

## Violation of Bell's inequalities in interference experiments

Włodzisław Duch

Max-Planck-Institut für Physik und Astrophysik, Institut für Astrophysik  
Karl-Schwarzschild-Str. 1, D-8046 Garching b. München, F.R.G.

Permanent address: Instytut Fizyki, Uniwersytet Mikołaja Kopernika,  
ul. Grudziądzka 5, 87-100 Toruń, Poland

### Abstract

An interference experiment is described to which the Bell inequality applies. The idea is very close to the original Einstein-Podolski-Rosen Gedankenexperiment and seems to allow faster-than-light communication.

The inequalities of Bell and the experiments aimed at their verification created a great interest in the foundations of physics, expressed in the numerous scientific and popular articles, books, as well as number of conferences on the subject. Despite clear experimental evidence<sup>1)</sup> in favour of quantum mechanics and against local and realistic theories still a significant percent of all physicists are not ready to accept the facts or rather common interpretation of the facts<sup>2)</sup>. It has been already pointed out by Franson<sup>3)</sup> that the original argument of Einstein, Podolski and Rosen (EPR)<sup>4)</sup> can be applied to a single particle and thus the existence of the EPR correlations between measurements on pairs of particles is not more mysterious than simple interference phenomena, provided that they persist at sufficiently large separation of the beam's paths and at sufficiently low intensities. As noted by Feynman<sup>5)</sup> the interference phenomena are mysterious and nobody really understands them (in the sense of not knowing the deeper workings of Nature), but their existence is an undisputable fact. It is the aim of this note to show that the Bell inequality<sup>6)</sup> may be formulated also for interference experiments in which pairs of particles instead of single particles are used. Moreover, such experiments are much closer to the original EPR idea than the spin or polarization measurement experiments are, showing the nonlocal character of interference in a striking way.

Let us start from a description of an experiment (Fig. 1) in which one measures the correlation between particle counts at two distant places. A source at the center sends pairs of particles moving in opposite directions, each particle reflected by a mirror and brought to interference with itself on a screen, with a particle counter placed in the middle of the screen. In the lower beams at left and right a phase shifting

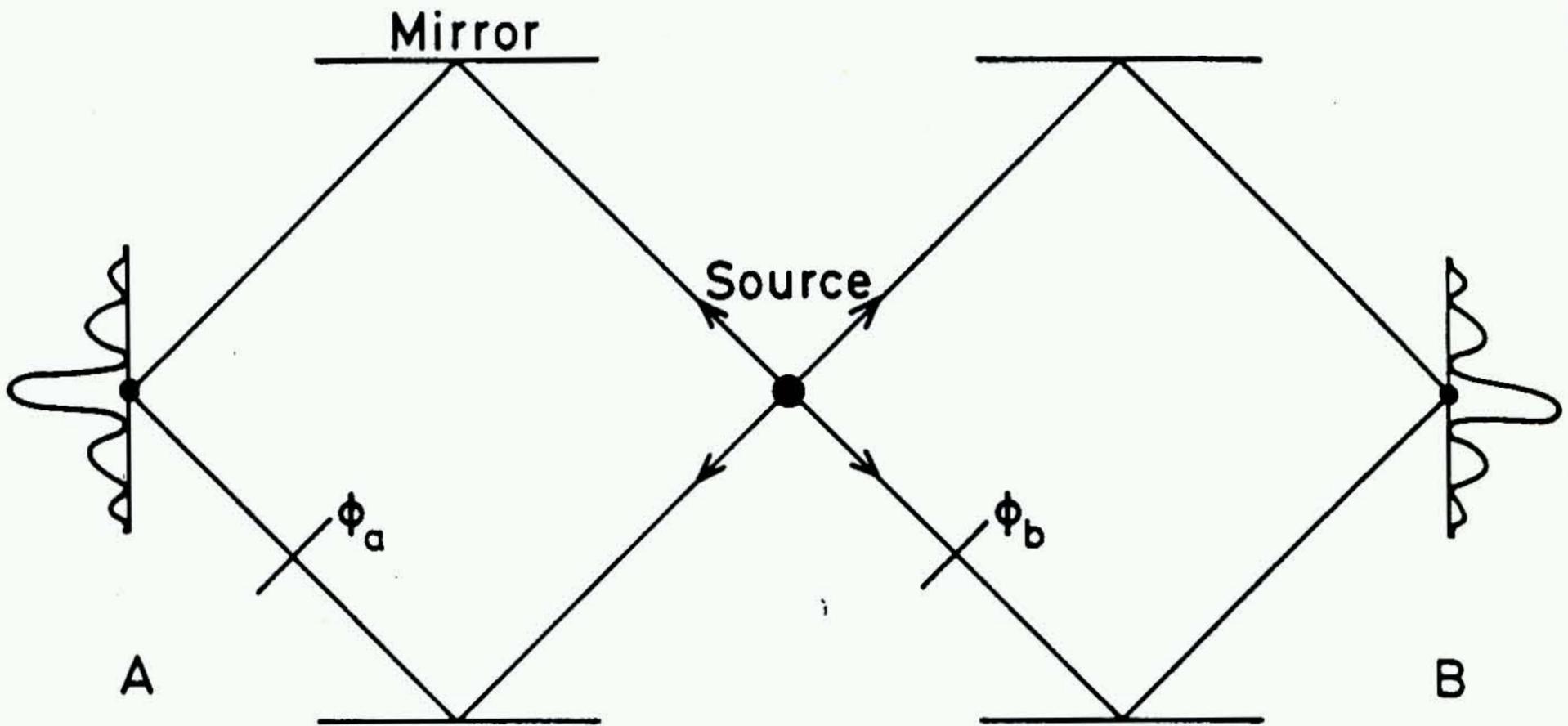


Fig 1. *Experimental setup. In the second experiment the mirrors on the right side are replaced by the detectors thus allowing to determine the particle's paths.*

devices are placed and, depending on their setting, the average number of counts the counters register is changed. The presence of particles moving to the right should not in any way influence the interference at the left screen and vice versa. All that is required of the source of the pairs of particles is that it always sends two particles in the directions "up-left and down-right" or "down-left and up-right" but otherwise no special correlation between the particles is assumed. However, the correlation between  $A$  and  $B$  counters for certain choices of the phase shifts violates a Bell-like inequality.

Let us designate by  $A(\phi_a)$  the probability of detection of a particle at the site  $A$  when the phase shift is set to  $\phi_a$ . It should be equal to:

$$A(\phi_a) = \frac{1}{4} |e^{ik_z z} (1 + e^{i\phi_a})|^2 = \frac{1}{2} (1 + \cos\phi_a)$$

where  $z$  is the coordinate of the  $A$  point along the  $AB$  line with the particle's source at  $z = 0$ , i.e. in the center. The probability  $AB(\phi_a, \phi_b)$  of a joint detection of a particle at site  $A$  and at site  $B$  is not  $A(\phi_a)B(\phi_b)$  but rather

$$AB(\phi_a, \phi_b) = \frac{1}{4} |e^{2ik_z z} (e^{i\phi_a} + e^{i\phi_b})|^2 = \frac{1}{2} (1 + \cos(\phi_a - \phi_b))$$

This function gives a non-trivial correlation while the product  $A(\phi_a)B(\phi_b)$  gives only trivial correlations in the sense that we may derive Bell's inequality that is violated by

$AB(\phi_a, \phi_b)$  but not by the  $A(\phi_a)B(\phi_b)$  product. In the derivation of Bell's inequality one assumes that since the two sites are separated we are allowed to write

$$AB(\phi_a, \phi_b) = \int d\lambda \rho(\lambda) A(\phi_a; \lambda) B(\phi_b; \lambda)$$

with  $1 \geq \rho(\lambda) \geq 0$  describing distribution of the internal parameters  $\lambda$  of the particles and  $A(\phi_a; \lambda) = 0, 1$  describing a single detection event. The assumptions used to write the equation above amount to realism ( $\lambda$  parameters) and locality (taking product under the integral above). Since

$$A(\phi_a; \lambda)[B(\phi_b; \lambda) - B(\phi'_b; \lambda)] + A(\phi'_a; \lambda)[B(\phi_b; \lambda) + B(\phi'_b; \lambda)] \leq 2$$

the same must be true for the correlation function, giving the celebrated inequality of Bell

$$AB(\phi_a, \phi_b) - AB(\phi_a, \phi'_b) + AB(\phi'_a, \phi_b) + AB(\phi'_a, \phi'_b) \leq 2$$

The true correlation function  $\frac{1}{2}(1 + \cos(\phi_a - \phi_b))$  violates this inequality, for example taking  $\phi_a = 0^\circ$ ,  $\phi'_a = 90^\circ$ ,  $\phi_b = 45^\circ$ , and  $\phi'_b = 135^\circ$  the left side is  $2\sqrt{2}$ .

The reason for the violation is obvious: the events at the two sites are not independent so the correlation coefficient  $AB(\phi_a, \phi_b)$  is not equal to the product  $A(\phi_a)B(\phi_b)$ . One can see it even better performing a second experiment: instead of allowing the beams at the right side (at  $B$ ) to interfere we will remove the  $\phi_b$  shifting device and put the particle detectors in place of mirrors. Now, since the detection of a particle at the "up-right" detector means that the corresponding particle arrived at  $A$  going through the lower route and detection by the "down-right" detector (or simply absence of the particle in the "up-right" detector) that it was going through the upper route the interference pattern has to disappear because this time we know which way the particle takes and so we have to sum the probabilities instead of the amplitudes. Thus, exchanging the mirrors with the detectors we create or destroy the interference pattern at  $A$  without in any way interacting with the particles moving to the left!

This experiment is much closer to the original EPR Gedankenexperiment<sup>4)</sup> because here the positions and the momenta are used and the detectors measuring positions determine the corresponding momenta. An interesting question arises: recent experiments<sup>1)</sup> have shown that EPR correlations are instantaneous in the sense that changing the experimental setup changes the appropriate correlation coefficients instantaneously. Can one send signals faster than light manipulating with one of the mirrors at  $B$  and observing the dependence of the interference pattern at  $A$ ? If such experiment worked one could also influence past events: removing the detectors and the mirrors very far the existence of an interference pattern at the left side should

depend on what will happen to the particles at the right side, whether we decide to use the mirrors or the detectors, long after the pattern was formed!

A faster-than-light communication device based on EPR correlations was proposed few years ago by Herbert<sup>7)</sup>. The reason why this particular device is not going to work was not trivial: Herbert's setup demanded "cloning of the single photons" and it was shown<sup>8)</sup> that quantum mechanics does not allow us to do that. However, the setup proposed here is much simpler and the reason why the experiment described above should not work seems also to be nontrivial.

### References

- [1] Aspect, A., Dalibard, J., Roger, G., Phys.Rev.Lett.49,1804(1982)
- [2] Duch, W., Aerts, D., Phys. Today. 6,13(1986)
- [3] Franson, J.D., Phys. Rev. D 26,787(1982)
- [4] Einstein, A., Podolsky, B., Rosen, N., Phys.Rev. 47,777(1935)
- [5] Feynman, R, Quantum Electrodynamics – A strange theory of light and matter. Princeton University Press (1985)
- [6] Bell, J.S., Physics 1,195(1964)
- [7] Herbert, N., Found. Phys. 12,1171(1982)
- [8] Wootters, W.K., Zurek, W.H., Nature 299,802(1982); *ibid.* 304,188(1983)  
Mandel, L., Nature 304,188(1983)