We demonstrate that the obtained result can be used for arbitrary directed graphs.

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Simulation of quantum resonance in a quantum register (poster)

In this paper, we examine whether a quantum computer can efficiently simulate resonant interaction between a pair of two-level quantum systems. We present an algorithm simulating time evolution of such system, implemented on standard two-input gates. We study the influence of accuracy of gates on the quality of the obtained results.

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Asymptotic properties of quantum states and channels

Properties of random mixed states of dimension N distributed uniformly with respect to the Hilbert-Schmidt measure are investigated. We show that for large N, due to the concentration of measure, the trace distance between two random states tends to a fixed number $\tilde{D} = 1/4 + 1/\pi$, which yields the Helstrom bound on their distinguishability. To arrive at this result we apply free random calculus and derive the symmetrized Marchenko–Pastur distribution. For quantum channels, we show that their level density is also described by the Marchenko-Pastur distribution. This allows us to deduce some properties of the diamond norm of large dimensional quantum channels and provide a new upper bound on the diamond norm. This results shed new light on the sets of quantum states and channels of large dimension N.

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Asymptotic properties of entanglement polytopes for large number of qubits and RMT

Entanglement polytopes have been recently proposed as the way of witnessing the SLOCC multipartite entanglement classes using single particle information. I will present first asymptotic results concerning feasibility of this approach for large number of qubits. In particular, I will discuss the witnessing power of entanglement polytopes for systems of large number of qubits. As finding entanglement polytopes, even for five qubits, is in fact intractable I will use the connection between the polytopes and the critical points of the linear entropy to provide at least brief characterization. This connection leads to a random matrix model involving the Bernoulli ensemble. I will discuss preliminary implications of this model for the witnessing power of entanglement polytopes. This is a joint work with Tomasz Maciążek (CFT PAN).

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A new formulation of space-time operations in quantum theory

In this talk, I shall address the question of the formulation of time in quantum theory. Here the essential problem is that, as pointed out by Pauli, quantum mechanics cannot support a representation of time as an observable which, together with energy, satisfies a canonical commutation relation. This therefore marks a crucial difference between position and time in quantum theory. In this talk I shall present a relativistically covariant formulation of space-time operations on local observables, NOT on the underlying Hilbert space, within the framework of the Borchers-Uhlmann algebraic picture of Wightman's axiomatic field theory. In this formulation, spatial and temporal operations are treated on the same footing and lead to time-energy and position-momentum uncertainty relations.

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Quantum semi-Markov evolution for the generalized Pauli channels

Quantum semi-Markov evolution is a quantum analogue of the semi-Markov stochastic classical evolution. This evolution is described by the memory kernel master equation and fully determined by the quantum semi-Markov map. In this framework, I analyze the generalized Pauli channels which solve the memory kernel master equation and provide several examples from within the quantum semi-Markov evolution and beyond.

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Entropy, special geometric configurations in the quantum state space and combinatorial designs

An intrinsic property of quantum measurement is the indeterminacy of its outcomes. Mathematically, each such measurement, described by a POVM, defines a family of probability distributions indexed by the quantum states. One can quantify the randomness of these distributions by calculating their entropy. The states for which the entropy is minimal (respectively, maximal) can be interpreted as the ones with minimal (resp. maximal) uncertainty with respect to given quantum measurement. While looking for such optimal configurations of quantum states is not easy in general, since the solutions can even not be analytical, it turns out that imposing some additional properties regarding symmetry of the POVM in question may enforce the optimal configurations to follow the pattern. What is more, the optimal configurations generate some special combinatorial designs if the effects (the elements of the POVM) constitute a 2-homogeneous space. In order to illustrate these ideas, we will focus mainly on the SIC-POVMs in dimension 3 and Hoggar's SIC-POVM in dimension 8.

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On transmitted complexity for quantum dynamical systems

In order to treat several complicated systems, it is important to study the dynamics of state change and the complexity of states of systems. Information Dynamics introduced by Ohya is a new concept synthesizing the research schemes of several complicated systems. In ID, there are two types of complexities, (1) a complexity of state describing the system itself and (2) a transmitted complexity between two systems. Classical and quantum entropies are the example of such complexities. Based on the relative entropy of Umegaki and the compound state, the quantum mutual entropy was defined by Ohya in 1983, and it was extended to general quantum systems by using the relative entropy of Araki and Uhlmann. One can discuss coding theorems by means of the mean entropy and the mean mutual entropy defined by the dynamical entropy. In this talk, we will discuss the transmitted complexity of quantum dynamical systems.

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On lattices of ground spaces

This talk addresses the topology and ordering of the lattice L of ground space projections of a vector space H of finite-dimensional energy operators. An extension of quantum information geometry from states of full support to states of non-maximal rank showed that L may not be closed in the Euclidean topology [1]. One of the consequences is that a maximum-entropy inference map, constrained on the fibers of the projection π onto H, may be discontinuous. We discuss examples of k-local Hamiltonians

[2,3], used in many-body physics, and of triples of qutrit observables [4]. In both examples, the geometry of the image $\pi(M)$ of the set M of mixed states provides a sufficient condition of discontinuity of the inference map [5]. Finally we point out some new ordering properties of L and explain them with (commutative) two-local three-bit Hamiltonians. Namely, L is coatomistic and a variation principle characterizes L within the projection lattice of the algebra.

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Subadditivity of violation of geometrical Bell inequalities for qudits

Bell inequalities (BIs) by their violation distinguish classically allowed statistics from strictly quantum mechanical ones. Geometrical BIs are based on the interpretation of the correlation function as a vector in space of functions and comparing its norm with an overlap with local realistic function. For qubits they provide the strongest violation among all inequalities (though, there is a protocol, in which even stronger violation is observed). However, it was a challenge to find similar inequalities for collections of qudits, i.e., higher dimensional quantum systems, due to lack of convenient formalism. We introduce a new formalism, in which observables have vector eigenvalues. This allows us to formulate a series of geometrical BIs, the violation of which grows exponentially with the number of particles and (approximately) polynomially with their dimensionality. It is then possible to compare two situations, one in which observers perform a Bell experiment on a large system, with the other, when they divide their subsystems into smaller parts.

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Theormodynamics of random walk (poster)

We analyze the random walk process as a thermodynamic phenomenon, and look for its thermodynamic description in a discrete manner, without passage to infinitesimal limit of the calculus. Thus, random walk is described by difference equations, instead of differential ones applied in treating the diffusion. Hence, some artifacts of differential treatment, such as infinite speed of signals, are avoided. In praticular, the first and second laws of thermodynamic are derived and the entropy growth of a random walk process is studied.

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Quantum entanglement, quantum orthogonal arrays and quantum Latin squares

Bell states are known to be maximally entangled among all two-qubit quantum states. Furthermore, GHZ states maximize 3-tangle and some other measures of entanglement for thee-qubit systems, as their one-party reductions are maximally mixed.

What are the most entangled states for quantum systems consisting of N systems with d levels each? On one hand the answer may depend on the entanglement measure used. On the other hand, already for four-qubit system there are no states, such that any of its two-party reduction, with respect to any possible splitting, is maximally mixed. We show that such states exist for four qutrits and some other higher dimensional systems. Construction of these stronly entnagled multipartite quantum states is shown to be linked with qeneralized combinatorial designes: quantum orthogonal arrays and quantum Latin squares.