Attractors in Neurodynamical Systems

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Abstract Analysis of the number, type and properties of attractors in complex neurodynamical systems is quite difficult. Fuzzy Symbolic Dynamics (FSD) creates visualization of trajectories that are easier to interpret than recurrence plots, showing basins of attractors. The variance of the trajectory within the attraction basin plotted against the variance of the synaptic noise provides some information about sizes and shapes of their basins. Semantic layer of dyslexia model implemented in the Emergent neural simulator is analyzed.

Keywords Neurodynamics · Attractor networks · Symbolic dynamics · Multidimensional time series visualization

1 Introduction

A general method for analysis of dynamical systems is based on recurrence plots [1]. A trajectory $\mathbf{x}_i = \mathbf{x}(t_i)$ returns to almost the same area (within ϵ distance) at some other point in time, creating non-zero elements (or black dots in the plot) of the recurrence matrix $\mathbf{R}_{ij}(\epsilon) = \mathbf{\Theta}(\epsilon - ||\mathbf{x}_i - \mathbf{x}_j||)$. With some experience such plots allow for identification of many interesting behaviors of the system. However, these plots depend strongly on the choice of ϵ and the choice of the time step $\Delta t = t_{i+1} - t_i$. They may become irregular, the patterns in the plot may wash out, show spurious correlations or tangential motions [1], especially if steps along the trajectory $\mathbf{x}(t_i)$ are not constant for consecutive t_i.

Complex dynamics is frequently modeled using attractor networks, but precise characterization of attractor basins and possible transitions between them is rarely attempted. Another point of view on global analysis is quite fruitful. Recurrence plots use as the reference previous points $\mathbf{x}_i$ on the trajectory. This is analogous to

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the use of the single-nearest neighbor method in classification [2]. Description of the density of the trajectories may be simplified and improved by selecting lower number of reference points and associating basis functions with these points. The recurrence rule $\mathbf{R}_i(\mathbf{x};\epsilon) = \mathbf{\Theta}(\epsilon - ||\mathbf{x}_i - \mathbf{x}||)$ may then be reinterpreted as the return of the trajectory $\mathbf{x}(t)$ into the neighborhood of the vector $\mathbf{x}_i$ associated with some metric function $||\cdot||$ and some parameters ϵ. In particular if L_∞ (Chebyshev) metric is used the neighborhoods are hyperboxes and positive $\mathbf{R}_i(\mathbf{x};\epsilon)$ entry is marked by a symbol s_i, changing the vector trajectory into a sequence of symbols. This approach, known as the symbolic dynamics [3] (SD), has been used to simplify description of relatively simple dynamical systems, providing very rough approximation to the description of the trajectory. However, such discretization may be too rough for most applications. The Fuzzy Symbolic Dynamics (FSD) introduced by us recently [4] replaces hyperboxes by membership functions that estimate how far is the trajectory from the reference points. This is in fact a projection of the trajectory on a set of k basis functions $G(\mathbf{x};\mu_i,\sigma_i), i = 1..k$, localized around μ_i with some parameters σ_i, strategically placed in important points of the phase space.

FSD provides a non-linear reduction of dimensionality suitable for visualization of trajectories, showing many important properties of neurodynamics, such as the size and the relative position of attractors. To analyze more precisely properties of individual attractors response of the system to different type of noise with increasing variance is studied. A model of word recognition implemented in the Emergent simulator [5] is used for illustrative purpose. Conclusions about the relations between properties of attractors for different words are drawn.

2 Visualization of Attractors

For FSD visualization two Gaussian membership functions $G(\mathbf{x};\mu_i,\sigma_i), i = 1,2$ are placed in different parts of the phase space [4]. These functions are activated each time system trajectories pass near their centers, providing a sequence of fuzzy symbols along the trajectory.

As an example of what one can learn from such mapping visualization of a large semantic layer in the model of dyslexia implemented in the Emergent simulator is presented (see [5], chapter 10). This model has full bidirectional connectivity between orthography (6×8 units), phonology (14×14), and semantic layers (10×14), with recurrent self-connections within each of these layers, and additional hidden layers of neurons between each of these 3 layers. The model has been trained on 40 words, half of them concrete and half abstract. Semantics has been captured by using 67 features for concrete words (with average of 18 active features per word) and 31 for the abstract ones (about 5 active features on average), with half of the semantic layer devoted to abstract and half to concrete features. Correlation dendrogram between all 40 words is presented in [5], Chapter 10, Fig. 10.7. In the model all lesion parameters were off because the goal was to show attractors which have evolved in the learning process for different words (effects of lesions are reflected in visualizations, but are not shown here).

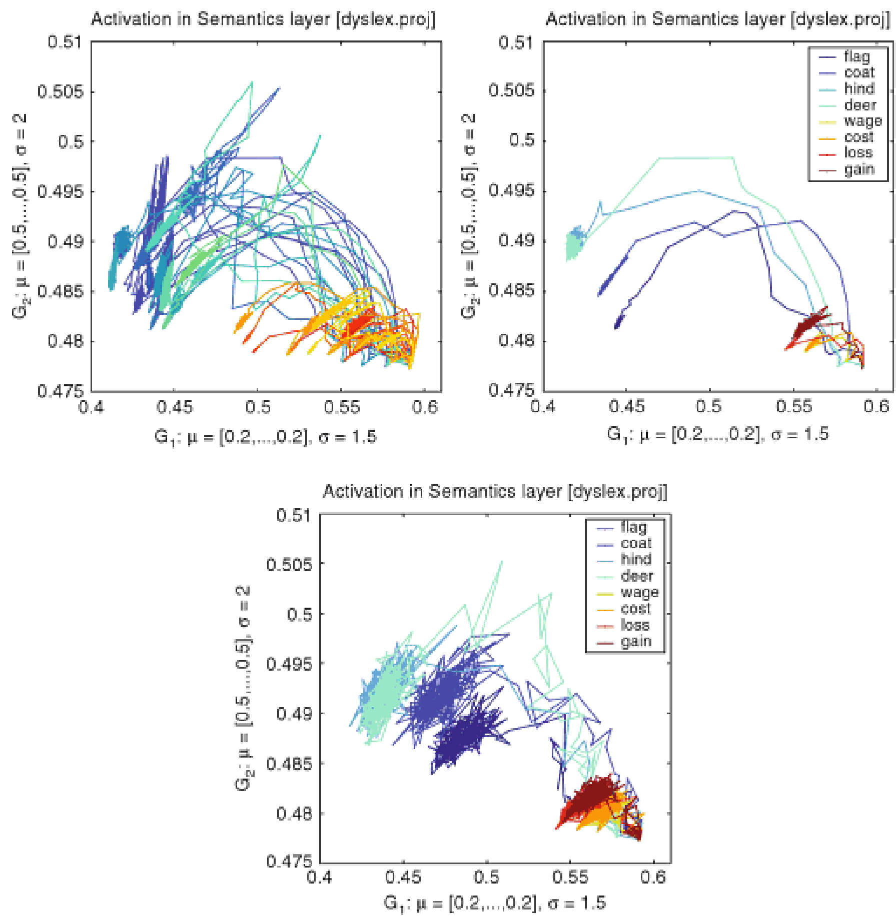

Fig. 1 FSD mapping showing trajectories and attractors of all 40 words used in the model (*left*); abstract words are plotted in the *right lower*, concrete words are in the *left-middle area*. Selected 8 words with the variance of the noise increased from 0.02 to 0.09

Two Gaussian functions with rather large dispersions have been placed in the 140 dimensional space of semantic layer activations. Parameters of these functions are given in Fig. 1, where trajectories for all 40 words are shown. The division between abstract and concrete words leads to quite different representations. Abstract words reach much faster their final configurations due to the significantly lower number of active units, while the meaning of concrete words is established only after a long trajectory, with more complex configurations.

Plots in Fig. 1 also show different shapes of attractor basins. The dyslexia model with default parameters (e.g. without any noise) is deterministic and converges to the point attractor, because the model has already been trained and starts from zero activity except for the input layer. To show attraction basins for a given word

Gaussian synaptic noise with zero mean and progressively growing variance has been introduced. In Fig. 1 (center, right) trajectories and attractor basins of 4 pairs of correlated words are displayed, with concrete words in the first two pairs (flag, coat, hind, deer) and abstract words (wage, coat, loss, gain) as the last two pairs. The variance of the noise has been increased from 0.02 to 0.09.

From such plots (due to the lack of space only a few are shown here) a number of interesting observations is made:

- convergence to sparse, simple representations (abstract words) is much faster than to more complex ones;
- some attractors may be difficult to reach – this is indicated by chaotic trajectories that lead to them;
- semantic representations from pairs of similar words are usually quite close to each other, with (coat, flag) being the most distinct pair;
- shapes of attraction basins differ for different words and noise levels – this is seen when trajectories explore larger areas with increasing noise levels;
- noise with a small variance explores only the bottom of attraction basin, for stronger noise the system may still show distinct attractors although patterns of semantic activity are severely distorted.
- for very strong synaptic noise (> 0.14) plots of all basins of attractors shrink.

The last observation has been quite surprising, but this is the result of quite complex dynamics of the model, with strong random saturation of the units and inhibition of most of them. Plot shown in Fig. 2 of the variance of trajectory near the attractor basin center as a function of increasing noise variance can serve as a measure of the basin depth. Initially there is little change in the trajectory variance,

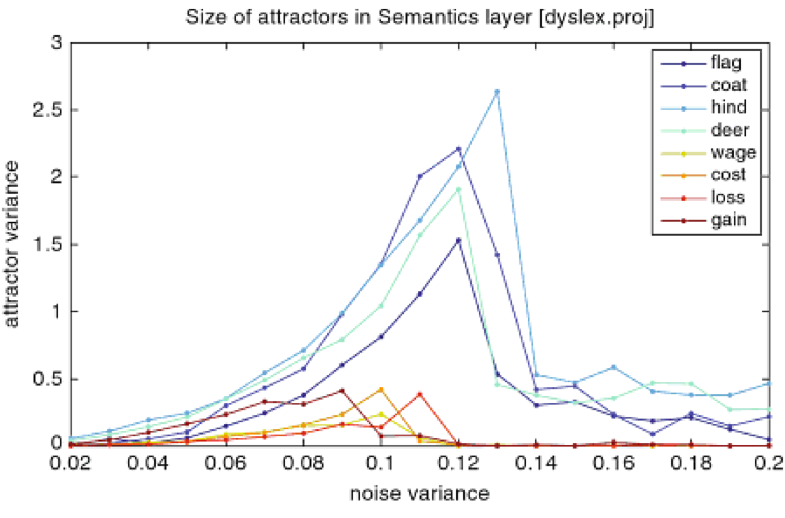

Fig. 2 Variance of the trajectory in the basin center as a function of increasing noise variance

showing that these basins are quite deep near the center, but for larger noise level there is sharp increase, as is also seen in Fig. 2, showing that the upper parts of the basin are much larger and additional energy helps to explore them. A sudden decrease of the trajectory variance is observed in all cases when synaptic noise is very strong.

3 Discussion

Cognitive science has reached the point where moving from finite automata models of behavior to continuous cognitive dynamics becomes essential [6]. Recurrent plots and symbolic dynamics may not be sufficient to show various important features of neurodynamics in complex systems. In this paper two new techniques have been explored: first, a fuzzy version of symbolic dynamics, that may also be treated as smoothed version of recurrent plot technique with large neighborhood, and second, analysis of variance of trajectories around point attractor as a function of noise. FSD depends on the choice of membership functions, and thus has more parameters than recurrent plots or the discrete symbolic dynamics [3] that may be obtained from the fuzzy version by thresholding the activations of membership functions. However, FSD retains more information about dynamics and creates plots that are easier to interpret than recurrence plots, less dependent on the sampling step.

For quasi-periodic attractors variance in the direction perpendicular to the trajectory may be estimated. Perhaps for the first time these techniques may show how attractor basins created by various learning procedures depend on the similarity of stimuli, their context, but also on the properties of neurons. For example, reducing the leaky ion channel currents creates basins attracting trajectories strongly and preventing attention shifts, a phenomenon that is of fundamental importance in autism.

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